



Inferential Statistics



Hypotheses

A hypothesis is a tentative statement about the relationship between two or more variables.

Formulating a Hypothesis

Step 1. Ask a Question

Start with a research question that you want to answer.

This question should be focused, specific, and researchable within the constraints of your project.



Formulating a Hypothesis

Step 2. Do Preliminary Research

Conduct some initial research to understand what is already known about the topic.

This helps you form educated assumptions about what your research will find.



Formulating a Hypothesis

Step 3. Define Variable

Identify the independent and dependent variables in your study.

The independent variable is what you change or control, while the dependent variable is what you observe and measure.



Formulating a Hypothesis

Step 4. Formulate the Hypothesis

Write your initial answer to the research question in a clear, concise sentence.

This hypothesis should be specific, testable, and based on existing theories and knowledge.



Testing the Hypothesis

Step 1. State your Hypotheses

Formulate your research hypothesis as a null hypothesis (H_0) and an alternate hypothesis (H_1).

The null hypothesis predicts no relationship between variables, while the alternate hypothesis predicts a relationship.



Testing the Hypothesis

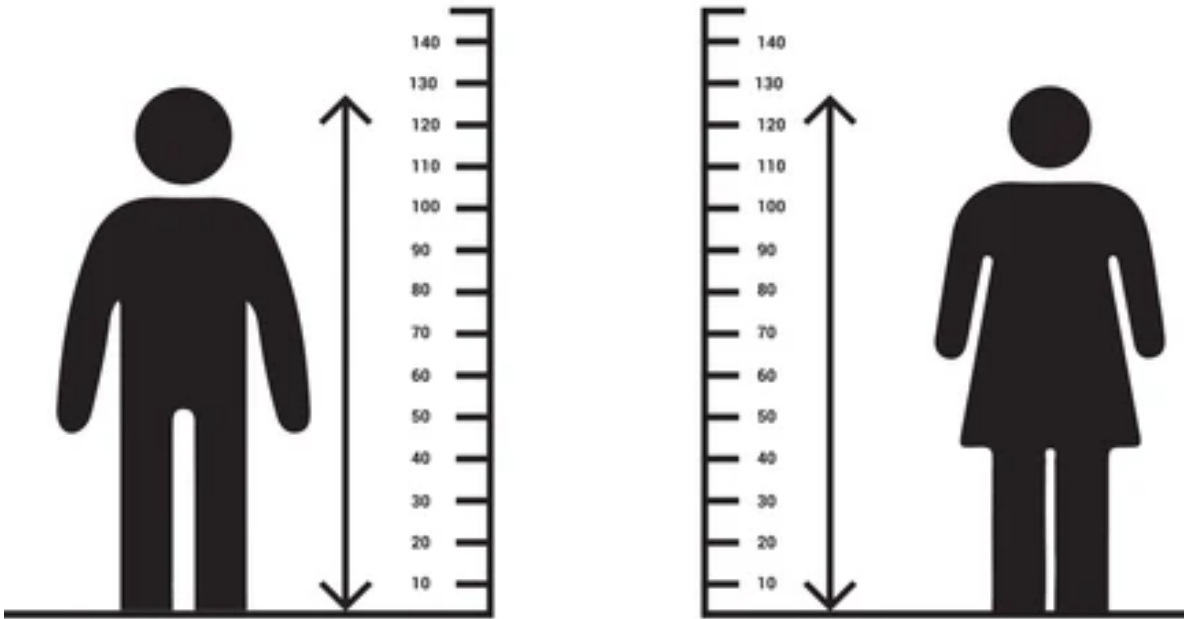
Example:

You want to test whether there is a relationship between gender and height.

Based on your knowledge of human physiology, you formulate a hypothesis that men are, on average, taller than women.

To test this hypothesis, you restate it as:

- H_0 : Men are, on average, not taller than women.
- H_a : Men are, on average, taller than women.



Testing the Hypothesis

Step 2. Collect Data

Gather data through experiments, surveys, or other research methods. Ensure that your data collection methods are reliable and valid.



Testing the Hypothesis

Step 3. Perform a Statistical Test

Use appropriate statistical tests to analyse your data.

Common tests include t-tests, ANOVA, regression analysis, and chi-square tests.

The choice of test depends on the type of data and the research question.



Statistical Tests

The t-test is one of the simplest statistical techniques that is used to evaluate whether there is a statistical difference between the means from up to two different samples.

The t-test is especially useful when you have a small number of sample observations (under 30 or so), and you want to make conclusions about the larger population.



Statistical Tests

Example:

You want to know whether the mean petal length of two flowers differs. You measure 25 petals of each.

The null hypothesis (H_0) is that the true difference between these group means is zero.

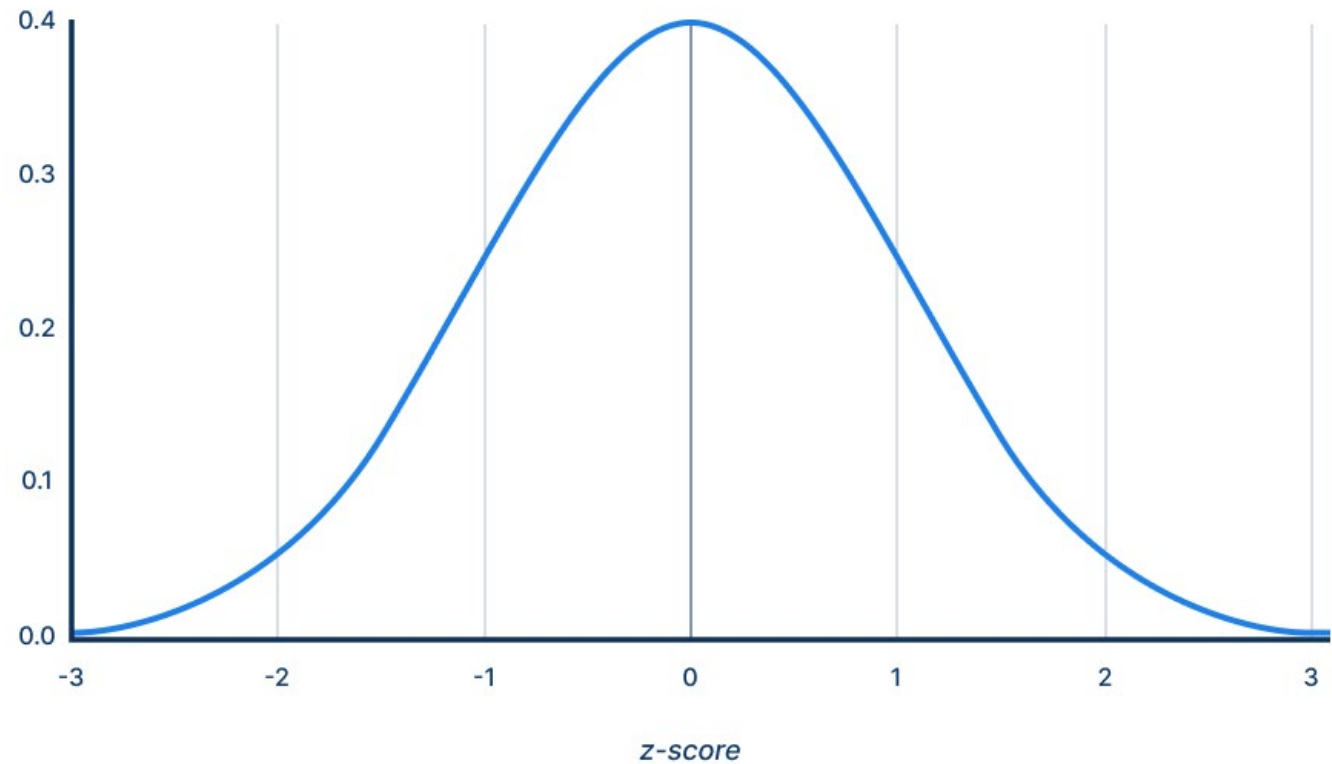
The alternate hypothesis (H_a) is that the true difference is different from zero.



When to use a t-test?

The t-test assumes your data:

- are independent
- are (approximately) normally distributed
- have a similar amount of variance within each group being compared





One-tailed or two-tailed?

If you only care whether the two populations are different from one another, perform a two-tailed t test.

If you want to know whether one population mean is greater than or less than the other, perform a one-tailed t test.

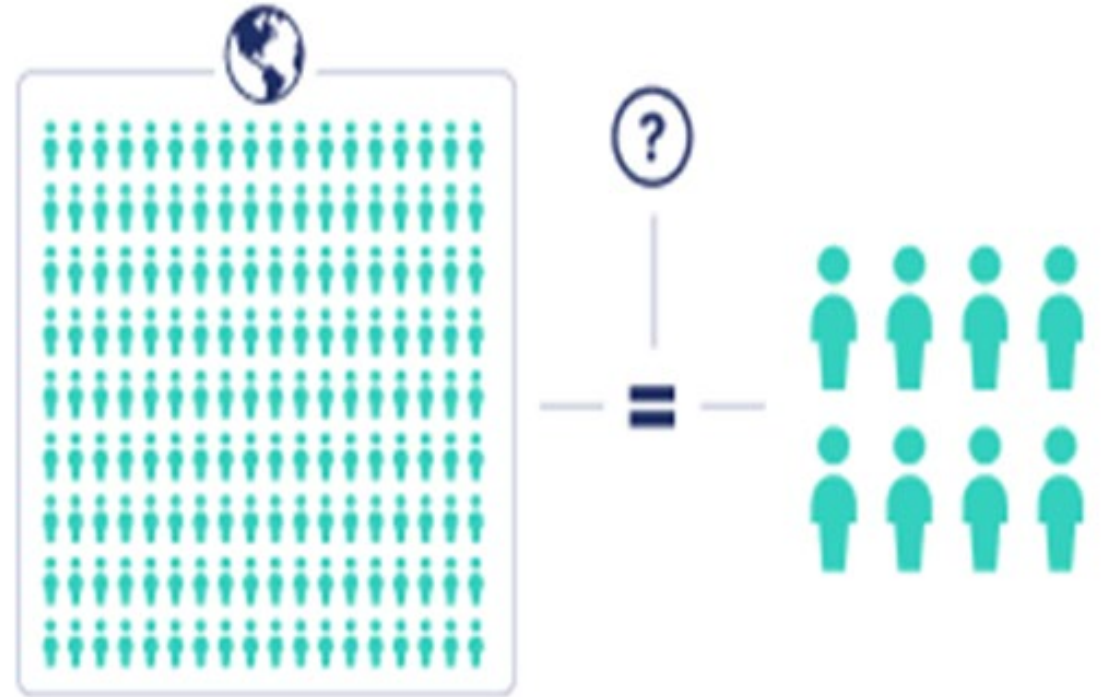
Statistical Tests

One Sample T-Test

You run a test to see if the average of your population is significantly different from a certain number or value.

Appropriate to use when you've collected a small, random sample from some statistical "population" and want to compare the mean from your sample to another value.

The value for comparison could be a fixed value (e.g., 10) or the mean of a second sample.



Investigate whether there's a difference

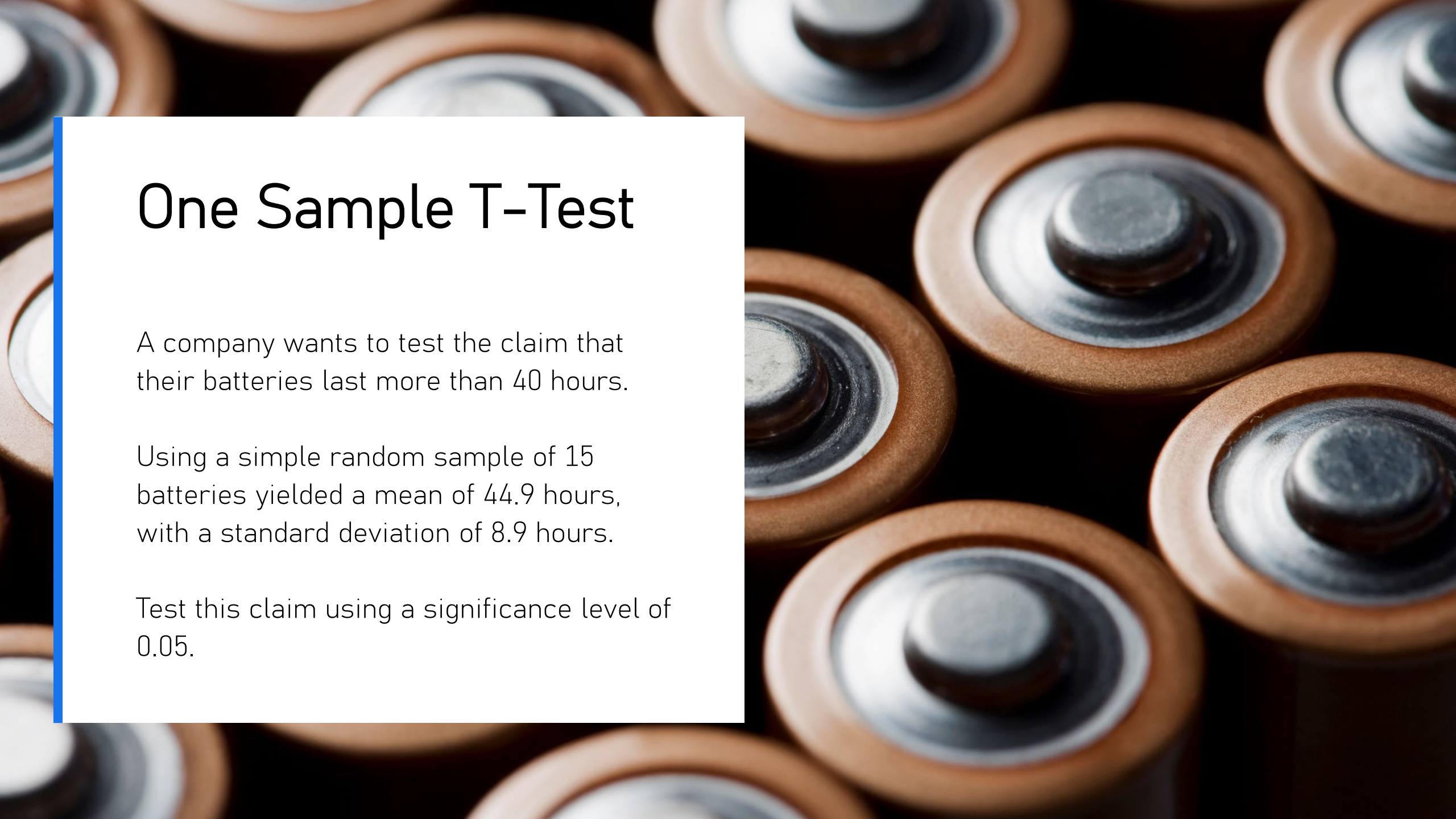


Statistical Tests

One Sample T-Test

$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$$

- $\bar{x}$ – Sample Mean
- μ – Reference Value
- s – Standard Deviation
- n – Number of samples

The background of the slide is a close-up, high-angle photograph of numerous AA batteries. The batteries are arranged in a dense, overlapping pattern, filling the entire frame. They have a characteristic copper-colored outer casing and a silver-colored inner ring with a central positive terminal. The lighting is soft, creating subtle highlights and shadows that emphasize the circular shapes and metallic textures of the batteries.

One Sample T-Test

A company wants to test the claim that their batteries last more than 40 hours.

Using a simple random sample of 15 batteries yielded a mean of 44.9 hours, with a standard deviation of 8.9 hours.

Test this claim using a significance level of 0.05.

One Sample T-Test

Null Hypothesis (H_0):

The mean battery life is less than or equal to 40 hours.

$$H_0: \mu \leq 40$$

Alternative Hypothesis (H_a):

The mean battery life is greater than 40 hours.

$$H_a: \mu > 40$$

One Sample T-Test

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{44.9 - 40}{\frac{8.9}{\sqrt{15}}}$$

$$t = 2.13$$

One Sample T-Test

$$df = n - 1$$

$$df = 15 - 1 = 14$$

Df = Degrees of Freedom

n = Sample Size

Critical values of t for one-tailed tests

Significance level (α)

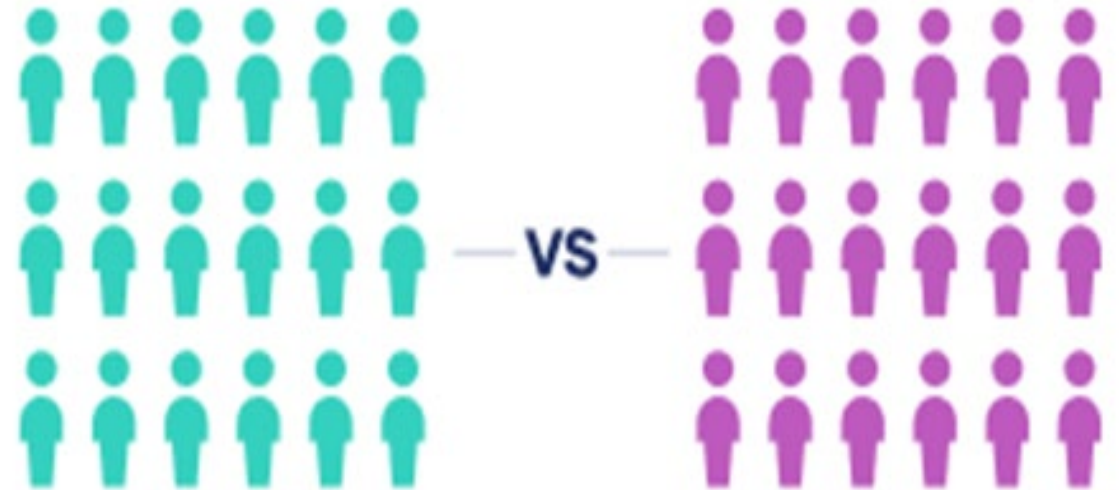
Degrees of freedom (df)	.2	.15	.1	.05	.025	.01	.005	.001
1	1.376	1.963	3.078	6.314	12.706	31.821	63.657	318.309
2	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327
3	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215
4	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173
5	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893
6	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208
7	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785
8	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501
9	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297
10	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144
11	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025
12	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930
13	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852
14	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787
15	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733
16	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686
17	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646
18	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610
19	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579
20	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552
21	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527

Independent-samples t test

Statistical Tests

Independent Samples T-Test

You run a test to see if the average of your population is significantly different from the average of a second population.



Investigate whether there's a difference between two groups (between-subjects).

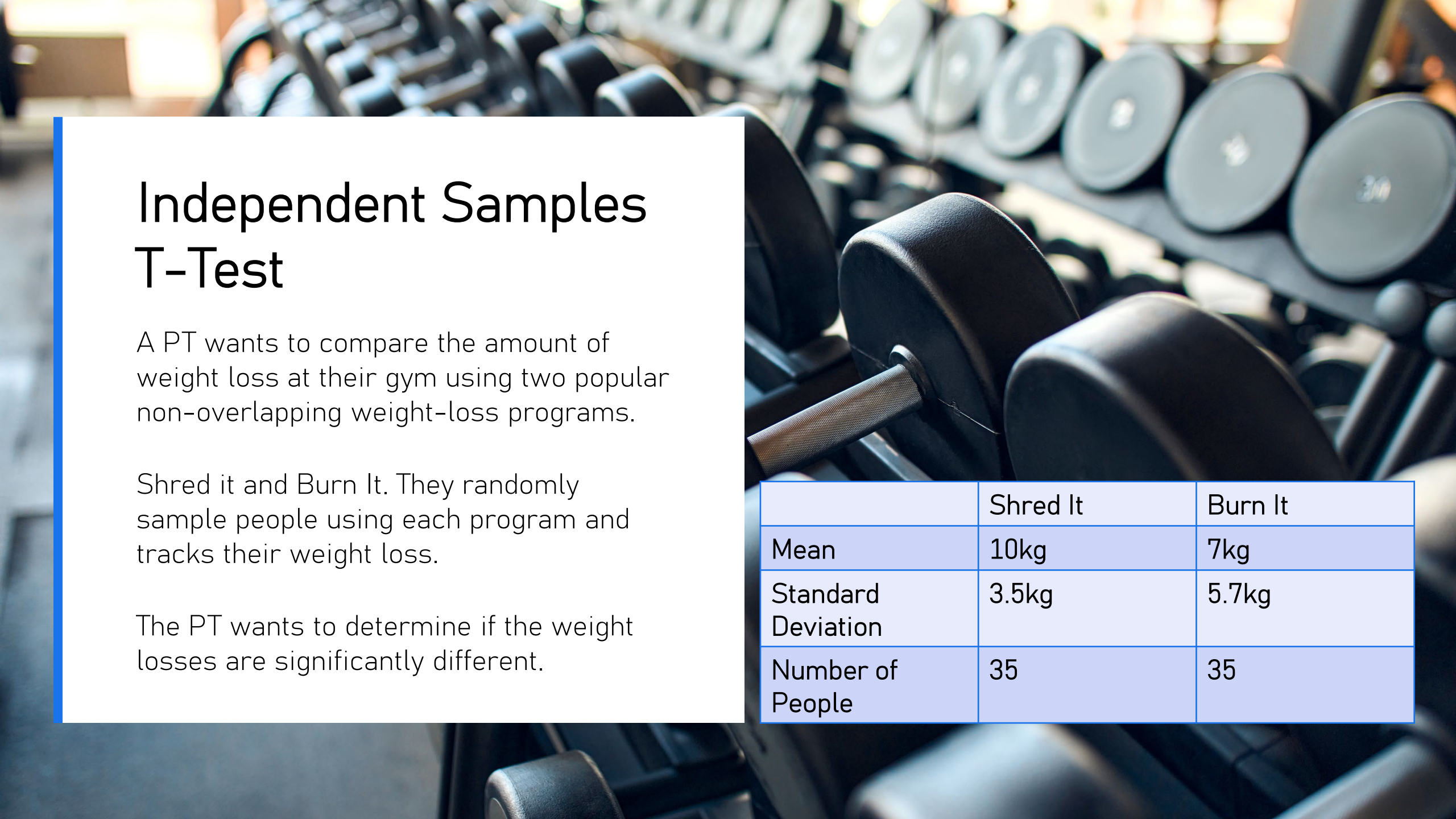


Statistical Tests

Independent Samples T-Test

$$t = \frac{\bar{x}_2 - \bar{x}_1}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

- $\bar{x}_1$ - Sample 1 Mean
- $\bar{x}_2$ - Sample 2 Mean
- s_1 - Standard Deviation Sample 1
- s_2 - Standard Deviation Sample 2
- n_1 - Number of samples 1
- n_2 - Number of samples 2



Independent Samples T-Test

A PT wants to compare the amount of weight loss at their gym using two popular non-overlapping weight-loss programs.

Shred it and Burn It. They randomly sample people using each program and tracks their weight loss.

The PT wants to determine if the weight losses are significantly different.

	Shred It	Burn It
Mean	10kg	7kg
Standard Deviation	3.5kg	5.7kg
Number of People	35	35

Independent Samples T-Test

Null Hypothesis (H_0):

There is no significant difference in the mean weight loss between the two programs.

$$H_0: \mu_1 = \mu_2$$

Alternative Hypothesis (H_a):

There is a significant difference in the mean weight loss between the two program

$$H_a: \mu_1 \neq \mu_2$$

	Shred It	Burn It
Mean	10kg	7kg
Standard Deviation	3.5kg	5.7kg
Number of People	35	35

Independent Samples T-Test

$$t = \frac{\bar{x}_2 - \bar{x}_1}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{10 - 7}{\sqrt{\frac{3.5^2}{35} + \frac{5.7^2}{35}}}$$

$$t = 2.653$$

	Shred It	Burn It
Mean	10kg	7kg
Standard Deviation	3.5kg	5.7kg
Number of People	35	35

Independent Samples T-Test

$$df = n_1 + n_2 - 2$$

$$df = 35 + 35 - 2 = 68$$

Df = Degrees of Freedom

n_1 = Sample 1 Size

n_2 = Sample 2 Size

13	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852
14	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787
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21	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527
22	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505
23	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485
24	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467
25	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450
26	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435
27	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421
28	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408
29	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396
30	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385
40	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307
50	0.849	1.047	1.299	1.676	2.009	2.403	2.678	3.261
60	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232
70	0.847	1.044	1.294	1.667	1.994	2.381	2.648	3.211
80	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195
100	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174
1000	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098
Infinite	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090

Paired-samples t test

Statistical Tests

Paired Samples T-Test

If the groups come from a single population (e.g., measuring before and after an experimental treatment), perform a paired t test



Investigate whether there's a difference within a group between two points in time



Statistical Tests

Paired Samples T-Test

$$t = \frac{\bar{X}_d - 0}{\frac{s}{\sqrt{n}}}$$

- $\bar{x}_d$ - Mean Sample Difference
- s - Standard Deviation
- n - Number of Samples

Paired Samples T-Test

A construction company has introduced a H&S training scheme, they want to see if this has reduced the number of incidents on sites.

	No of Incident Before	No of Incidents After	Difference
Site 1	10	7	3
Site 2	4	4	0
Site 3	8	9	-1
Site 4	6	4	2
Site 5	12	9	3
Site 6	5	4	1

Paired Samples T-Test

Null Hypothesis (H_0):

The H&S training has not reduced the number of incidents.

$$H_0: \mu_{after} \geq \mu_{before}$$

Alternative Hypothesis (H_a):

The H&S training has reduced the number of incidents.

$$H_a: \mu_{after} < \mu_{before}$$

	No of Incident Before	No of Incidents After	Difference
Site 1	10	7	3
Site 2	4	4	0
Site 3	8	9	-1
Site 4	6	4	2
Site 5	12	9	3
Site 6	5	4	1

Paired Samples T-Test

$$t = \frac{\bar{x}_d - 0}{\frac{s}{\sqrt{n}}} = \frac{1.333}{\frac{1.633}{\sqrt{6}}}$$

$$t = 1.995$$

	No of Incident Before	No of Incidents After	Difference
Site 1	10	7	3
Site 2	4	4	0
Site 3	8	9	-1
Site 4	6	4	2
Site 5	12	9	3
Site 6	5	4	1

Paired Samples T-Test

$$df = n - 1$$

$$df = 6 - 1 = 5$$

Df = Degrees of Freedom

n = Sample Size

Critical values of t for one-tailed tests

Significance level (α)

Degrees of freedom (df)	.2	.15	.1	.05	.025	.01	.005	.001
1	1.376	1.963	3.078	6.314	12.706	31.821	63.657	318.309
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21	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527



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