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$$= -10 \frac{20x+6}{(2x-1)^3}$$

$$= \frac{2(10x+3)}{(2x-1)^3}$$

$$b) y = \frac{1}{x} + \cos 2x = x^{-1} + \cos 2x$$

$$\frac{dy}{dx} = -x^{-2} - 2\sin 2x$$

$$\frac{d^2y}{dx^2} = 2x^{-3} - 4\cos 2x$$

$$\frac{d^3y}{dx^3} = -6x^{-4} + 8\sin 2x$$

$$\frac{d^4y}{dx^4} = 49x^{-5} + 16\cos 2x$$

$$\frac{d^5y}{dx^5} = -120x^{-6} - 32\sin 2x$$

$$= \frac{-120 - 32\sin 2x}{x^6}$$

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(4)

$$c) f(x) = \ln(\operatorname{cosec}(x) - \cot(x))$$

$$f'(x) = \frac{1}{\operatorname{cosec}(x) - \cot(x)} \cdot \frac{d}{dx}(\operatorname{cosec} x - \cot x)$$

$$= \frac{1}{\operatorname{cosec}(x) - \cot(x)} \cdot (-\operatorname{cosec}(x)\cot(x) - \operatorname{cosec}^2(x))$$

$$= \frac{-\operatorname{cosec}(x)\cot(x) + \operatorname{cosec}^2(x)}{\operatorname{cosec}(x) - \cot(x)}$$

$$= \frac{\operatorname{cosec}^2 x - \operatorname{cosec}(x)\cot(x)}{\operatorname{cosec}(x) - \cot(x)}$$

$$Q2) a) i) \int \frac{2x}{(4+3x^2)} dx$$

$$= \int (2x)(4+3x^2)^{-1} dx$$

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$$\text{ii) } \int_0^4 e^{\frac{1}{2}x} dx$$

$$= \int_0^4 e^u dx$$

$$u = \frac{1}{2}x$$

$$\frac{du}{dx} = \frac{1}{2}$$

$$dx = 2du$$

$$= 2 \int_0^4 e^u du$$

$$= 2 \int_0^4 e^u du = 2 \left[ 2e^u \right]_0^4$$

$$= 4 \left[ 2e^{\frac{1}{2}x} \right]_0^4$$

$$= 2e^2 - 2e^0$$

$$= 12.78$$

$$\text{iii) } \int_0^4 \frac{5x+3}{(2x+1)(x+4)} dx$$

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6)

3) a) i)  $\left(\frac{1+i}{1-i}\right)^3$

$$= \left(\frac{1+i^2+2i}{1+i^2-2i}\right) \left(\frac{1+i}{1-i}\right) = \left(\frac{2i}{-2i}\right) \left(\frac{1+i}{1-i}\right)$$

~~$$\frac{2i+2i^2}{2-2i^2-2i+2i}$$~~

$$= -1 \left(\frac{1+i}{1-i}\right)$$

~~$$\frac{(2-2i)(1-i)}{2+2i^2-2i-2i}$$~~

$$= \frac{2-2-2i+2i^2}{2-2-4i}$$

~~$$= \frac{2i-2}{-4i}$$~~

~~$$= \frac{-2+2i}{-4-4i}$$~~

~~$$= \frac{-1+i}{-2-i}$$~~

$$= -1 \left(\frac{1+i}{1-i} \cdot \frac{1+i}{1+i}\right)$$

~~$$\frac{-1+i}{2-2i} \cdot \frac{2+2i}{2+2i}$$~~

~~$$= \frac{-2+2i^2-2i+2i}{4-4i+4i-2i^2}$$~~

~~$$= \frac{-4}{6} = -\frac{2}{3}$$~~

~~$$\frac{-1+i}{-2i} \cdot \frac{2i}{2i}$$~~

~~$$= \frac{i-2i}{-4i^2}$$~~

~~$$= \frac{-i}{4}$$~~

$$= -1 \left(\frac{1+i^2+2i}{1-i^2+2i}\right)$$

$$= -1 \left(\frac{2i}{2}\right)$$

$$\underline{\underline{= -i}}$$

ii)  $\frac{3}{(1+i)(2+i)}$

$$= \frac{3}{2+2i+i+i^2}$$

$$= \frac{3}{1+3i}$$

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$$\text{iii) } \frac{1+i}{1-i} - \underbrace{(1+2i)(2+2i)} + \frac{3-i}{1+i}$$

$$= \frac{1+i}{1-i} - (2 + 4i^2 + 2i + 4i) + \frac{3-i}{1+i}$$

$$= \frac{1+i}{1-i} - (2 + 0 + 6i) + \frac{3-i}{1+i}$$

$$= \frac{1+i}{1-i} + \frac{3-i}{1+i} + 2 - 6i$$

$$\text{th } \frac{1+i}{1-i} \cdot \frac{1+i}{1+i} = \frac{1+i^2-2i}{1-i^2-i+i} = \frac{-2i}{2} = -i$$

$$= -i + \frac{3-i}{1+i} + 2 - 6i$$

$$= \frac{3-i}{1+i} + 2 - 5i$$

$$\left( \frac{3-i}{1+i} \cdot \frac{1-i}{1-i} = \frac{3+i^2-3i-i}{1-i^2-i+i} = \frac{2-4i}{2} = 1-2i \right)$$

$$= 1-2i + 2 - 5i$$

$$= \underline{\underline{3-7i}}$$

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$$\text{iv) } 2i(1-i) + (\sqrt{3} - i) + (1+i) + \frac{?}{1+i}$$

Question ~~item~~ incomplete

$$\begin{aligned} \text{b) } z &= (4+7i)(3-2i) \\ &= 12 - 14i^2 - 8i + 21i \\ &= 26 + 13i \end{aligned}$$

$$r = \sqrt{26^2 + 13^2} = \sqrt{845}$$

$$\begin{aligned} \arg(z) &= \tan^{-1} \frac{13}{26} \\ &= 0.463 \end{aligned}$$

$$z = \sqrt{845} \angle 0.463 \text{ radians} \quad \text{or } z = \sqrt{845} \angle 26^\circ \text{ degrees}$$

$$\begin{aligned} \text{c) } \frac{1+3i}{1+i} &= \frac{1+3i}{1+i} \cdot \frac{1-i}{1-i} \\ &= \frac{1-3i-i-3i^2}{1-i^2+i-i} = \frac{4-4i}{2} \\ &= \underline{\underline{2-2i}} \end{aligned}$$

FIN

$$Q1) a) i) y = \frac{x^2}{2x+1} = \cancel{x^2(2x+1)^{-1}}$$

$$\begin{aligned} f &= x^2 & \frac{df}{dx} &= 2x \\ g &= 2x+1 & \frac{dg}{dx} &= 2 \end{aligned}$$

Quotient rule

$$\frac{d}{dx} \left( \frac{f}{g} \right) = \frac{\frac{df}{dx} g - f \frac{dg}{dx}}{g^2}$$

$$\frac{(2x)(2x+1) - (x^2)(2)}{(2x+1)^2}$$

$$= \frac{4x^2 + 2x - 2x^2}{4x^2 + 4x + 1}$$

$$= \frac{2x^2 + 2x}{(2x+1)^2}$$

$$= \frac{2x(x+1)}{2x+1}$$

$$\cancel{= (2x+1)^{-1}}$$

$$ii) y = \sqrt{x^2+1} = (x^2+1)^{\frac{1}{2}}$$

$$y' = \frac{1}{2} (x^2+1)^{-\frac{1}{2}} 2x$$

$$= \frac{x}{\sqrt{x^2+1}}$$

Chain rule

$$\frac{dy}{du} \frac{du}{dx}$$

$$iii) y = 3x^{\frac{3}{2}} + \frac{3}{x^2} - 6x = 3^{\frac{3}{2}} + 3x^{-2} - 6x$$

$$y' = \frac{9}{2} x^{\frac{1}{2}} - 6x^{-3} - 6$$

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iv)  $y = \sec^3 4x$

$$\begin{aligned} y' &= 3 \sec^2 4x \cdot \frac{d}{dx} \sec 4x && \text{chain rule} \\ &= 3 \sec^2 4x \cdot (\sec 4x \tan 4x \cdot 4) \\ &= \underline{\underline{12 \sec^3 4x \tan 4x}} \end{aligned}$$

v)  $y = (x + \sin 2x)$

$$y' = \frac{d}{dx} x + \frac{d}{dx} \sin 2x \quad \text{Sum rule}$$

$$y' = \underline{\underline{1 + 2 \cos 2x}}$$

vi)  $y = \frac{1-10x}{(2x-1)^2}$  Quotient rule

$$f = 1-10x \quad \frac{df}{dx} = -10$$

$$\begin{aligned} g &= (2x-1)^2 \quad \frac{dg}{dx} = 8x+4 \\ &= 4x^2 + 4x + 1 \end{aligned}$$

$$y' = \frac{-10(2x-1)^2 - (1-10x)(8x+4)}{((2x-1)^2)^2}$$

$$= \frac{-10(2x-1)^2 - (1-10x)(4)(2x-1)}{(2x-1)^4}$$