

Engine Analysis: Con-Rod Acceleration, Forces, Displacement

Advanced Engine Design

M6X00410

Assignment: 1

Cornelius May

1701329

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Introduction

Since as far back as 1680, people around the world have been designing, developing and building internal combustion engines (InfoPlease, 2019). As most people who are interested in IC engines will know, in some way, no two engines are the same. Each one has its own, sometimes subtle, pros and cons. In this analysis the author will investigate the behaviour and performance of a Ford engine as well as analysing potential drawbacks for common modification methods.

The engine being investigated is a 1.6 Litre GDI Ford engine. This engine is turbo charged. The modifications that are commonly carried out on this engine are a change in bore, crank throw and con-rod length. These changes are thought to improve the power of the engine, whilst maintaining total engine volume. These changes are preferred as, maintaining the total engine volume often means that the engine still complies with regulations for a range of racing series.

Gas Forces

The first area of interest are the gas forces that act on the piston in the cylinder. Depending on cylinder volume more or less fuel can be injected (Balzer, 2000), dictating the amount of energy that can act on the piston face to drive the crank. Furthermore, different dimensions of cylinder and piston assembly, will produce different compression ratios defining how the fuel burns and when it acts on the piston.

To calculate the gas force, two knowns are required. The first is the pressure in the cylinder at the crank angle being investigated, and the other is the bore of the piston. With this we can calculate gas force using the equation below:

$$\text{Gas Force} = \text{Cylinder Pressure} \times \frac{\pi \times \text{Bore}^2}{4}$$

Equation 1: Gas Force

Since the engine dimensions are known (see Appendix) and the engine pressure is given at each degree of crank angle swept, it is possible to work out what the gas force is at each degree of crank angle and at each RPM. Plotting gas force against crank angle, as was done in the figure below, makes it possible to determine at which RPM and at which crank shaft position the maximum gas force acts on the cylinder. This plot shows that the maximum gas pressure of 46862.91N occurs at 6000 RPM at 18° past top dead centre (TDC).

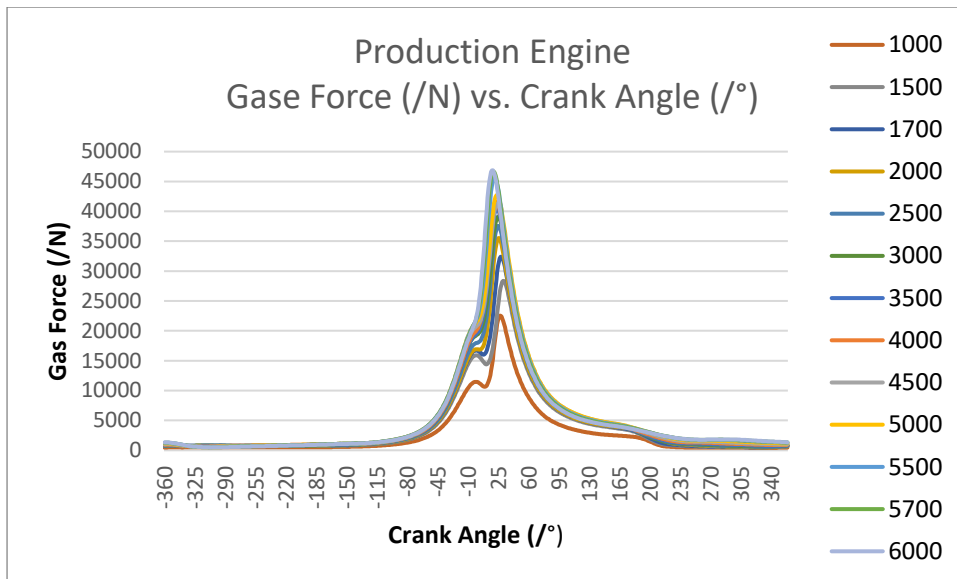


Figure 1: Production Engine Gas Force vs. Crank Angle

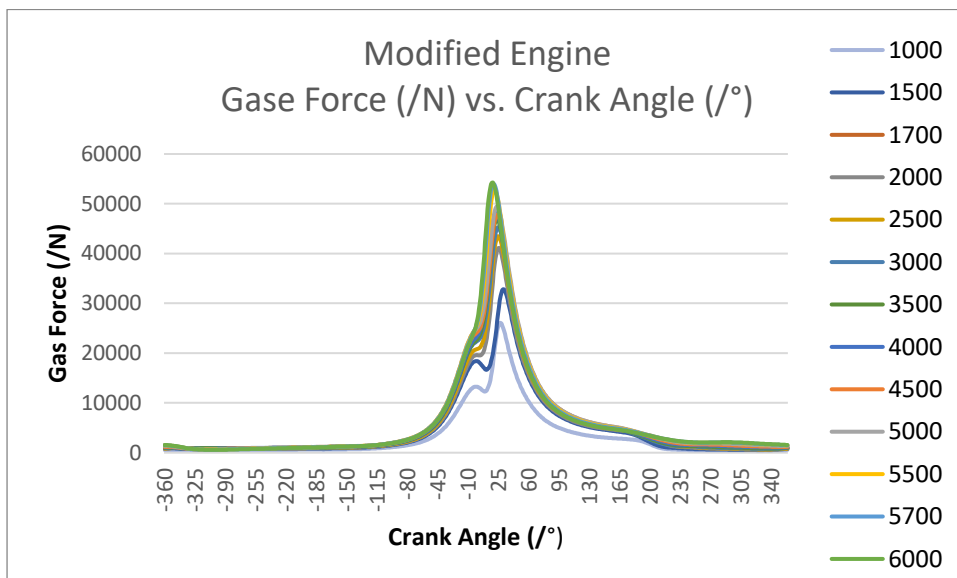


Figure 2: Modified engine Gas Force vs. Crank Angle

As Figure 2 shows, the modified engine produces its peak gas force at 6000 RPM. However, modification has increased the gas force to 54251.64N, almost 14% more than the production engine. This is as a result of the increased surface area of the piston being acted on by the same pressure.

The shape of all the gas force curves in the figure above, shows that the gas force without combustion peaks around 10 degrees before top dead centre. At RPMs below 5000 the gas force briefly decreases before again increasing. At this point combustion begins (Figure 13), causing the gas pressure to continue increasing.

Con-Rod Forces

Another area to investigate is the internal forces within the engine. These must be overcome before any usable force can be produced. This resistive force consists of the friction within the engine, and the inertia of its components. Looking at only one cylinder, this is the inertia of the piston assembly and the friction between the side of the piston and the side of the cylinder face.

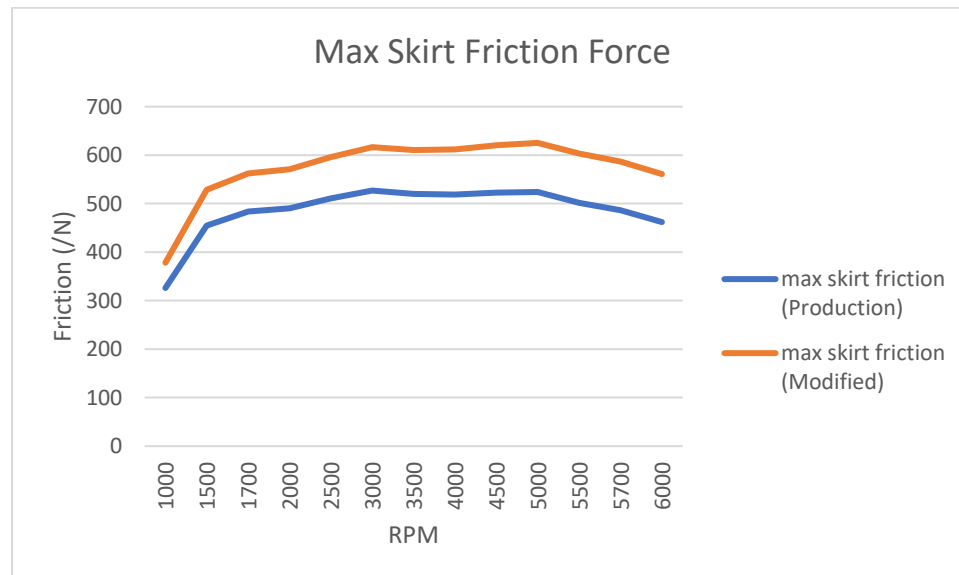


Figure 3: Skirt Friction Force

The figure above shows how the skirt friction increases, peaking at 5000 RPM, before beginning to decrease again. Equation 9 shows how skirt friction can be calculated.

Inertia is defined as the tendency to do nothing or to remain unchanged (Lexico Dictionaries | English, 2019). This could also be interpreted as an objects tendency to resist movement or acceleration. The equation below shows how the inertia loads are calculated. This value relies solely on the mass of the components being investigated and on the angular acceleration of those components.

$$Inertia = Mass \times Acceleration$$

Equation 2: Inertia

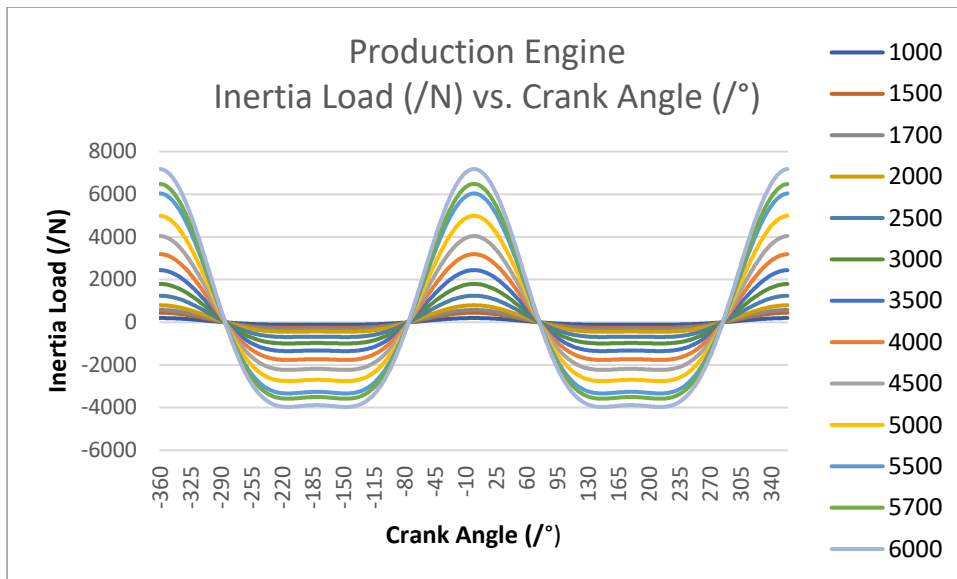


Figure 4: Production Engine Inertia Load vs. Crank Angle

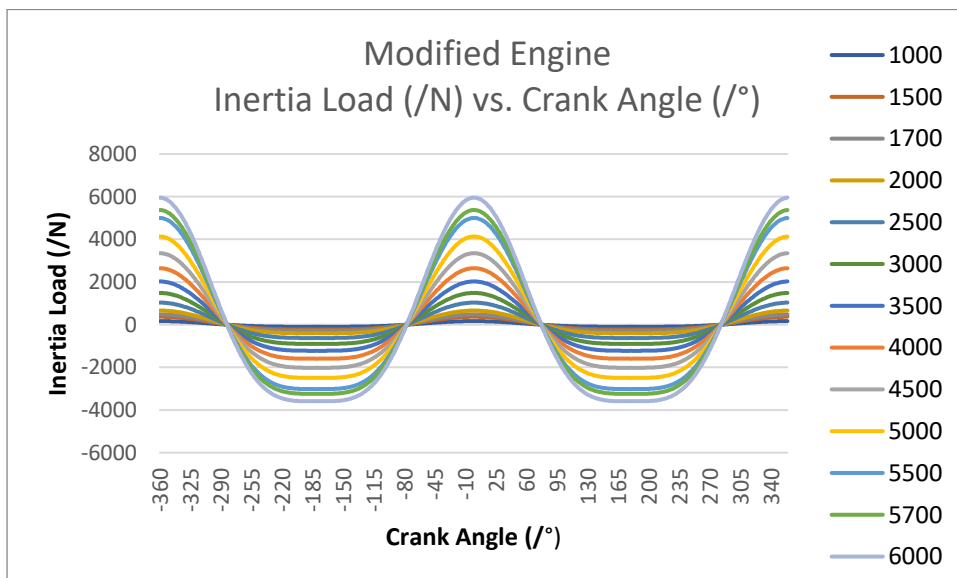


Figure 5: Modified Engine Inertia Load vs. Crank Angle

These graphs show that the maximum inertia forces occur at top dead centre of every crank rotation. The shape of the curve below zero inertia with a flat base, is accounted for when considering the second moment of inertia forces.

With this value, it is now possible to calculate the net compressive force on the rod. This is the force, at each degree of crank angle, that acts vertically down through the face of the piston, onto the con-rod. It consists of the difference between the forces due to inertia and the gas forces. The inertial forces are acting in the same direction as the gas forces when the piston is moving down the cylinder and when the cylinder is moving back up it acts against them. Figure 9 shows that the maximum net compressive force occurs at 6000 RPM where both the gas force and the inertial loads are also at their maximum. From the figure below it is evident that the maximum net force at one RPM occurs when the piston is at top dead centre, again the point where gas forces are at their maximum and acting in the opposite direction to the inertia force of the piston moving up towards the cylinder head.

The tensile load can be defined as the force acting on the rod per piston face surface area. This acts upwards, stretching the con-rod. The production and modified engine's net tensile load peaks at 6000 RPM, however the tensile load on the modified engine peaks at 350.17 N/mm², which is significantly less than the load on the production engine at 5865.14 N/mm².

The stress in an object is the force being applied across a surface area. The equation below shows that as surface area increases, stress decreases. It also shows that a higher force results in higher stress. It is important to note that as we are interested in is the stress within the con-rod, we are using the force acting on the rod, and the surface area of the con-rod, perpendicular to the direction in which the force is acting.

$$\text{Stress} = \frac{\text{Force}}{\text{Surface Area}}$$

Equation 3: Stress

To find the maximum compression and tensile stresses within the con-rod we first have to find the maximum and minimum forces acting on the con-rod. Figure 10 shows that the force acting on the rod peaks at 38293.48 N for a production engine and at 46371.5 for a modified engine. This positive value is the compressive force, acting down the conrod to compress it. To calculate the tensile stress, the lowest value of F rod force is used as this negative value acts up wards to stretch the con-rod. The maximum tensile force is 7638.2 N for the production engine and 7933.3 N for the modified engine. Knowing that we are looking for the maximum values and that stress increases as surface area decreases, it is important to take the surface area to be the point at which the con-rod is thinnest. For the production engine this value is 70mm² and for the purposes of comparison will be assumed to be the same for the modified engine. Using these values in Equation 3, the maximum compressive stress is found to be 547 N/mm² for the production engine and 662.5 N/mm² for the modified engine. Tensile stress is found to be 109.1 N/mm² for the production engine and 113.3 N/mm² for the modified engine.

Engine Design

When building an engine, it is important to ensure that the materials being used will be able to withstand the forces acting on individual components. Any given material has a UTS (ultimate tensile stress). Knowing a given material's UTS and the maximum tensile stress that will be acting on the con-rod, it is possible to evaluate if the material is likely to be safe for the purpose it is intended for. This is called the static safety factor and is calculated using the equation below.

$$\text{Safety Factor} = \frac{\text{Ultimate Tensile Load}}{\text{Max. Tensile Load}}$$

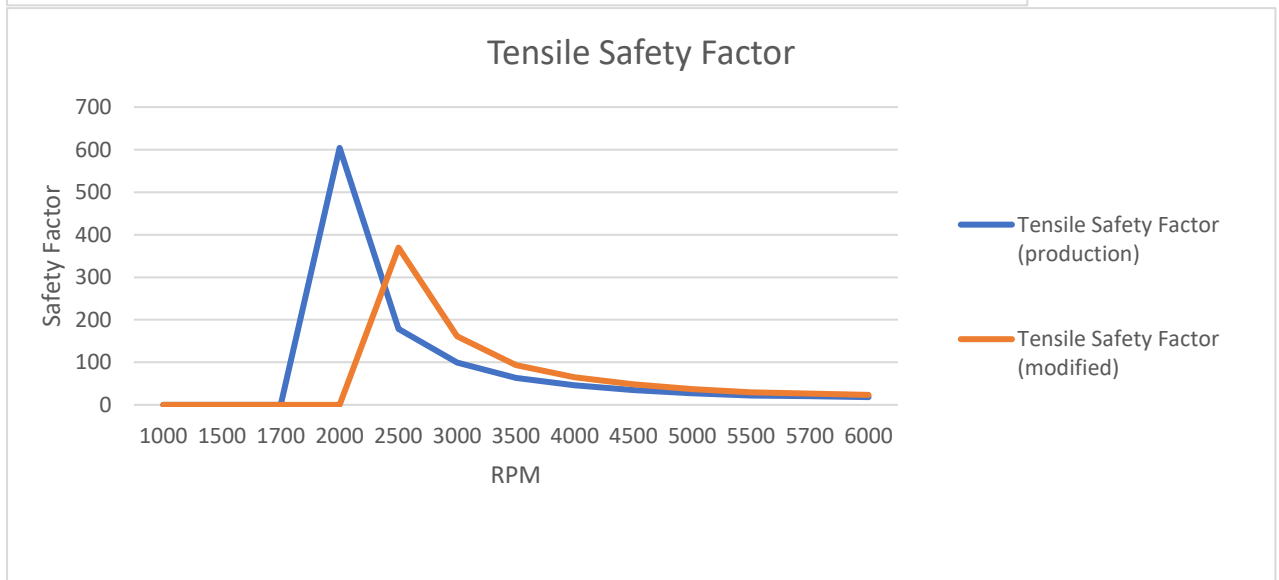
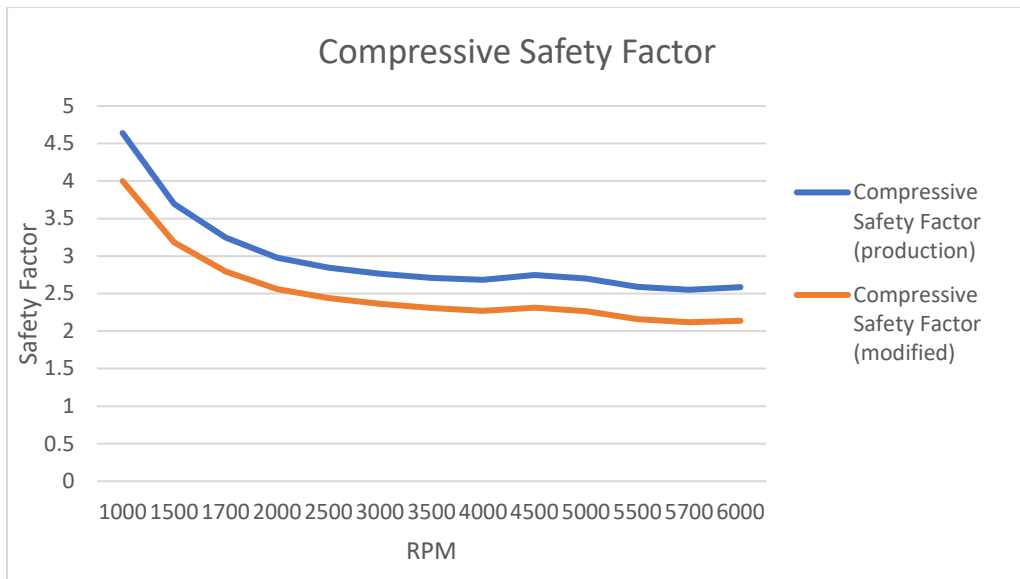


Figure 6: Safety Factor at each RPM

The above figure shows that the safety factor under compression and stretch decreases as the RPM increases. This is due to the higher loads, caused by higher gas forces, the con-rod is experiencing. At this point the con-rod is much closer to its breaking point.

Knowing the stress in the con-rod also allows us to calculate the distance by which it stretches or compresses at each RPM and each angle of crank rotation. This is calculated using the equation:

$$Displacement = \frac{Stress \times ConRod Length}{Young's Modulus \times Surface Area}$$

The graphs below show that the maximum displacement for both stretch and compression occurs at 5700 RPM. These two graphs also show a comparison between the use of steel and titanium. These two materials have a different young's modulus which explains why the titanium has a much higher displacement. Its Young's Modulus is lower than that of the steel used by ford which in keeping with the equation above, results in a higher displacement.

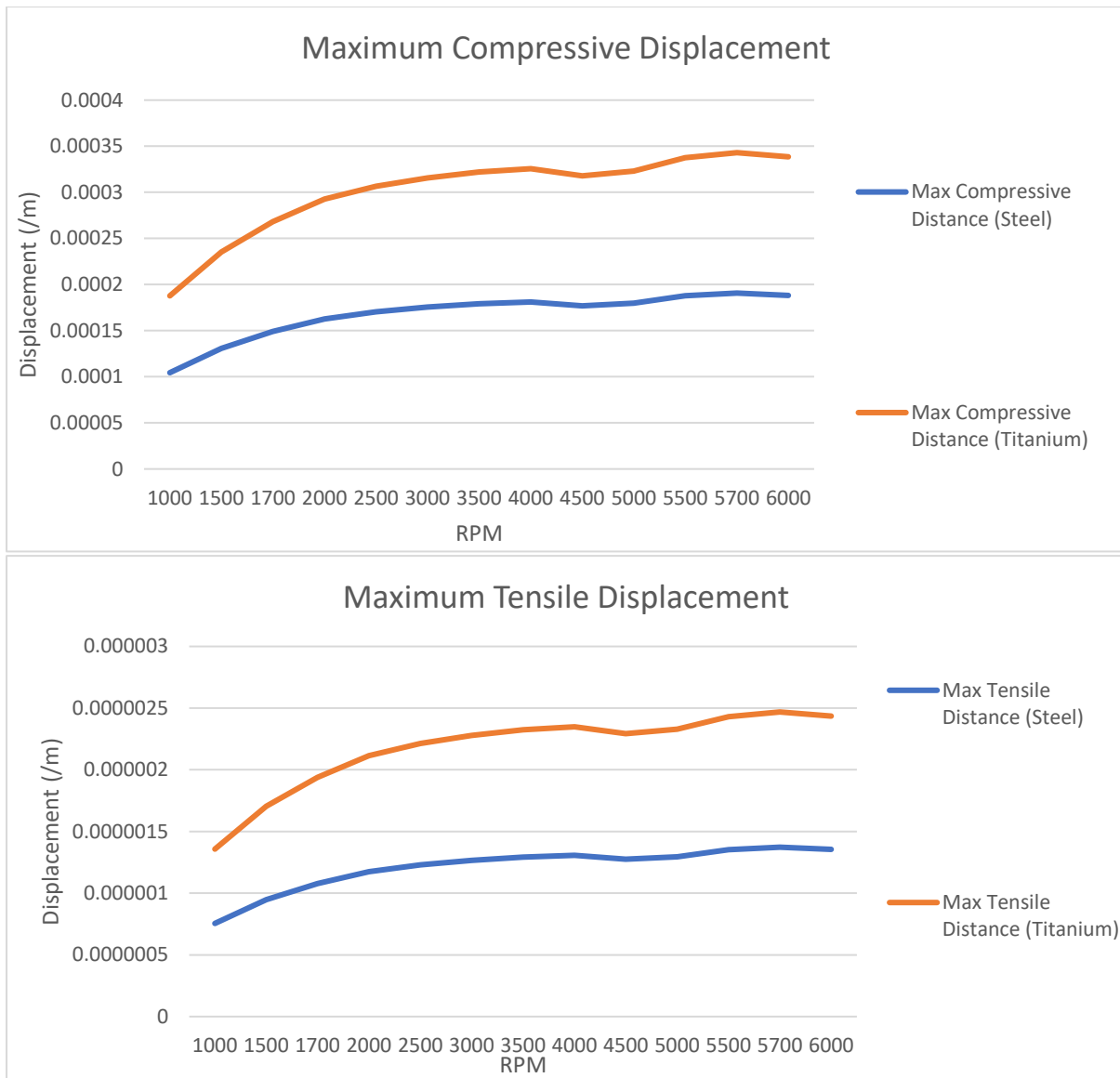


Figure 7: Maximum displacement under stress

When designing an engine, this fact needs to be taken into consideration. This excess stretch under tensile stress could result in the piston hitting valves as the piston moves up to top dead centre, due to the operating clearances. If the increased safety factor that comes through using titanium is required, the minimum cross-sectional area of the con-rod could be increased. Due to the lower density of titanium compared to steel, this increase in surface area could be achieved without the mass of this new con-rod exceeding that of the original steel con-rod. This increased surface area may be sufficient to bring the tensile displacement to within tolerance of the cylinder geometry. The safety factor would be increased and the mass of the engine could remain constant.

Conclusion

There are pros and cons to using the modifications the author has proposed. The reduced safety factor could be a concern as it is likely to mean a reduced lifetime of the engine components, however, in motorsport, this might not be an issue. The figure below shows that the modifications dictate a marginal increase in torque, which might make the reduced lifetime worth while. It would be worth while for anybody considering this upgrade to ensure that the length of the con-rod is adjusted to take into account the increased tensile displacement under the increased loads. Depending on engine tolerances, this could become an issue.

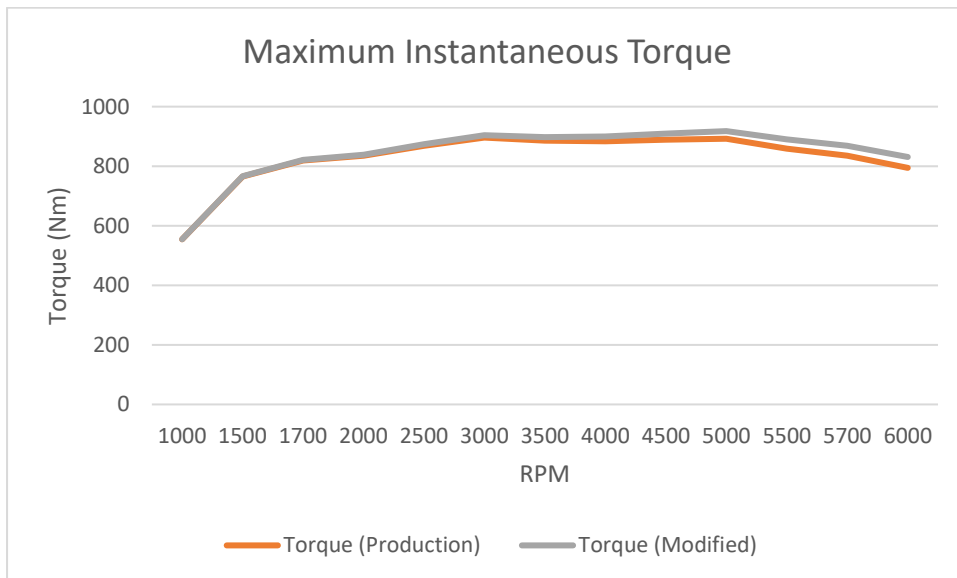


Figure 8: Maximum Instantaneous Torque against RPM

The use of titanium vs. steel can be useful, if the power of your engine means that steel does not have a high enough safety factor, or if its tensile/compressive displacement is outside of the engine's tolerances.

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Appendix

Assignment questions

For clarity, this defines where each question from the assignment brief has been answered.

- Part A is answered in “Gas Forces”
- Part B is answered in “Con-Rod Forces”
- Part C is answered in “Con-Rod Forces”
- Part D is answered in “Con-Rod Forces”
- Part E is answered in “Con-Rod Forces”
- Part F is answered in “Engine Design”
- Part G is answered in “Engine Design”
- Part H is answered in “Engine Design”

Extra Graphs

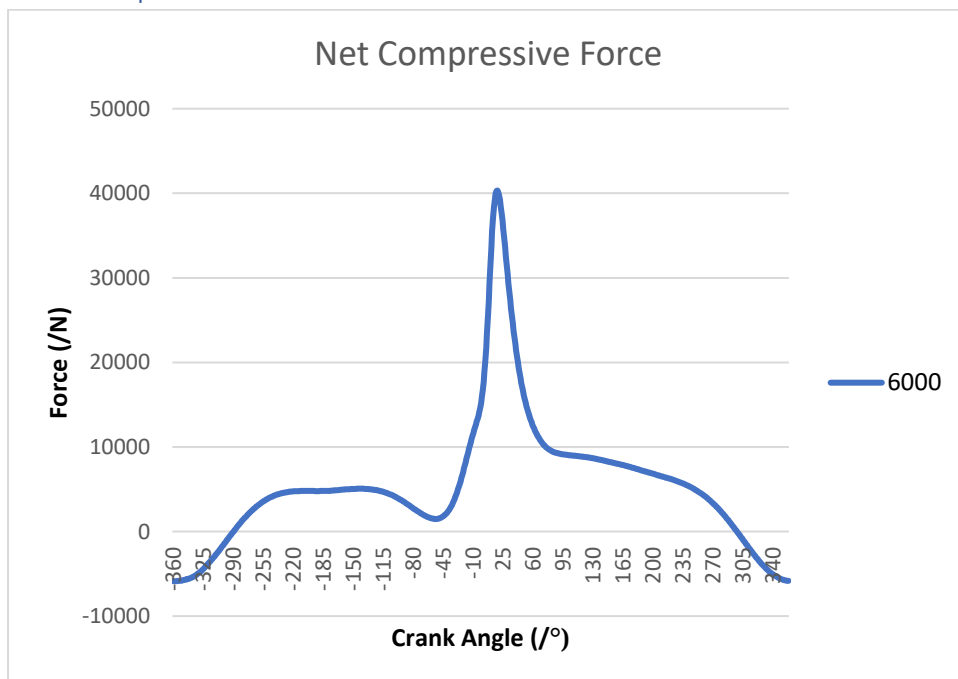


Figure 9: Net Compressive force at 6000 RPM

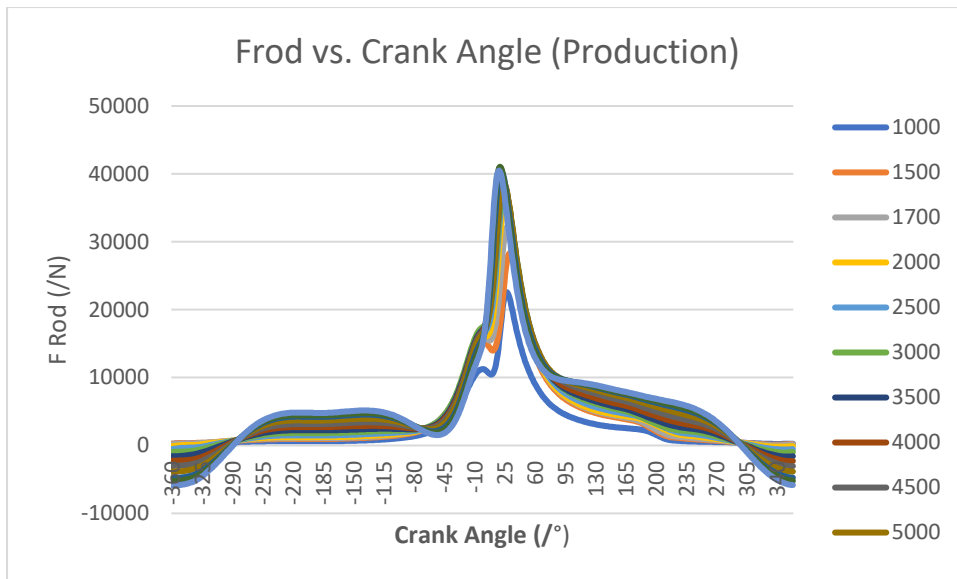


Figure 10: F Rod for Production Engine

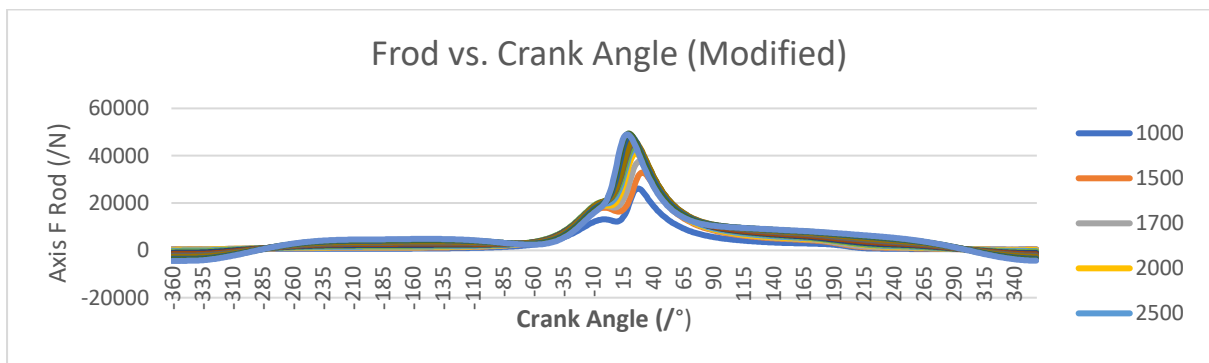


Figure 11: F Rod for modified engine

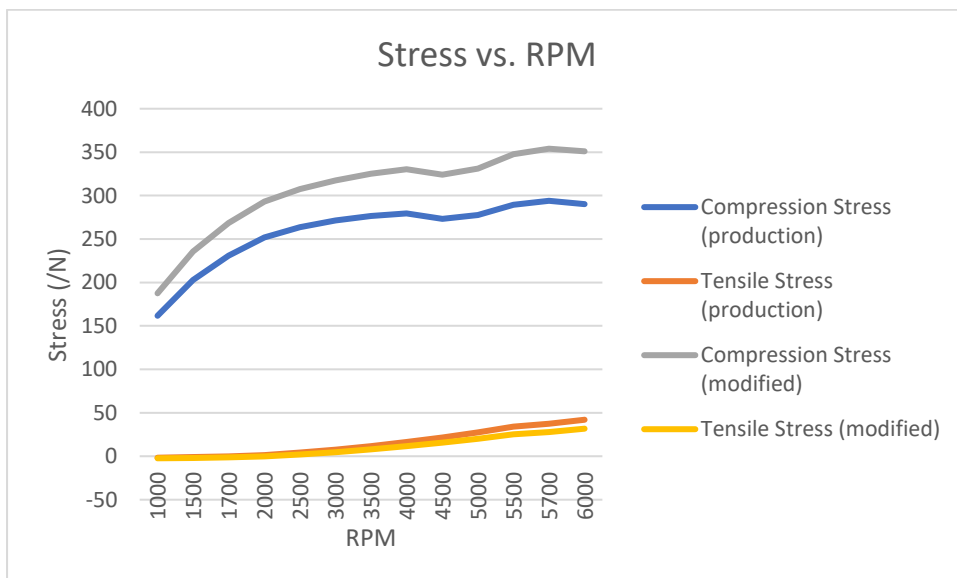


Figure 12: Stress at each RPM

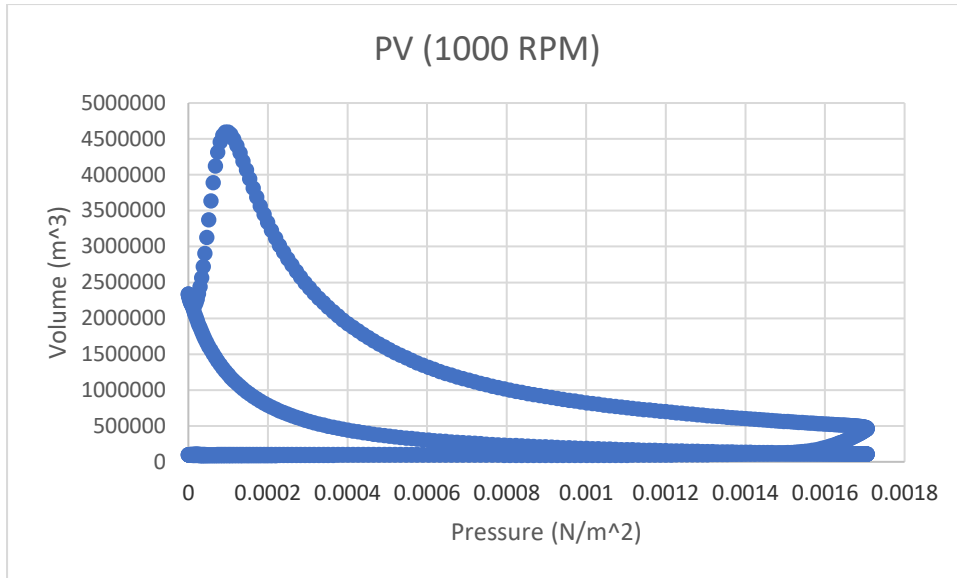


Figure 13: P-V Diagram

Equations

μ	Coefficient of Friction
ω	Angular Velocity
θ	Crank Angle
$\emptyset$	Rod Angle
R	Crank Throw
L	Con-Rod Length

Figure 14: Table of Symbols

$$\text{Gas Force} = \text{Average Cylinder Pressure} \times \frac{\pi \times R^2}{4}$$

Equation 4: Gas Force

$$\text{Net Compressive Force} = \text{Gas Force} - \text{Inertia Load}$$

Equation 5: Net Compressive Force

$$F_{\text{Rod}} = \frac{\text{Net Compressive Force}}{\cos(\emptyset)}$$

Equation 6: F Rod

$$\text{Tensile Stress} = \frac{F_{\text{Rod}}}{\text{ConRod Surface Area}}$$

Equation 7: Tensile Stress

$$F_{\text{side}} = \text{Net Compressive Force} \times \tan(\emptyset)$$

Equation 8: Piston Side Force

$$\text{Skirt Friction} = F_{\text{Side}} \times \mu$$

Equation 9: Force due to Skirt Friction

$$\text{Radial Force} = \text{Pressure} \times \pi \times \text{Rind Diameter} \times \text{Ring Height}$$

Equation 10: Radial Force

$$\text{Top Ring Friction Force} = \mu \times \text{Radial Force}$$

Equation 11: Top Ring Friction Force

$$\text{Inertia Load} = \text{Piston Acceleration} \times \text{Reciprocating Assembly Mass}$$

Equation 12: Inertia Load Force

$$\text{Instantaneous Torque} = F_{\text{Rod}} \times R \times \sin(\theta + \emptyset)$$

Equation 13: Instantaneous Torque

$$\text{Inertia Torque} = \text{Inertia Load} \times R \times \sin(\theta + \emptyset)$$

Equation 14: Inertia Torque

$$\text{Gas Torque} = \text{Gas Force} \times R \times \sin(\theta + \emptyset)$$

Equation 15: Gas Torque

$$\text{Piston Displacement} = R + L - (R \times \cos(\theta)) - (\text{ConRod Length} \times \cos(\theta))$$

Equation 16: Piston Displacement

$$\text{Piston Angular Acceleration} = (R \times \omega \times \cos(\theta)) + \left(\frac{R}{L} \times R \times \omega \times \cos(2 \times \theta)\right)$$

Equation 17: Piston Angular Acceleration

$$\text{Work Done} = \text{abs}\left(\frac{\text{Top Ring Friction}_1 + \text{Top Ring Friction}_2}{2}\right) \times \text{abs}(\text{Piston Travel}_2 - \text{Piston Travel}_1)$$

Equation 18: Work Done

Other Data

RPM	max skirt friction (Production)	max skirt friction (Modified)	Net Fompressive Force (Production)	Net Compressive Force (Modified)	Tensile Load (production)	Tensile Load (modified)
1000	326.102185	378.258615	22365.23661	25942.92297	232.5910153	187.0482588
1500	455.0359043	528.4531823	28018.12294	32544.62969	122.497292	235.0988084
1700	483.6712519	562.0783221	31938.93756	37120.46378	41.52005974	267.8062296
2000	490.6981362	571.012931	34865.89246	40576.26493	-173.3917212	292.2012777
2500	510.5936433	595.7278296	36520.50471	42617.24052	-586.0234998	306.7211727
3000	527.0219433	616.7837061	37591.89218	44006.55908	-1051.861352	316.7202579
3500	519.8369804	610.8259963	38372.76367	45097.23213	-1646.866645	324.3876026
4000	518.492695	612.0333913	38799.45134	45810.9828	-2290.652057	329.3424516
4500	522.7577569	620.1819139	37859.06264	44942.92815	-3032.240011	323.277683
5000	524.1081503	625.264322	38481.84303	45925.38251	-3843.59017	330.3445472
5500	501.9070843	602.9650108	40207.66471	48285.32799	-4770.925743	346.77299
5700	486.7279291	586.7465311	40856.28928	49189.07002	-5208.055678	353.0916208
6000	462.2199037	560.8847679	40326.30542	48805.41741	-5865.141246	350.1743712
RPM	Compression Stress (production)	Tensile Stress (production)	Compression Stress (modified)	Tensile Stress (modified)	Compressive Safety Factor (production)	Compressive Safety Factor (modified)
1000	161.6573132	-1.665528216	187.5170514	-2.341842321	4.639443681	3.99963627
1500	202.9070916	-0.877173591	235.6880284	-1.898760505	3.696272979	3.182172658
1700	231.0015226	-0.297315143	268.4774231	-1.453223148	3.246731846	2.793530984
2000	251.7085252	1.241616335	292.9336118	-0.283324376	2.979636861	2.560307079
2500	263.5010177	4.196373074	307.4898974	2.028972491	2.846288817	2.439104524
3000	271.2312419	7.532125684	317.514043	4.652612176	2.765168182	2.362100249
3500	276.7097965	11.79281522	325.2006041	8.002968425	2.710420843	2.306268779
4000	279.6345215	16.40280742	330.1678713	11.53188271	2.682072285	2.271571722
4500	273.0550835	21.71314007	324.0879028	15.6326217	2.746698543	2.314186964
5000	277.4963786	27.52302306	331.1724784	20.10138752	2.702737974	2.264680941
5500	289.4849717	34.16344965	347.6420952	25.38672877	2.590808067	2.157391209
5700	294.0118368	37.29363178	353.9765622	27.91560829	2.550917705	2.118784349
6000	290.0626809	41.9988632	351.0520012	31.73513625	2.585648032	2.136435621
RPM	Tensile Safety Factor (production)	Tensile Safety Factor (modified)	Max Compressive Distance (Steel)	Max Compressive Distance (Titanium)	Max Tensile Distance (Steel)	Max Tensile Distance (Titanium)
1000	0	0	0.000104423	0.000187778	7.54776E-07	1.35727E-06
1500	0	0	0.000130816	0.00023524	9.47371E-07	1.70361E-06
1700	0	0	0.000149123	0.000268159	1.07854E-06	1.93949E-06
2000	604.0513311	0	0.000162788	0.000292734	1.17522E-06	2.11334E-06
2500	178.7257679	369.6452285	0.000170514	0.000306626	1.23028E-06	2.21235E-06
3000	99.5734845	161.1997673	0.000175516	0.000315621	1.26638E-06	2.27725E-06
3500	63.59804562	93.71522667	0.000179162	0.000322177	1.29196E-06	2.32325E-06
4000	45.7238801	65.03708186	0.000181154	0.00032576	1.30561E-06	2.34781E-06
4500	34.54129607	47.97659756	0.000176764	0.000317864	1.27489E-06	2.29257E-06
5000	27.24991359	37.31085723	0.000179671	0.000323093	1.29563E-06	2.32986E-06
5500	21.95328656	29.54299495	0.000187729	0.000337583	1.3516E-06	2.43051E-06
5700	20.1106721	26.86669022	0.000190758	0.000343029	1.37274E-06	2.46852E-06
6000	17.8576262	23.63311108	0.000188283	0.000338579	1.3543E-06	2.43536E-06

Figure 15: Other Data

Production Engine		
Bore	0.079	m
Crank Throw (R)	0.040	m
Con-Rod Length (L)	0.134	m
Conrod	0.267	kg
Con-rod Cap	0.104	kg
Conrod Bolts	0.035	kg
Conrod Assembly	0.406	kg
Gudgeon pin	0.080	kg
Piston	0.258	kg
Piston Ring	0.005	kg
Circlip	0.001	kg
Reciprocating Assembly	0.350	kg
Conrod SA	140	mm ²
Conrod Minimal SA	139.7	mm ²
UTS (Steel)	750	MPa
Young's Modulus (Steel)	20500	N/mm ²
UTS (Titanium)	1000	MPa
Young's Modulus (Titanium)	11400	N/mm ²
Ring Height	0.0012	m
Ring Diameter	0.076	m
Ring Thickness	0.0029	m
Coefficient of Friction	0.1	mu

Figure 16: Production Engine Constants

Modified Engine		
Bore	0.085	m
Crank Throw (R)	0.0345	m
Con-Rod Length (L)	0.1395	m

Figure 17: Modified Engine Constants