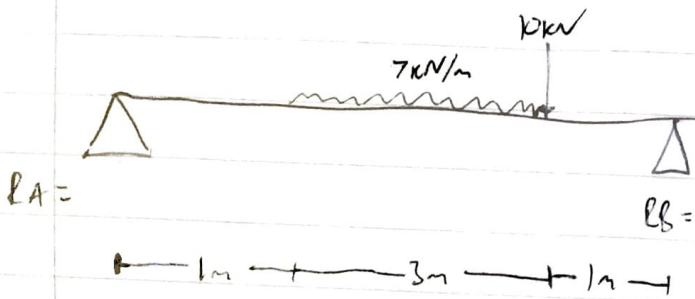


① Deflection at midpoint.



$$(7 \text{ kN/m} \times 3 \text{ m} \times 2.5 \text{ m}) + (10 \text{ kN} \times 2.5 \text{ m}) = R_B \times 5 \text{ m}$$

$$52.5 \text{ kNm} + 25 \text{ kNm} = R_B \times 5 \text{ m}$$

$$77.5 \text{ kNm} = R_B \times 5 \text{ m}$$

$$R_B = \frac{77.5 \text{ kNm}}{5 \text{ m}} = 15.5 \text{ kN}$$

$$\therefore R_A = 10 \text{ kN} + 21 \text{ kN} - 15.5 \text{ kN} = 15.5 \text{ kN}$$

$$EI \frac{d^2 y}{dx^2} = 15.5x - 7(x-1)\frac{(x-1)}{2} - 10(x-4) + 7(x-4)\frac{(x-4)}{2}$$

$$= 15.5x - \frac{7(x-1)^2}{2} - 10(x-4) + \frac{7(x-4)^2}{2}$$

$$EI \frac{dy}{dx} = \frac{15.5x^2}{2} - \frac{7(x-1)^3}{2 \times 3} - \frac{10(x-4)^2}{2} + \frac{7(x-4)^3}{2 \times 3} + A$$

$$EI y = \frac{15.5x^3}{6} - \frac{7(x-1)^4}{24} - \frac{10(x-4)^3}{6} + \frac{7(x-4)^4}{24} + Ax + B$$

Applying boundary conditions when $x=0$ $y=0$

$$0 = 0 + 0 + B \quad \boxed{B=0}$$

$$\text{When } x=5 \quad y=0 \quad 0 = \frac{15.5(5)^3}{6} - \frac{7(5-1)^4}{24} - \frac{10(5-4)^3}{6} + \frac{7(5-4)^4}{24} + 5A + B$$

$$5A = -184.375 \quad A = -36.875$$

$$EI y = \frac{15.5(x)^3}{6} - \frac{7(x-1)^4}{24} - \frac{10(x-4)^3}{6} + \frac{7(x-4)^4}{24} - 36.875(x)$$

Midpoint deflection: when $x = 2.5$

$$EI_y = \frac{12.5 (2.5)^3}{6} - \frac{7 (1.5)^4}{24} - 36.875 (2.5)$$

$$\begin{aligned} EI_y &= -61.112 & y &= -61.112 \times 10^3 / (200 \times 10^9) \times (3 \times 10^{-5}) \\ & & &= -0.010185 \text{ m} \\ & & &= -10.185 \text{ mm} \end{aligned}$$

③

Question 2a

According to Case, et al., (1999) Euler's method is only used for beams that have a slenderness ratio of larger than 120 where the failure mode will be pure buckling, compared with the Rankine-Gordon method which is used for beams with slenderness ratios of less than 120 down to approximately a 30 which will result in the failure mode being a combination of buckling and crushing. This is since columns with a slenderness ratio of less than 30 tend to fail from a compressive force overcoming the strength of the material causing it to fail, rather than the column buckling prior to the compressive stress overwhelming the yield strength of the material. Since the two methods are more accurate for specific types of failures and struts with specific boundaries for slenderness ratios, it has become good practice to use Euler's method for long beams and the Rankine-Gordon for shorter beams as when this is done, values closer to the true practical values are found due to different, specific properties taken into account when using the formulae.

$$\textcircled{2} \textcircled{6}) \text{ OD} = 100 \text{ mm} \quad \text{ID} = 93 \text{ mm} \quad l = 3.5 \text{ m} \quad E = 200 \text{ GN/mm}^2 \quad n = 1$$

$$F_e = \frac{\pi^2 E I}{l^2}$$

$$\text{where } I = \frac{\pi D^4}{64} - \frac{\pi d^4}{64}$$

$$= \frac{\pi \times 100 \times 10^{-3}{}^4}{64} - \frac{\pi \times 93 \times 10^{-3}{}^4}{64}$$

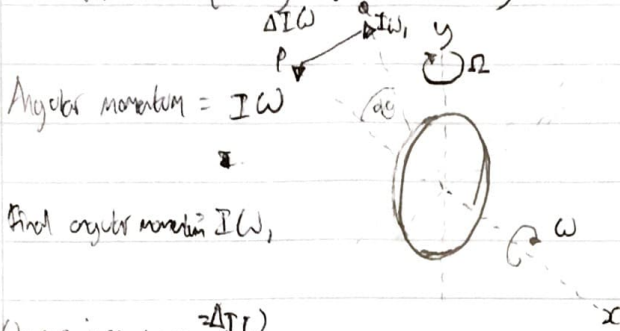
$$= 1.237 \times 10^{-6}$$

$$F_e = \frac{\pi^2 \times 200 \text{ GN/m}^2 \times 1.237 \times 10^{-6}}{3.5^2} = 199.326 \text{ kN}$$

(3) a)

Gyroscopic Motion

Consider a spinning disc rotating at a velocity (ω) about the x axis.



Angular momentum = $I\omega$

Final angular momentum $I(\omega + \Delta\omega)$

Change in angular momentum = $\Delta(I\omega)$

$I = mk^2$
mass radius of gyration

Also it is moving around the y axis at an angular velocity Ω

$$\Delta(I\omega) \cdot d\theta = p \cdot q$$

$$\text{Rate of change} = I\omega \frac{d\theta}{dt}$$

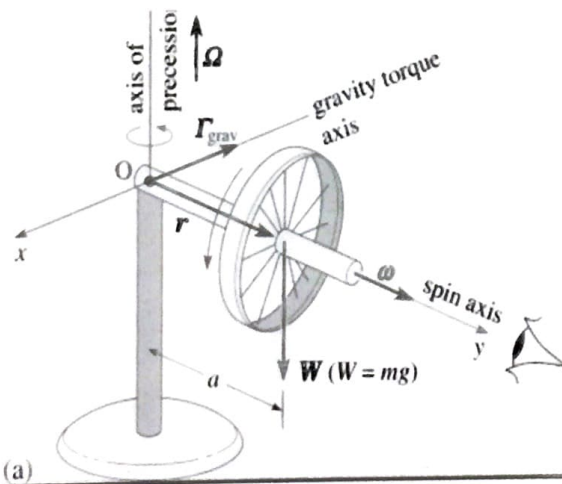
$$\text{where } \frac{d\theta}{dt} = \Omega$$

In order to change the direction a torque is needed.

$$\tau = I \cdot \omega \cdot \Omega = N \cdot m = \text{Gyroscopic torque} \quad I = \text{moment of inertia}$$

$(N \cdot m) (kg \cdot m^2) (rad/s) (rad/s)$

Question 3a



Rotating the wheel seen above, will result in angular momentum ω axially. There will also be a resultant torque which means that the gyroscope will rotate counter-clockwise on the axis of precession about axis O which is the pivot point. and the radius of this rotation is the distance from the centre of the axis of precession to the centre of the wheel which is labelled as a. This causes the wheel to rotate counter clockwise about axis O since the resultant torque is acting at a tangent to the angular momentum of the wheel. The direction that the wheel is rotating in on the axis of precession will be a result of the direction that it is spinning in on the spin axis. (Akshay, 2017)

5b) moment of inertia = 1.4 kg m^2 CW 1600 RPM

What is the couple? What is its effect when 200 km/h vel turn or 200m

$$T = I \cdot \omega \cdot \Omega = \text{Nm}$$

rad/s of prop rad/s of plane

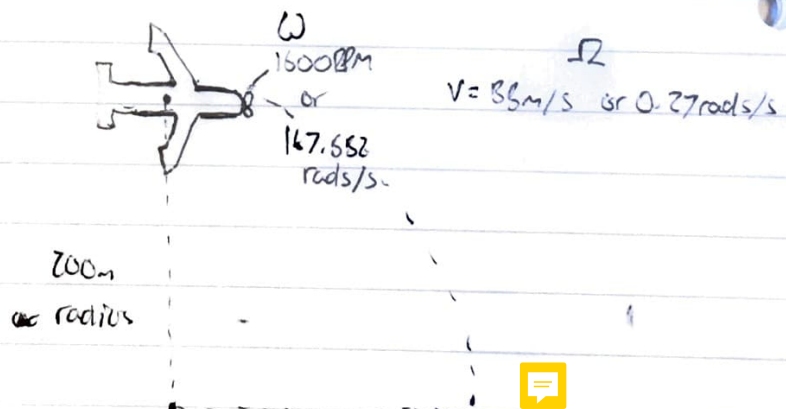
$$\omega = 1600 \text{ RPM} \times \frac{2\pi}{60} = 167.552 \text{ rad/s}$$

$$\Omega = \frac{v}{r} = 200 \text{ km/h} \times \frac{1000}{3600} = 55.5 \text{ m/s}$$

$$r = 200 \text{ m} \quad \text{so } \Omega = 55 \text{ m/s} / 200 \text{ m} = 0.27 \text{ rad/s}$$

$$T = 1.4 \text{ kg m}^2 \times 167.552 \text{ rad/s} \times 0.27 \text{ rad/s} = 65.159 \text{ Nm}$$

c)



④

(A) $m = 15 \text{ kg}$ $r = 0.25 \text{ m}$ $mr = 3.75$

(B) $135^\circ \text{ CW from (A)}$ $m = 27 \text{ kg}$ $r = 0.25 \text{ m}$ $mr = 6.75$

Plane	Mass	Radius	Mr	θ
A	15	0.25	3.75	0
B	27	0.25	6.75	135
C	m_c	0.25	$m_c \cdot 0.25$	θ_c

Horizontal Components = $3.75 \cos(0) + 6.75 \cos(135) + 0.25 m_c \cos C = 0$
 $-1.0229 + 0.25 m_c \cos C = 0$

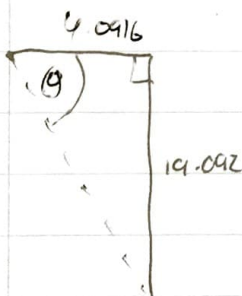
$0.25 m_c \cos C = 1.0229$

$m_c \cos C = 1.0229 \times 4 = 4.0916$

Vertical components = $3.75 \sin(0) + 6.75 \sin(135) + 0.25 m_c \sin C = 0$
 $4.773 + 0.25 m_c \sin C = 0$

$0.25 m_c \sin C = -4.773$

$m_c \sin C = -4.773 \times 4 = -19.092$



$\tan^{-1} \left(\frac{19.092}{4.0916} \right) = 77.904^\circ = C$

So $m_c \cos(77.904^\circ) = 4.0916$

$m_c = 4.0916 / \cos(77.904^\circ)$

$= \underline{19.526 \text{ kg}}$



~~$180^\circ - 77.904^\circ = 102.096^\circ$~~

Question 4

Imbalance is an important factor when looking at rotating equipment such as pumps and turbines. Imbalance is a force that results in increased vibration throughout the machine which therefore effects the performance of the machine and can result in losses such as heat and premature component failure.

“Static imbalance is a condition of unbalance where the central principal axis is displaced.” (Cleng Ltd, 2017) This can be seen when holding a rotor on parallel knife edges as the heavy side will swing to the bottom as the central principle axis and greater mass is now on the lower side of the rotor due to gravity.

“Dynamic unbalance is perhaps the most common type of unbalance which occurs when the central principal axis and the rotating centreline do not coincide or touch”. (Cleng Ltd, 2017) This results in imbalance on multiple planes and the central principle axis being tilted and displaced.

Balancing machines which can be set up for specific balance tolerances for certain rotor design and applications can be calibrated with trial weights of known masses which are put at known radii from the centreline and then factored into the machine. This factored in imbalance results in the machine knowing how it affected the imbalance and therefore how to reduce it, it then gives an imbalance reading and a mass to add or remove at a certain angle at the same radius which be manipulated to bring the rotor into tolerance.



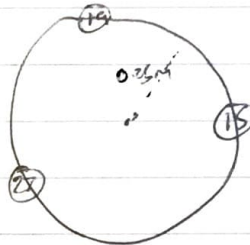
④ Continued

$$F = m r \omega^2$$

$$= 19.526 \text{ kg} \times 0.25 \text{ m} \times \left(150 \times \frac{2\pi}{60}\right)^2$$

$$= \cancel{76.178 \text{ N}}$$

$$1204.462 \text{ N}$$



⑤ $\phi = 40 \text{ mm}$ $B_m = 400 \text{ Nm}$ $T = 350 \text{ Nm}$

$$\tau = \frac{Tr}{J}$$

$$J = \frac{\pi d^4}{32} = \frac{\pi \times 40 \times 10^{-3} \text{ m}^4}{32} = 0.251 \times 10^{-6} \text{ m}^4$$

$$\text{So } \tau = \frac{350 \text{ Nm} \times 20 \times 10^{-3} \text{ m}}{0.251 \times 10^{-6} \text{ m}^4} = 27.852 \times 10^6 = 27.852 \text{ MN/m}^2$$

$$\text{Principle stress } \sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$\sigma = \frac{my}{I} = \frac{400 \times 20 \times 10^{-3} \text{ m}}{I} \quad \text{where } I = \frac{\pi D^4}{64} = \frac{\pi \times 40 \times 10^{-3} \text{ m}^4}{64} = 0.126 \times 10^{-6}$$

$$\therefore \sigma_x = 63.662 \text{ MN/m}^2$$

$$\begin{aligned} \text{Principle stress } \sigma_{1,2} &= \frac{63.662 \text{ MN/m}^2}{2} \pm \frac{1}{2} \sqrt{(63.662 \text{ MN/m}^2)^2 + 4\tau^2} \\ &= 31.831 \text{ MN/m}^2 \pm 31.831 \text{ MN/m}^2 \quad 42.320 \text{ MN/m}^2 \\ \sigma_1 &= 74.151 \text{ MN/m}^2 \quad \sigma_2 = -10.489 \text{ MN/m}^2 \end{aligned}$$

$$\begin{aligned} \text{Max Shear stress } \tau &= \frac{\sigma_1 - \sigma_2}{2} = \frac{74.151 \text{ MN/m}^2 - (-10.489 \text{ MN/m}^2)}{2} \\ &= 42.320 \text{ MN/m}^2 \end{aligned}$$

$$f = 2.5 \text{ Hz}$$

⑥ mass = 5 kg 4 seconds = 10 oscillations ratio of first to sixth is 2.

What is the stiffness and damping coefficient.

$$\omega_n = \sqrt{\frac{k}{m}} \quad \omega_n = 2\pi \times 2.5 \text{ Hz} = 15.708 \text{ rad/s} \quad \omega_n^2 = \frac{k}{m} \quad 246.741 = \frac{k}{m}$$

$$k = \omega_n^2 m = 246.741 \times 5 \text{ kg} = 1233.706$$

$$C_c = 2m\omega_n = 2 \times 5 \text{ kg} \times 15.708 \text{ rad/s} = 157.08 \text{ N/m/s}$$

$$\begin{aligned} \text{Logarithmic decrement} &= \ln\left(\frac{x_1}{x_6}\right) = \frac{2\pi\zeta m}{\sqrt{1-\zeta^2}} \\ &= \ln(2) = \frac{2\pi\zeta m}{\sqrt{1-\zeta^2}} = 0.693 \end{aligned}$$

$$1 - \zeta^2 = \left(\frac{10\pi\zeta}{0.693}\right)^2 \rightarrow \frac{100\pi^2\zeta^2}{0.480} = 1 - \zeta^2$$

$$2055.101\zeta^2 = 1 - \zeta^2 \quad \text{so } 1 = \zeta^2 + 2055.101\zeta^2 = 2056\zeta^2$$

$$\frac{1}{2056} = \zeta^2 \quad \text{so } \sqrt{\frac{1}{2056}} = \zeta = 0.0221$$

$$\begin{aligned} \zeta &= \frac{c}{C_c} \quad \text{so } c = \zeta \times C_c \\ &= 0.022 \times 157.08 \text{ N/m/s} \\ &= 3.464 \end{aligned}$$



Question 6

Logarithmic decrement is the rate at which the amplitude of a free damped vibration decreases. It is defined as the natural logarithm of ratio of any two successive amplitudes. (cmu.edu, 2014) To determine this, the amount that the amplitude is reduced by over a certain number of amplitudes must be known, for example "the ratio of the first to this sixth amplitude is 2." Meaning that the amplitude has halved over 5 amplitudes.



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