

Exam Solutions/Mark Scheme

SECTION A				Qu total
a)	$ \begin{array}{cccc} x & f & xf & f(x-\bar{x})^2 \\ 3 & 3 & 9 & 26.4627 \\ 4 & 12 & 48 & 46.5708 \\ 5 & 21 & 105 & 19.7589 \\ 6 & 28 & 168 & 0.0252 \\ 7 & 23 & 161 & 24.4007 \\ 8 & 11 & 88 & 45.3299 \\ 9 & 2 & 18 & 18.3618 \\ \hline \Sigma & 100 & 597 & 180.91 \end{array} $			
	$\bar{x} = \frac{597}{100} = 5.97$			
	$\sigma = \sqrt{\frac{180.91}{100}} = 1.35$			10
b)	$P(x=r) = \frac{e^{-1.9} \times 1.9^r}{r!}$			
	$(i) P(x=3) = \frac{e^{-1.9} \times 1.9^3}{3!}$			
	$= 0.171$			
	$(ii) P(>1) = 1 - P(0) - P(1)$			
	$P(0) = \frac{e^{-1.9} \times 1.9^0}{0!} = 0.1496$			
	$P(1) = \frac{e^{-1.9} \times 1.9^1}{1!} = 0.2842$			

$$\therefore P(>1) = 1 - 0.1496 - 0.2842 = 0.5662$$

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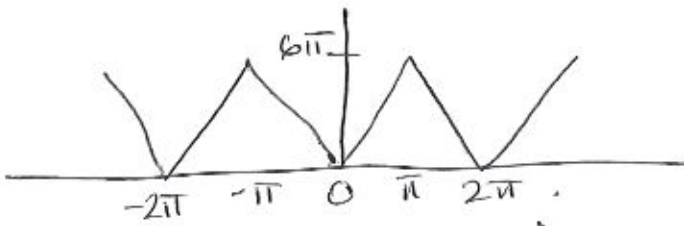
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			Qu total
1.	$\frac{dy}{dt} + 2y = 10e^{-t} \quad y(0)=0.$ $s\bar{y} - y_0 + 2\bar{y} = \frac{10}{s+1}$ $s\bar{y} - 0 + 2\bar{y} = \frac{10}{s+1}$ $\bar{y}(s+2) = \frac{10}{s+1}$ $\bar{y} = \frac{10}{(s+1)(s+2)}$ $\frac{10}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$ $10 = A(s+2) + B(s+1)$ put $s = -2$ $B = -10$ put $s = -1$ $A = 10$ $\mathcal{L}^{-1}\left\{\frac{10}{s+1} - \frac{10}{s+2}\right\}$ $x = 10e^{-t} - 10e^{-2t}$	 3 3 3 3 1 4 4 4	25

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				Qu total
2	$R=40 \quad L=0.5 \quad E=30$ $L \frac{dI}{dt} + RI = E$ $0.5 \frac{dI}{dt} + 40I = 30$ $\frac{dI}{dt} + 80I = 60$ $\int p dt = \int 80 dt = 80t$ $IF = e^{80t}$ $Ie^{80t} = \int e^{80t} \cdot 60 dt$ $= \int 60e^{80t} dt$ $Ie^{80t} = 0.75e^{80t} + C$ When $I=0 \quad t=0$ $0 = 0.75e^0 + C$ $C = -0.75$ $\therefore Ie^{80t} = 0.75e^{80t} - 0.75$	3		
		2		
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		3		25

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			Qu total
3.	 even function $b_n = 0$ $a_0 = \frac{2}{\pi} \int_0^{\pi} 6x \, dx$ $= \frac{2}{\pi} \left[\frac{6x^2}{2} \right]_0^{\pi}$ $= \frac{2}{\pi} [3\pi^2 - 0] = 6\pi$ $a_n = \frac{2}{\pi} \int_0^{\pi} 6x \cos nx \, dx$ Let $u = 6x$ $\frac{dv}{dx} = \cos nx$ $\frac{du}{dx} = 6$ $v = \frac{1}{n} \sin nx$ $\int 6x \cos nx \, dx = 6x \left(\frac{1}{n} \sin nx \right) - \int \frac{1}{n} \sin nx \, dx$ $= \frac{6x}{n} \sin nx + \frac{6}{n^2} \cos nx$ $\therefore a_n = \frac{2}{\pi} \left[\frac{6x}{n} \sin nx + \frac{6}{n^2} \cos nx \right]_0^{\pi}$ $= \frac{12}{\pi n^2} [\cos n\pi - 1]$ $n=1, a_1 = -\frac{24}{\pi(1)^2}$ $a_2 = 0$ $a_3 = -\frac{24}{\pi(3)^2}$	3 2 4 11	25

$$\therefore f(x) = \frac{1}{2}(6\pi) - \frac{24}{\pi(1)^2} \cos x - \frac{24}{\pi(3)^2} \cos 3x + \frac{24}{\pi(5)^2} \cos 5x \dots$$

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			Qu total
4.	$\ddot{I} + 2\dot{I} + 0.5I = 10\cos 10t$ CF $m^2 + 2m + 0.5 = 0$ $m = \frac{-2 \pm \sqrt{2^2 - 4(1)(0.5)}}{2(1)}$ $= -0.293 \text{ or } -1.707$ real + different $I_c = Ae^{-0.293t} + Be^{-1.707t}$ $I_p = A\sin 10t + B\cos 10t$ $\dot{I}_p = 10A\cos 10t - 10B\sin 10t$ $\ddot{I}_p = -100A\sin 10t - 100B\cos 10t$ $(-100A\sin 10t - 100B\cos 10t) + 2(10A\cos 10t - 10B\sin 10t) + 0.5(A\sin 10t + B\cos 10t) = 10\cos 10t$ Taking sines $-100A - 20B + 0.5A = 0$ $-99.5A - 20B = 0$ $A = \frac{20B}{-99.5} = -0.201B$ Taking cosines $-100B + 20A + 0.5B = 10$ $-100B + 20(-0.201B) + 0.5B = 10$ $-103.52B = 10$ $B = \frac{10}{-103.52} = -0.097$	5.	
		3.	
		3.	
		2.	

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			Qu total
$GS = I Ae^{-0.293t} + Be^{-1.707t}$ $+ 0.019 \sin 10t - 0.097 \cos 10t$	2.		
$I(0) = 0$ $0 = A + B - 0.097$ ①	2		
$\frac{dI}{dt} = -0.293 Ae^{-0.293t} - 1.707Be^{-1.707t}$ $+ 0.019 \cos 10t + 0.097 \sin 10t$	2		
$I'(0) = 0$ $0 = -0.293A - 1.707B + 0.019$ ②	2.		
solving ① + ② simultaneously $B = -0.005$ $A = 0.102$	2		
$\therefore I = 0.102e^{-0.293t} - 0.005e^{-1.707t}$ $+ 0.019 \sin 10t - 0.097 \cos 10t$	2		25.