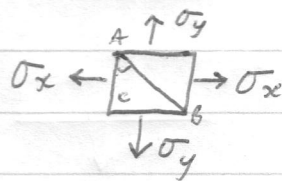


NORMAL AND SHEAR STRESS ON ANY PLANE IN AN ELEMENT SUBJECT TO NORMAL STRESSES ONLY

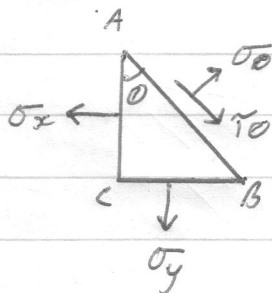
- (a) CONSIDER A STRESSED ELEMENT OF UNIT THICKNESS IN EQUILIBRIUM



NOTE POSITIVE DIRECTIONS

AND WHERE θ IS

- (b) IF WE CUT BY THE PLANE AB, EQUILIBRIUM IS MAINTAINED BY THE NORMAL STRESS σ_θ AND THE SHEAR STRESS τ_θ



- (c) LETS CALCULATE THE FORCE ON EACH FACE.

$$F = \text{AREA} \times \text{STRESS}$$

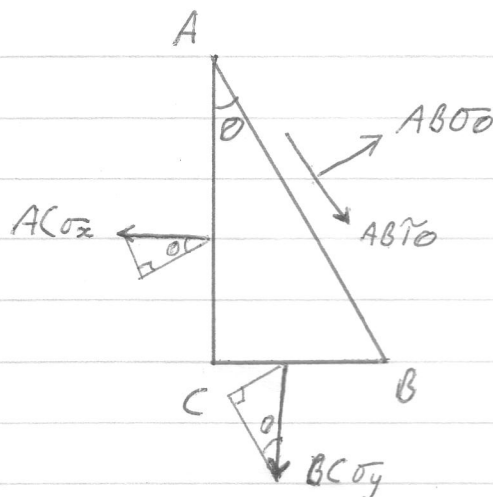
$$AB \text{ DUE TO } \sigma_\theta \quad F = AB \times 1 \times \sigma_\theta = AB\sigma_\theta$$

$$AB \text{ DUE TO } \tau_\theta = AB \times 1 \times \tau_\theta = AB\tau_\theta$$

$$AC \text{ DUE TO } \sigma_x = AC \times 1 \times \sigma_x = AC\sigma_x$$

$$BC \text{ DUE TO } \sigma_y = BC \times 1 \times \sigma_y = BC\sigma_y$$

- (d) IF WE REDRAW THE ELEMENT WITH THE FORCES ON



② ~~LET'S~~ ~~CONSIDER~~

FOR EQUILIBRIUM

$$\sum \text{FORCES } \perp \text{r TO } AB = 0$$

$$\therefore AB \sigma_\theta = AC \sigma_x \cos \theta + BC \sigma_y \sin \theta$$

DIVIDE BY AB:

$$\sigma_\theta = \frac{AC}{AB} \sigma_x \cos \theta + \frac{BC}{AB} \sigma_y \sin \theta$$

LOOKING AT OUR FORCE DIAGRAM $AB \cos \theta = AC$

$$AB \sin \theta = BC$$

$$\therefore \frac{AC}{AB} = \cos \theta$$

$$\therefore \frac{BC}{AB} = \sin \theta$$

$$\therefore \sigma_\theta = \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta$$

REMEMBER $\cos^2 \theta = \frac{1}{2} (1 + \cos 2\theta)$

AND $\sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta)$

$$\begin{aligned} \therefore \sigma_\theta &= \frac{1}{2} \sigma_x (1 + \cos 2\theta) + \frac{1}{2} \sigma_y (1 - \cos 2\theta) \\ &= \frac{1}{2} \sigma_x + \frac{1}{2} \sigma_x \cos 2\theta + \frac{1}{2} \sigma_y - \frac{1}{2} \sigma_y \cos 2\theta \end{aligned}$$

$$\boxed{\sigma_\theta = \frac{1}{2} (\sigma_x + \sigma_y) + \frac{1}{2} (\sigma_x - \sigma_y) \cos 2\theta} \quad (1)$$

(f) CONSIDER Σ FORCES PARALLEL TO AB:

$$\therefore \cancel{ABT_\theta} = \cancel{AC\sigma_x \sin\theta} + \cancel{BC\sigma_y \cos\theta}$$

$$ABT_\theta + BC\sigma_y \cos\theta = AC\sigma_x \sin\theta$$

DIVIDE BY AB

$$T_\theta + \frac{BC}{AB} \sigma_y \cos\theta = \frac{AC}{AB} \sigma_x \sin\theta$$

$$T_\theta + \sin\theta \sigma_y \cos\theta = \cos\theta \sigma_x \sin\theta$$

$$T_\theta + \sigma_y \sin\theta \cos\theta = \sigma_x \sin\theta \cos\theta$$

REMEMBER $\sin\theta \cos\theta = \frac{1}{2} \sin 2\theta$

$$T_\theta + \sigma_y \times \frac{1}{2} \sin 2\theta = \sigma_x \times \frac{1}{2} \sin 2\theta$$

$$T_\theta = \sigma_x \times \frac{1}{2} \sin 2\theta - \sigma_y \times \frac{1}{2} \sin 2\theta$$

$$T_\theta = \frac{1}{2} (\sigma_x - \sigma_y) \sin 2\theta \quad (2)$$

HENCE $T_{\max}$ IS WHEN $\sin 2\theta = 1$ i.e. WHEN $2\theta = 90^\circ$
OR $\theta = 45^\circ$

AND

$$T_{\max} = \frac{1}{2} (\sigma_x - \sigma_y) \quad (3)$$

(g) NOTE THAT IF $\sigma_y = 0$

$$\begin{aligned} \text{FROM (1): } \sigma_\theta &= \frac{\sigma_x}{2} + \frac{\sigma_x}{2} \cos 2\theta \\ &= \frac{\sigma_x}{2} (1 + \cos 2\theta) \end{aligned}$$

$$= \frac{\sigma_x}{2} (2 \cos^2 \theta)$$

$$\sigma_\theta = \sigma_x \cos^2 \theta$$

FROM (3)

$$T_{\max} = \frac{\sigma_x}{2}$$

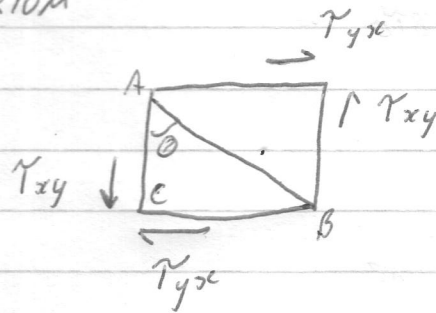
SIGNIFICANCE:

ORIGINAL MEMBER WAS IN
PURE TENSION BUT STILL
SEES SOME SHEAR

~~NOT A TENSION MEMBER~~
NORMALLY STRESSES IT SEES SOME SHEAR

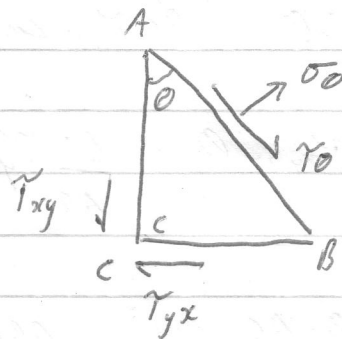
NORMAL AND SHEAR STRESS ON ANY PLANE IN AN ELEMENT SUBJECT TO SHEAR STRESSES ONLY

(a) CONSIDER A STRESSED ELEMENT OF UNIT THICKNESS IN EQUILIBRIUM



NOTE POSITIVE DIRECTIONS AND WHERE θ IS

(b) IF WE CUT BY THE PLANE AB, EQUILIBRIUM IS MAINTAINED BY THE NORMAL STRESS σ_θ AND THE SHEAR STRESS τ_θ



(c) LETS CALCULATE THE FORCE ON EACH FACE

$$F = \text{AREA} \times \text{STRESS}$$

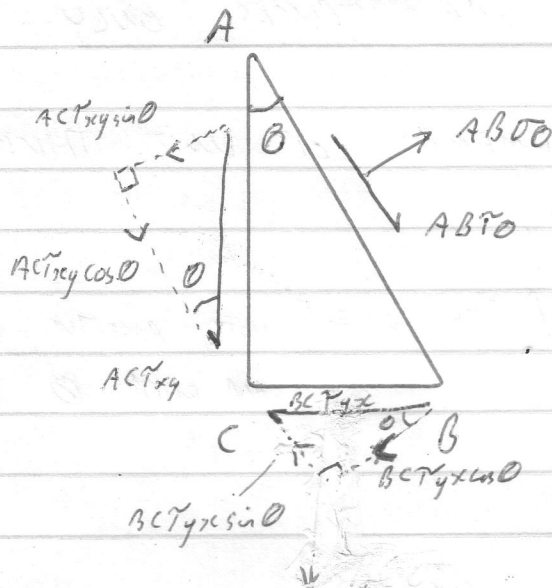
$$AB \text{ DUE TO } \sigma_\theta = AB \times 1 \times \sigma_\theta = AB \sigma_\theta$$

$$AB \text{ DUE TO } \tau_\theta = AB \times 1 \times \tau_\theta = AB \tau_\theta$$

$$AC \text{ DUE TO } \tau_{xy} = AC \times 1 \times \tau_{xy} = AC \tau_{xy}$$

$$BC \text{ DUE TO } \tau_{yx} = BC \times 1 \times \tau_{yx} = BC \tau_{yx}$$

d) IF WE REDRAW THE ELEMENT WITH THE FORCES ON



e) FOR EQUILIBRIUM:

$$\sum \text{FORCES } \perp \text{ TO } AB = 0$$

$$\therefore AB\sigma_0 = ACT_{xy} \sin \theta + BCT_{yx} \cos \theta$$

$$\text{DIVIDE BY } AB: \quad \sigma_0 = \frac{AC}{AB} T_{xy} \sin \theta + \frac{BC}{AB} T_{yx} \cos \theta$$

$$\text{LOOKING AT OUR DIAGRAM } AB \cos \theta = AC$$

$$AB \sin \theta = BC$$

$$\therefore \cos \theta = \frac{AC}{AB}$$

$$\therefore \sin \theta = \frac{BC}{AB}$$

$$\begin{aligned} \therefore \sigma_0 &= T_{xy} \sin \theta \cos \theta + T_{yx} \sin \theta \cos \theta \\ &= 2T_{xy} \sin \theta \cos \theta \end{aligned}$$

$$\text{REMEMBER } \sin \theta \cos \theta = \frac{1}{2} \sin 2\theta$$

$$\therefore \boxed{\sigma_0 = T_{xy} \sin 2\theta} \quad (4)$$

(F) Σ FORCES PARALLEL TO AB = 0

$$\therefore AB\tau_0 + AC\tau_{xy}\cos\theta = BC\tau_{yx}\sin\theta$$

DIVIDE BY AB :

$$\tau_0 + \frac{AC}{AB}\tau_{xy}\cos\theta = \frac{BC}{AB}\tau_{yx}\sin\theta$$

$$\tau_0 + \tau_{xy}\cos^2\theta = \tau_{yx}\sin^2\theta$$

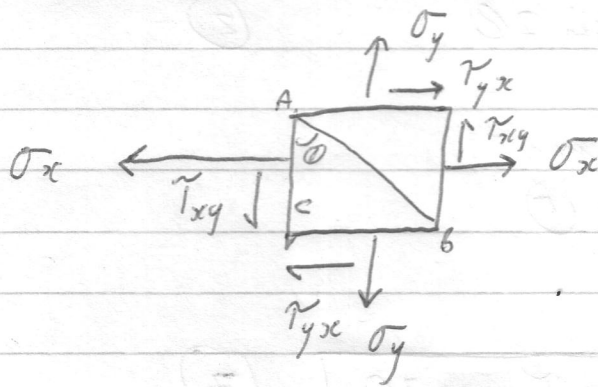
$$\tau_0 = \tau_{yx}\sin^2\theta - \tau_{xy}\cos^2\theta$$

$$= \tau_{xy}(\sin^2\theta - \cos^2\theta)$$

REMEMBER $\sin^2\theta - \cos^2\theta = -\cos 2\theta$

$$\therefore \boxed{\tau_0 = -\tau_{xy}\cos 2\theta} \quad (5)$$

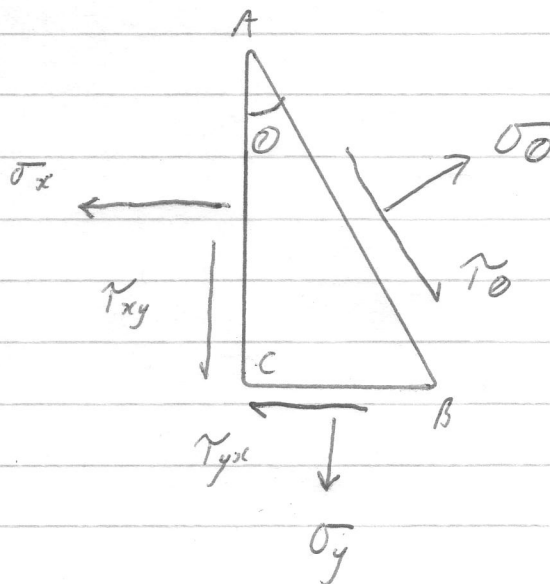
NORMAL AND SHEAR STRESSES ON ANY PLANE IN A GENERAL 2D STRESS SYSTEM



NOTE POSITIVE DIRECTIONS
AND WHERE θ IS

(i) CONSIDER A STRESSED ELEMENT OF UNIT THICKNESS IN EQUILIBRIUM SUBJECT TO STRESS & STRAIN.

(ii) IF WE CUT THE PLANE AB, EQUILIBRIUM IS MAINTAINED BY σ_θ AND τ_θ :



WE HAVE SHOWN FOR AN ELEMENT EXPERIENCING NORMAL STRESS : $\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y) \cos 2\theta$ (1)

WE HAVE ALSO SHOWN FOR AN ELEMENT EXPERIENCING SHEAR STRESS : $\sigma_\theta = \tau_{xy} \sin 2\theta$ (4)

FOR COMBINED CASE

$$\therefore \sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y) \cos 2\theta + \tau_{xy} \sin 2\theta \quad (6)$$

WE

FOR NORMAL STRESS ELEMENT:

$$\tau_{\theta} = \frac{1}{2}(\sigma_x - \sigma_y) \sin 2\theta \quad (2)$$

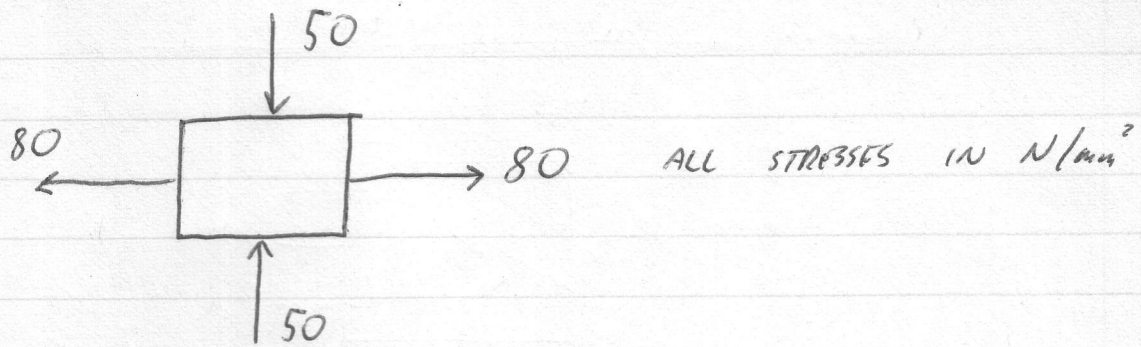
FOR SHEAR STRESS ELEMENT:

$$\tau_{\theta} = -\tau_{xy} \cos 2\theta \quad (5)$$

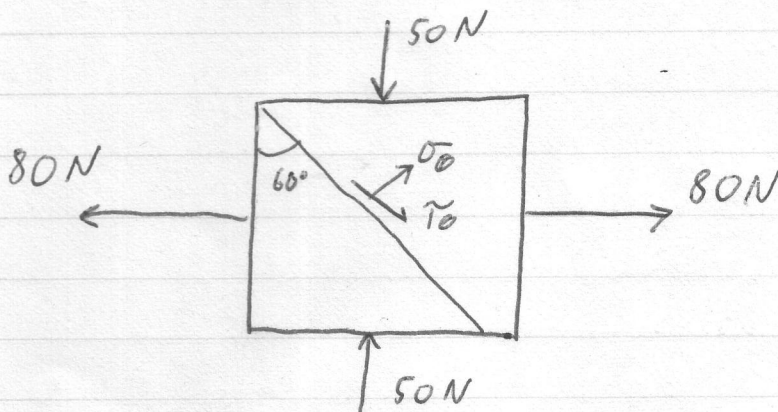
∴ FOR COMBINED CASE

$$\tau_{\theta} = \frac{1}{2}(\sigma_x - \sigma_y) \sin 2\theta - \tau_{xy} \cos 2\theta \quad (7)$$

EXAMPLE 2



FIND THE NORMAL & SHEAR STRESSES ACTING ON A PLANE INCLINED AT 60° TO THE VERTICAL,



$$\sigma_x = 80 \text{ Nmm}^{-2}$$

$$\sigma_y = -50 \text{ Nmm}^{-2}$$

$$\theta = 60^\circ$$

$$\tau_{xy} = 0$$

$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{1}{2}(80 - 50) + \frac{1}{2}(80 + 50)\cos 120 + 0$$

$$= 15 + 65\cos 120$$

$$= \underline{\underline{-17.5 \text{ Nmm}^{-2}}}$$

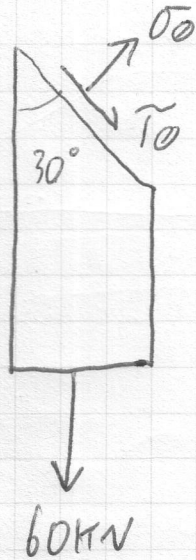
$$\tau_\theta = \frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$= \frac{1}{2}(80 + 50)\sin 120 - 0$$

$$= 65\sin 120$$

$$= \underline{\underline{56.29 \text{ Nmm}^{-2}}}$$

③



CROSS SECTIONAL AREA = 850 mm^2

$$\sigma_x = \frac{P}{A}$$

$$= \frac{60 \times 10^3}{850 \times 10^{-6}}$$

$$= \underline{\underline{70.6 \text{ MPa}}}$$

$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y) \cos 2\theta$$

$$= \cancel{\frac{1}{2}(70.6 \times 10^6 + 0)} + \cancel{\frac{1}{2}(70.6 \times 10^6 - 0) \cos 60}$$

$$= \frac{1}{2}(0 + 70.6 \times 10^6) + \frac{1}{2}(0 - 70.6 \times 10^6) \cos 60$$

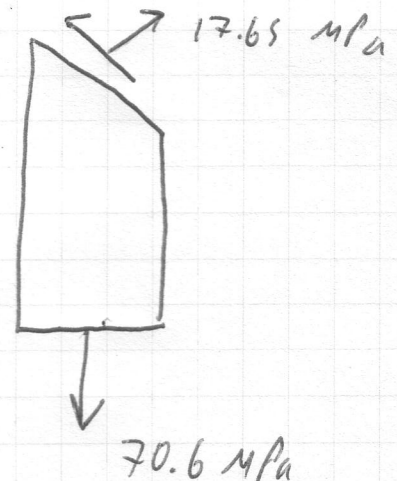
$$= \underline{\underline{17.65 \text{ MPa}}}$$

$$\tau_\theta = \frac{1}{2}(\sigma_x - \sigma_y) \sin 2\theta$$

$$= \frac{1}{2}(0 - 70.6 \times 10^6) \sin 60$$

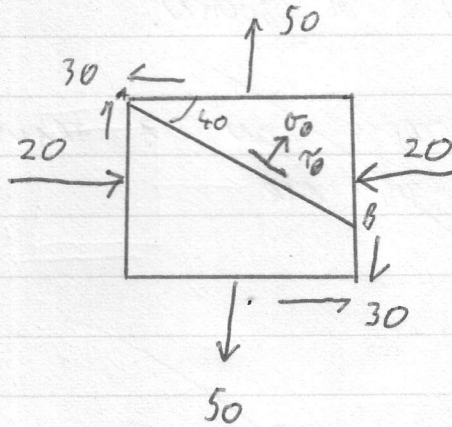
$$= \underline{\underline{-30.6 \text{ MPa}}}$$

30.6 MPa



COMPLEX STRESS EXAMPLES

①



ALL STRESSES IN MN/m^2

$$\theta = 90 - 40 = 50^\circ$$

WHAT ARE THE NORMAL & SHEAR STRESSES ON PLANE AB?

④

$$\sigma_x = -20 \text{ MNm}^{-2}$$

$$\sigma_y = 50 \text{ MNm}^{-2}$$

$$\theta = 90 - 40 = 50^\circ$$

$$\tau_{xy} = \tau_{yx} = -30 \text{ MNm}^{-2}$$

$$\sigma_\theta = \frac{1}{2} (\sigma_x + \sigma_y) + \frac{1}{2} (\sigma_x - \sigma_y) \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{1}{2} (-20 + 50) + \frac{1}{2} (-20 - 50) \cos 100^\circ + -30 \sin 100^\circ$$

$$= \underline{\underline{-8.5 \text{ MNm}^{-2}}}$$

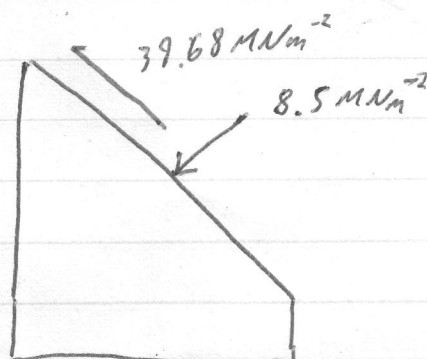
↑ τ_{xy} IS NEGATIVE AS
~~IT IS IN THE OTHER ORIENTATION TO OUR DERIVED ASSUMPTION~~

$$\tau_\theta = \frac{1}{2} (\sigma_x - \sigma_y) \sin 2\theta - \tau_{xy} \cos 2\theta$$

$$= \frac{1}{2} (-20 - 50) \sin 100^\circ + 30 \cos 100^\circ$$

$$= \underline{\underline{-39.68 \text{ MNm}^{-2}}}$$

IT IS IN THE OTHER ORIENTATION TO OUR DERIVED ASSUMPTION



PRINCIPAL PLANES AND PRINCIPAL STRESSES

PRINCIPAL STRESSES - MAX AND MIN NORMAL STRESSES, σ_1 & σ_2

PRINCIPAL PLANES - PLANES THE PRINCIPAL STRESSES ACT ON.

FOR AN ELEMENT SUFFERING COMBINED NORMAL AND SHEAR STRESS:

$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y) \cos 2\theta + \tau_{xy} \sin 2\theta$$

TO FIND THE MAX OR MIN $\frac{d\sigma_\theta}{d\theta} = 0$
~~NORMAL~~ STRESS;

~~$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y) \cos 2\theta + \tau_{xy} \sin 2\theta$$~~

REMEMBER: IF $y = \cos 2\theta$ THEN $\frac{dy}{d\theta} = -2 \sin 2\theta$
 AND $y = \sin 2\theta$ THEN $\frac{dy}{d\theta} = 2 \cos 2\theta$

$$\frac{d\sigma_\theta}{d\theta} = -(\sigma_x - \sigma_y) \sin 2\theta + 2\tau_{xy} \cos 2\theta$$

$$\therefore 0 = -(\sigma_x - \sigma_y) \sin 2\theta + 2\tau_{xy} \cos 2\theta$$

$$(\sigma_x - \sigma_y) \sin 2\theta = 2\tau_{xy} \cos 2\theta$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\boxed{\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}} \quad (8)$$

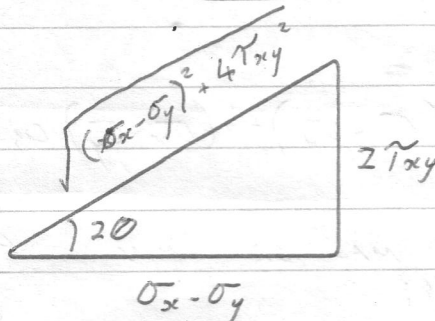
WE HAVE SHOWN FOR MAX OR MIN DIRECT STRESS

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

WE KNOW

$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta \quad (6)$$

FOR MAX OR MIN



$$\begin{matrix} \text{SOH} \\ \text{CAH} \\ \tan \theta = \frac{O}{A} \end{matrix}$$

$$\therefore \cos 2\theta = \frac{\sigma_x - \sigma_y}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}$$

$$\sin 2\theta = \frac{2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}$$

$\therefore$ FROM (6) FOR MAX OR MIN VALUES $\sigma_{1,2}$

$$\begin{aligned} \sigma_{1,2} &= \frac{\frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}} + \frac{\tau_{xy} \times 2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}} \\ &= \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2} \left[\frac{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}} \right] \end{aligned}$$

THE
PRINCIPAL
STRESSES
OF THE
SYSTEM

$$\therefore \sigma_{1,2} = \frac{1}{2} \left[(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \right] \quad (9)$$

$$\left(\text{CONSIDER } \frac{25}{5} = 5 \quad \text{ALSO } \frac{25}{-5} = -5 \right) \text{ THE ANSWER IS PLUS OR MINUS THE BOTTOM TERM! }$$

WE NOW KNOW THE MAX AND MIN DIRECT STRESSES AT THAT POINT

$$\tau_0 = \frac{1}{2}(\sigma_x - \sigma_y) \sin 2\theta - \tau_{xy} \cos 2\theta \quad (7)$$

SUBST. IN OUR VALUES FOR $\sin 2\theta$ & $\cos 2\theta$ AT THIS POINT

$$\tau_0 = \frac{\frac{1}{2}(\sigma_x - \sigma_y) \times 2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}} - \frac{\tau_{xy}(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}$$

$$\tau_0 = 0$$

I.E. ON A PRINCIPAL PLANE WHEN THE NORMAL STRESS IS A MAX OR A MIN ~~THE~~ THE ~~SHEAR STRESS~~ IS SHEAR STRESS = 0.

IF AT THE PRINCIPAL PLACES THE SHEAR STRESS IS ZERO τ_{max} IS GIVEN BY OUR EXAMPLE SUBJECT TO NORMAL STRESSES ONLY: ~~Then~~

$$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2)$$

$$\leftarrow \begin{matrix} \text{WHEN } \sin 2\theta = 1 \\ \text{OR } \theta = 45^\circ \end{matrix}$$

$$= \frac{1}{2} \left[\frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} - \frac{1}{2}(\sigma_x + \sigma_y) - \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \right]$$

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

SHEAR STRESS IS A MAX ON PLANES AT 45° TO PLANES OF PRINCIPAL STRESS.

SUMMARY:

- ① ELEMENT SUBJECT TO NORMAL STRESSES ONLY:

$$\sigma_{\theta} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y) \cos 2\theta$$

- ② ~~ELEMENT SUBJECT TO SHEAR STRESSES ONLY:~~

$$\tau_{\theta} = \frac{1}{2}(\sigma_x - \sigma_y) \sin 2\theta$$

$$\tau_{\max} = \frac{1}{2}(\sigma_x - \sigma_y) \quad \text{WHEN } \theta = 45^\circ$$

- ③ ELEMENT SUBJECT TO SHEAR STRESSES:

$$\sigma_{\theta} = \tau_{xy} \sin 2\theta$$

$$\tau_{\theta} = -\tau_{xy} \cos 2\theta$$

- ③ ELEMENT SUBJECT TO GENERAL 2-D STRESS SYSTEM

$$\sigma_{\theta} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y) \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{\theta} = \frac{1}{2}(\sigma_x - \sigma_y) \sin 2\theta - \tau_{xy} \cos 2\theta$$

- ④ PRINCIPAL STRESSES AND PRINCIPAL STRAINS

$$\sigma_{1,2} = \frac{1}{2} \left[(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \right]$$

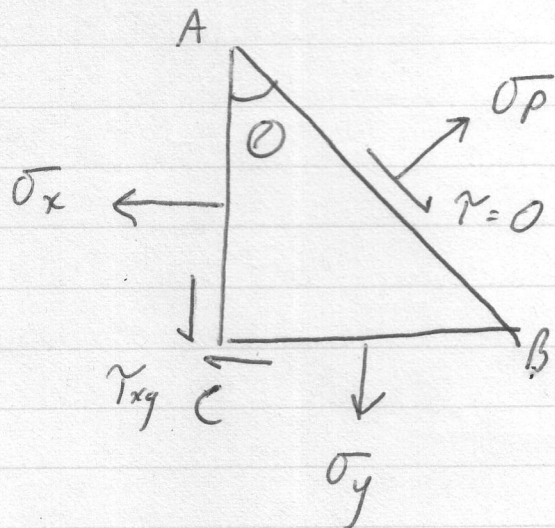
$$\tau_{\max} = \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

SHEAR STRESSES ON PRINCIPLE PLANES ARE ZERO

SHEAR STRESSES ARE A MAXIMUM ON PLANES AT 45° TO THE PRINCIPAL PLANES.

ANGLE OF PRINCIPAL PLANES

C.S. 10a



$$AB \cos \theta = AC \quad \therefore \frac{AC}{AB} = \cos \theta$$
$$AB \sin \theta = BC \quad \therefore \frac{BC}{AB} = \sin \theta$$

RESOLVE HORIZONTALLY

$$(\sigma_x \times AC \times 1) + \tau_{xy} \times BC \times 1 = (\sigma_p \times AB \times 1) \cos \theta$$

$$AC \sigma_x + BC \tau_{xy} = AB \sigma_p \cos \theta$$

$$\frac{AC}{AB} \sigma_x + \frac{BC}{AB} \tau_{xy} = \sigma_p \cos \theta$$

$$\sigma_x \cos \theta + \tau_{xy} \sin \theta = \sigma_p \cos \theta$$

$$\sigma_x + \tau_{xy} \frac{\sin \theta}{\cos \theta} = \sigma_p$$

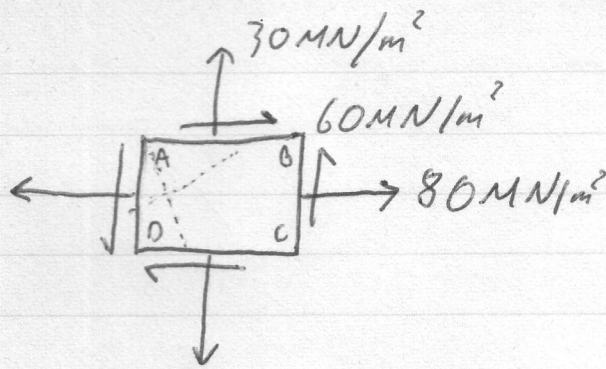
$$\sigma_x + \tau_{xy} \tan \theta = \sigma_p$$

$$\therefore \tan \theta = \frac{\sigma_p - \sigma_x}{\tau_{xy}}$$

HEARN 1

P.343

EXAMPLE 1



$$\begin{aligned}\sigma_1, \sigma_2 &= \frac{1}{2} \left[(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \right] \\ &= \frac{1}{2} \left[(80 + 30) \pm \sqrt{(80 - 30)^2 + 4 \times 60^2} \right] \\ &= \frac{1}{2} (110 \pm 130)\end{aligned}$$

$$\sigma_1 = 120 \text{ MN/m}^2$$

$$\sigma_2 = -10 \text{ MN/m}^2 \text{ i.e. COMPRESSIVE.}$$

$$\sigma_1 \text{ PLANE: } \tan \theta_1 = \frac{\sigma_p - \sigma_x}{\tau_{xy}}$$

$$\theta_1 = \tan^{-1} \left(\frac{120 - 80}{60} \right)$$

$$= \underline{33.69^\circ} \quad \begin{array}{l} \text{COUNTER CLOCKWISE} \\ \text{FROM AD} \end{array}$$

$$\sigma_2 \text{ PLANE: } \tan \theta_2 = \frac{\sigma_p - \sigma_x}{\tau_{xy}}$$

$$= \tan^{-1} \left(\frac{-10 - 80}{60} \right)$$

COUNTER CLOCKWISE
FROM AD

$$= -56.3^\circ \text{ or } \underline{123.7^\circ}$$

NOTE σ_1 AND σ_2 ARE 90° TO EACH OTHER, AS EXPECTED.

IF LOADING CHANGES.

$$\sigma_1, \sigma_2 = \frac{1}{2} \left[(-80 + 30) \pm \sqrt{(-80 - 30)^2 + 4 \times 60^2} \right]$$

$$\sigma_1 = \underline{56.5 \text{ MN/m}^2} \quad \frac{1}{2} (-50 \pm 162.8)$$

$$\sigma_2 = \underline{-106.5 \text{ MN/m}^2} \quad (\text{ie COMPRESSIVE})$$

Also

$$\tan \theta_1 = \frac{56.5 - (-80)}{60}$$

$$\theta_1 = \underline{66.27^\circ}$$

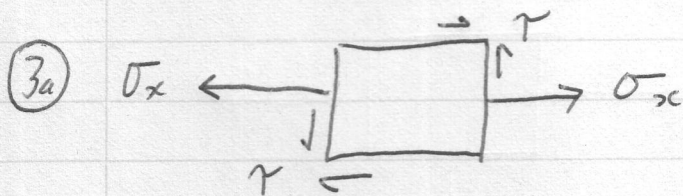
COUNTER CLOCKWISE
FROM AD

$$\tan \theta_2 = \frac{-106.5 - (-80)}{60}$$

$$\theta_2 = -23.8 \quad \text{or} \quad \underline{156.2^\circ} \quad \text{COUNTER CLOCKWISE FROM AD}$$

AGAIN NOTICE THAT σ_1 & σ_2 ARE 90° TO EACH OTHER AS EXPECTED.

EXAM QUESTION MAY 2006



$$\sigma_x = \frac{\text{FORCE}}{\text{AREA}} = \frac{20}{\pi \left(\frac{d}{2}\right)^2} = \frac{20}{\pi \frac{d^2}{4}} = \frac{80}{\pi d^2} \text{ kNm}^{-2}$$

$$\tau = \frac{\text{SHEAR LOAD}}{\text{SHEAR AREA}} = \frac{10}{\pi \left(\frac{d}{2}\right)^2} = \frac{10}{\pi \frac{d^2}{4}} = \frac{40}{\pi d^2} \text{ kNm}^{-2}$$

PRINCIPAL STRESSES

$$\sigma_{1,3} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$= \frac{1}{2}\left(\frac{80}{\pi d^2} + 0\right) \pm \frac{1}{2}\sqrt{\left(\frac{80}{\pi d^2} - 0\right)^2 + 4\left(\frac{40}{\pi d^2}\right)^2}$$

$$= \frac{40}{\pi d^2} \pm \frac{1}{2}\sqrt{\left(\frac{80}{\pi d^2}\right)^2 + 4\left(\frac{40}{\pi d^2}\right)^2}$$

$$= \frac{40}{\pi d^2} \pm \frac{1}{2}\sqrt{\left(\frac{80}{\pi d^2}\right)^2 + \left(\frac{2 \times 40}{\pi d^2}\right)^2}$$

$$= \frac{40}{\pi d^2} \pm \frac{1}{2}\sqrt{\left(\frac{80}{\pi d^2}\right)^2 + \left(\frac{80}{\pi d^2}\right)^2}$$

$$= \frac{40}{\pi d^2} \pm \frac{1}{2}\sqrt{2\left(\frac{80}{\pi d^2}\right)^2}$$

$$= \frac{40}{\pi d^2} \pm \frac{1}{2} \times \frac{80}{\pi d^2} \sqrt{2}$$

$$= \frac{40}{\pi d^2} + \frac{40}{\pi d^2} \sqrt{2}$$

$$= \frac{40}{\pi d^2} (1 + \sqrt{2})$$

$$\therefore \sigma_1 = \frac{40}{\pi d^2} (1 + \sqrt{2})$$

$$= \frac{40 \times 2.414}{\pi d^2}$$

$$= \frac{30.7}{d^2} \text{ KNm}^{-2}$$

$$\therefore \sigma_3 = \frac{40}{\pi d^2} (1 - \sqrt{2})$$

$$= \frac{40 \times -0.414}{\pi d^2}$$

$$= \frac{-5.27}{d^2} \text{ KNm}^{-2}$$

(h)

$$\sigma_{max} = 270 \text{ MN m}^{-2}$$

$$\text{FACTOR OF SAFETY} = 4$$

$$\therefore \sigma = \frac{270}{4} = 67.5 \text{ MN m}^{-2}$$

$$\nu = 0.283$$

MAX PRINCIPAL STRESS THEORY

$$\text{FAILURE WHEN } \sigma_1 = 67.5 \times 10^6$$

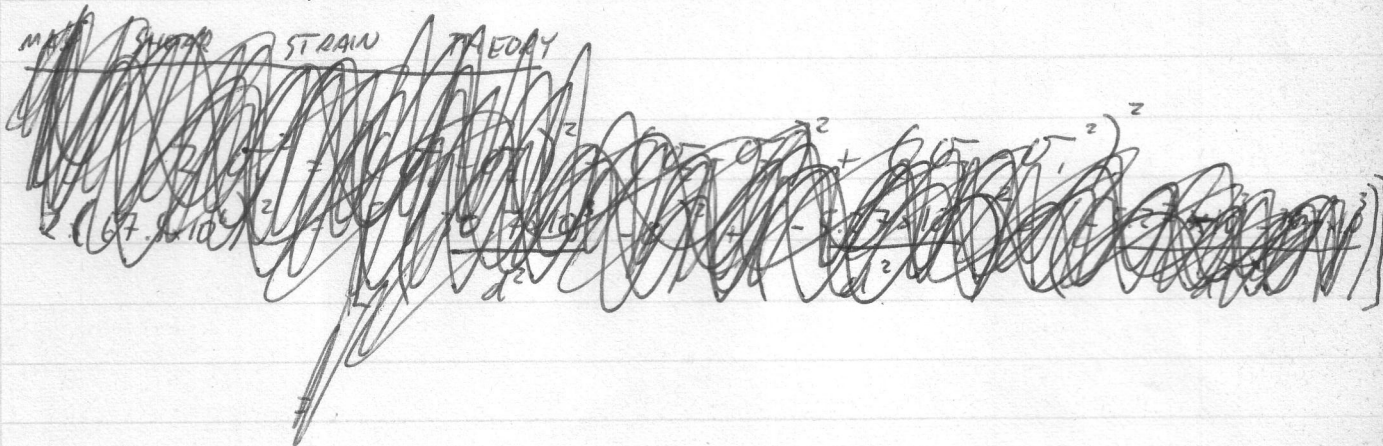
$$i \quad \frac{30.7 \times 10^3}{d^2} = 67.5 \times 10^6$$

$$d^2 = \frac{30.7 \times 10^3}{67.5 \times 10^6}$$

$$= \text{[scribbled out]}$$

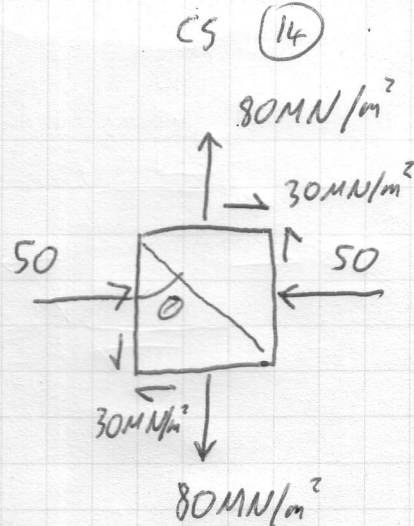
$$\therefore d = 0.0213 \text{ m}$$

$$= \underline{\underline{21.3 \text{ mm}}}$$



13.3

EXAMPLE 3



(a)

$$\sigma_{1,2} = \frac{(\sigma_x + \sigma_y)}{2} \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{1}{2} \left[(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \right]$$

$$= \frac{1}{2} \left[(-50 + 80) \pm \sqrt{(-50 - 80)^2 + 4 \times 30^2} \right]$$

$$= \frac{1}{2} \left(+30 \pm \sqrt{16900 + 3600} \right)$$

$$= \frac{1}{2} \left(30 \pm 143.18 \right)$$

$$\sigma_2 = \underline{\underline{-56.59 \text{ MN/m}^2}}$$

$$\sigma_1 = \underline{\underline{86.55 \text{ MN/m}^2}}$$

(b)

$$\tau_{max} = \frac{1}{2} (\sigma_1 - \sigma_2)$$

$$= \frac{1}{2} (86.55 + 56.59)$$

$$= \underline{\underline{71.6 \text{ MN/m}^2}}$$

$$= \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$= \frac{1}{2} \sqrt{(-50 + 80)^2 + 4 \times 30^2}$$

$$= \underline{\underline{71.6 \text{ MPa}}}$$

(c)

$$\tan \theta_1 = \frac{\sigma_p - \sigma_x}{\tau_{xy}}$$

$$= \frac{86.55 - (-50)}{30}$$

$$= 4.552$$

$$\therefore \theta_1 = \underline{\underline{77.61^\circ}}$$

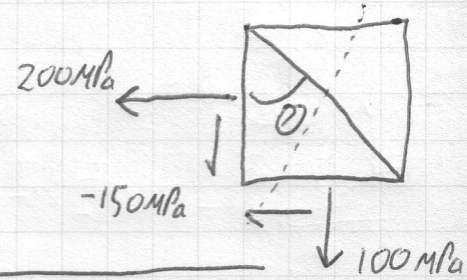
$$\tan \theta_2 = \frac{-56.59 - (-50)}{30}$$

$$= -0.22$$

$$\therefore \theta_2 = -12.4 \text{ or } \underline{\underline{167.6^\circ}}$$

(4)

$$\sigma_{1,2} = \frac{1}{2} \left[(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \right]$$



$$= \frac{1}{2} \left[(200 + 100) \pm \sqrt{(200 - 100)^2 + 4 \times (-150)^2} \right]$$

$$= \frac{1}{2} \left(300 \pm \sqrt{100,000} \right)$$

$$\sigma_1 = \underline{\underline{308.1 \text{ MPa}}}$$

$$\sigma_2 = \underline{\underline{-8.1 \text{ MPa}}}$$

$$\tan \theta_1 = \frac{\sigma_p - \sigma_x}{\tau_{xy}}$$

$$\theta_1 = \tan^{-1} \left(\frac{308.1 - 200}{-150} \right)$$

$$= -35.8^\circ \quad \text{or} \quad \cancel{324.2^\circ} \quad \underline{\underline{54.2^\circ}}$$

$$\therefore \theta_2 = \underline{\underline{144.2^\circ}} \quad (90^\circ \text{ to } \theta_1)$$