

INTERNAL RATE OF RETURN (IRR)

Internal rate of return (IRR) calculations are in no sense a 'fourth' criterion for investment appraisal, but simply another way of employing the DCF technique which can be extremely useful in particular circumstances.

Consider the following.

A machine will cost £80,000 immediately, and will generate cash inflows of:

<i>Time</i>	£
1	40,000
2	50,000
3	10,000

If the cost of capital is 10%, should we accept or reject the project?

<i>Time</i>	£	<i>DF @ 10%</i>	<i>PV</i> £
0	(80,000)	1	(80,000)
1	40,000	0.909	36,360
2	50,000	0.826	41,300
3	10,000	0.751	7,510
			NPV = 5,170

Since the net present value is positive, we should **accept** the project.

However, since our calculation is based on forecasts it could easily be the case that in retrospect the NPV is negative and that we should have rejected. One element that could obviously be incorrect is the cost of capital that we have used of 10%. This could affect our decision in two ways:

Firstly, it may not stay at 10% throughout the life of the investment and secondly, the calculation of the current rate being 10% may be suspect. In either case, the NPV would stand to alter and if ever it ended up being negative we would have made the wrong decision. At first glance, the first problem would appear the most important. However, we will assume for the time being that the cost of capital will remain constant over the investment's life.

The second problem is, in practice, a very severe difficulty. For an individual, the cost of borrowing is easy to identify, but a company has several sources of finance – shareholders' funds, debentures, bank borrowing – and the exact cost can be virtually impossible to calculate.

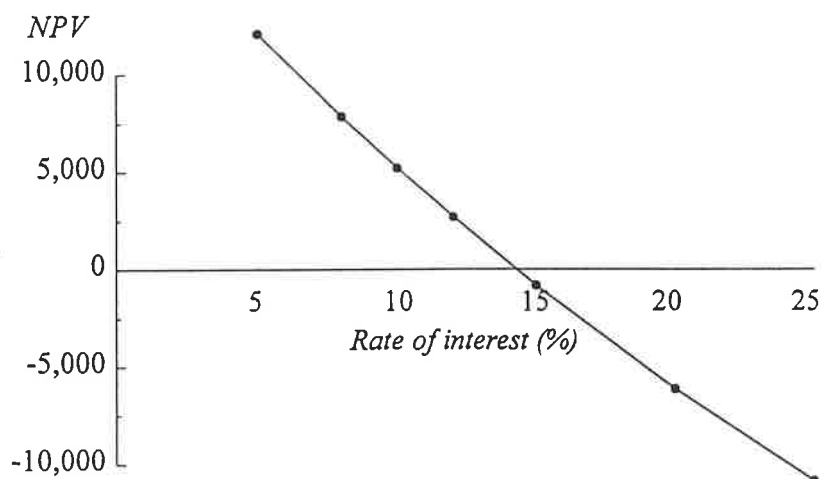
On the basis of the above, we may therefore be in the position where all we can say is that the cost of capital is about 10%. At exactly 10% the NPV is positive and we accept, but what if the cost were actually 10.5% or 11% – would the NPV still be positive, and would we still be correct in accepting? One approach to deal with this problem would be to calculate the NPV at various rates of interest and investigate the results.

For the above example, the NPV at various rates of interest is as follows:

	£
5%	12,070
8%	7,830
10%	5,170
12%	2,690
15%	(820)
20%	(6,190)

These have been calculated in the normal way, and, as one intuitively would expect, the project becomes less worthwhile as the cost of money increases.

If we plot these on a graph:



From the graph, we can now identify the ranges of interest for which the NPV will be positive and negative and hence for which we should accept or reject the project.

Since the NPV will be zero for a rate of interest of approximately 14%, we can say that for a cost of capital of up to 14% we should accept the project, whereas if the cost of capital exceeds 14% we should reject the project.

We have now an idea of our margin for error, as a result of this exercise. The break-even rate of 14% is known as the internal rate of return, for which the definition is:

The IRR is the rate of interest that discounts the project flows to a net present value of zero.

Although the above approach is really the only way of finding the exact internal rate of return, it is obviously time-consuming and, as a result, we generally use a quick approach to arrive at an approximation. Very rarely would we need a precise figure for the IRR and normally a reasonable approximation will be quite sufficient.

Interpolation

This approach involves calculating the NPV for two different rates of interest and then interpolating linearly between the two to estimate the rate of interest that gives a net present value of zero. Using the same example as before:

at 10% the net present value is + £5,170

10% is clearly not the IRR and, since the NPV at 10% is positive, the IRR must in fact be greater than 10%.

We therefore make a second guess at a rate higher than 10% – eg. 15%

at 15% the net present value is – £820

We now know that the IRR lies certainly between 10% and 15%. We need to estimate the rate that will yield zero.

Over a range of 5% (ie. from 10% to 15%) the net present value has fallen by £5,990 (ie. from + £5,170 to –£820).

We therefore assume that £5,990 equates to 5% and that there is a £1 fall in NPV for every $\frac{5}{5,990}\%$ change in the rate of interest.

If we take 10% as our starting point – for which the NPV is £5,170 – we need a change of £5,170 to get to a NPV of zero. This will require a change in the rate of interest of:

$$5,170 \times \frac{5\%}{5,990} \text{ or } \frac{5,170}{5,990} \times 5\%$$

The internal rate of return is therefore:

$$10\% + \left(\frac{5,170}{5,990} \times 5\% \right) = 14.32\%$$

It is of vital importance to appreciate that this is only an approximation to the IRR since we have assumed that the NPV changes linearly with the rate of interest, which we already know from the earlier graph is not, in fact, the case.

Generally, however, as already stated, we do not require a precise IRR and it is normally quite sufficient to be able to say that it is about 14%. Two further points should be noted about this approach:

- (1) It is rather ludicrous to state that the IRR is approximately 14.32% (to two decimal places). In the examination, normally leave it at 14.32% so that your calculations may be checked, but do make the point that this is only an approximation since we have assumed a linearity that does not exist.
- (2) As a result of the above, a different pair of guesses would yield a different IRR. Again, using the same example.

If we had used guesses of 10% and 20% we would have arrived at the following:

$$\begin{aligned} \text{at 10\% NPV} &= +£5,170 \\ \text{at 20\% NPV} &= -£6,190 \end{aligned}$$

Therefore over a range of 10% (10% to 20%) the NPV has fallen by £11,360 (+£5,170 to –£6,190).

$$\text{Therefore IRR} = 10\% + \left(\frac{5,170}{11,360} \times 10\% \right) = 14.55\%$$

The closer the two guesses are to the true IRR, the better the approximation will be. Do not, however, worry over this point in the examination since credit will be given for any two guesses provided they are not ludicrous (for instance do not use 0% as a guess!).

The above calculation may be expressed as a formula:

$$IRR = R_1 + \left[\frac{N_1}{N_1 - N_2} \times (R_2 - R_1) \right]$$

where R_1 , N_1 and R_2 , N_2 are the rate of interest and NPV for the first and second guesses respectively. However, you will not be provided with this formula in the examination and (except in desperation) it would seem rather unnecessary to learn this formula for its own sake.