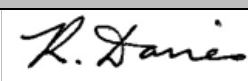


EXAMINATION INTERNAL VERIFICATION FORM (2017-2018)

MODULE :	PSDE5008 Further Engineering Mathematics	LEVEL:	5
PROGRAMME :	HNC Electrical and Electronic Engineering	TERM OF EXAM: (SPRING OR SUMMER)	n/a
LEARNING OUTCOME(S) BEING ASSESSED:	MILO1, MILO 2 (partially covered)	EXAM PAPER:	Main Class Test 2
		EXAMINATION AUTHOR :	A. Thomas

VERSION:	DATE SUBMITTED FOR VERIFICATION:	INTERNAL VERIFIER COMMENTS (Approval or detail of amendments required):	DATE:
1.	16-02-18	Conforms to the assessment pattern and tests the learning outcomes appropriately.	06-03-18
2.			

INTERNAL VERIFIER PLEASE TICK TO INDICATE THAT THE EXAMINATION:	
☒	Conforms to the assessment pattern described in the module syllabus
☒	Tests the module learning outcome(s) appropriately

SIGNATURES OF FINAL APPROVAL:	SIGNATURE	DATE
INTERNAL VERIFIER: I confirm that this version of the exam paper is ready to be sent to the external examiner		06-03-18
EXAM AUTHOR: I confirm that internal verifier and external examiner approval have been received and that this version of the exam paper is ready for printing.		

HNC Electrical and Electronic Engineering
PSDE5008 Further Engineering Mathematics
Level 5, Year 2

Class Test 2 (2017-18)

This is a closed book test, worth **45%** of the module mark and addresses Module Intended Learning Outcomes 1 & 2.

Date of test =
Time allowed = 1.5 hour

Answer All Questions

1. Find the Fourier Transform of the function

$$V(t) = 7e^{4t} \quad \text{for } t < 0$$
$$V(t) = 7e^{-4t} \quad \text{for } t > 0$$

(35 Marks)

2. State whether the following functions have odd, even or no symmetry, showing how you arrived at the decision for full marks:

- (i) $6\sin x + 2$
- (ii) $x^7 - 3x$
- (iii) $5 - 3\cos x$
- (iv) x^3
- (v) $\sin^2(x)$

(25 Marks)

3. Sketch the graph and determine the Fourier Series expansion of the function

$$f(x) = -6x \quad (-\pi < x < 0)$$
$$f(x) = 6x \quad (0 < x < \pi) \text{ and } f(x + 2\pi) = f(x)$$

and find the first 4 non-zero terms of the series.

(40 Marks)

PSDE5008 Test 2

Formulae

Fourier Series

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

where

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) dx$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx \, dx$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx$$

Integration by parts'

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Fourier Transform

Here, the Fourier transform of $f(x)$, denoted by $\hat{f}(\omega)$, is given by:

$$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-j\omega x} dx$$

Tables of Integrals and Derivatives

y	$y' \left(= \frac{dy}{dx} \right)$	$\int y dx$
$\sin(kx)$	$k \cos(kx)$	$-\left(\frac{1}{k}\right) \cos(kx) + c$
$\cos(kx)$	$-k \sin(kx)$	$\left(\frac{1}{k}\right) \sin(kx) + c$
e^{kx}	ke^{kx}	$\left(\frac{1}{k}\right) e^{kx} + c$

Laplace transforms

Function, $f(t)$	Laplace transform,
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^2	$\frac{2}{s^3}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
e^{-at}	$\frac{1}{s+a}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\sin bt$	$\frac{b}{s^2+b^2}$
$\cos bt$	$\frac{s}{s^2+b^2}$
$e^{-at} \sin bt$	$\frac{b}{(s+a)^2+b^2}$
$e^{-at} \cos bt$	$\frac{s+a}{(s+a)^2+b^2}$

$$L\{x\} = \bar{x}$$

$$L\{\dot{x}\} = s\bar{x} - x_0$$

$$L\{\ddot{x}\} = s^2\bar{x} - sx_0 - \dot{x}_1$$

R-L Circuit

$$E = RI + L \frac{dI}{dt}$$

R-C Circuit

$$E(t) = RI + \frac{1}{C} \int I dt$$

RLC Circuit

$$E = RI + L \frac{dI}{dt} + \frac{1}{C} \int I dt$$

Standard derivatives

y	$\frac{dy}{dx}$
ax^n	nax^{n-1}
$a \sin(kx)$	$ka \cos(kx)$
$a \cos(kx)$	$-ka \sin(kx)$
ae^{kx}	kae^{kx}
$a \ln x$	$\frac{a}{x}$

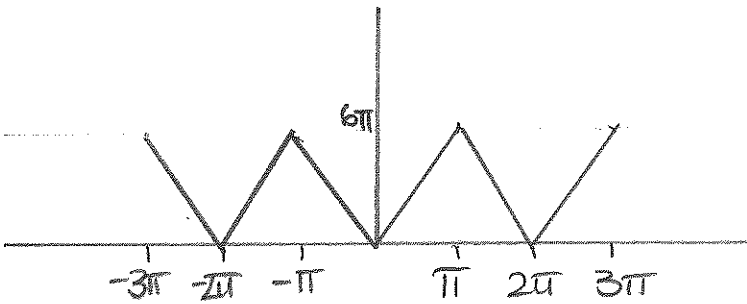
Standard integrals

$\int ax^n dx$	$\frac{ax^{n+1}}{n+1} + c$ (except when $n = -1$)
$\int \cos ax dx$	$\frac{1}{a} \sin ax + c$
$\int \sin ax dx$	$-\frac{1}{a} \cos ax + c$
$\int e^{ax} dx$	$\frac{1}{a} e^{ax} + c$
$\int \frac{1}{x} dx$ or $\int x^{-1} dx$	$\ln x + c$
$\int \frac{1}{ax+b} dx$	$\frac{1}{a} \ln(ax+b) + c$

Test 2 Solutions/Mark Scheme

						Qu total	
1.	$v(t) = 7e^{4t} \quad t < 0$ $v(t) = 7e^{-4t} \quad t > 0$ $\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} v(t) e^{-j\omega t} dt$ $= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^0 7e^{4t} \cdot e^{-j\omega t} dt + \frac{1}{\sqrt{2\pi}} \int_0^{\infty} 7e^{-4t} \cdot e^{-j\omega t} dt$ $= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^0 7e^{(4-j\omega)t} dt + \frac{1}{\sqrt{2\pi}} \int_0^{\infty} 7e^{-(4+j\omega)t} dt$ $= \frac{1}{\sqrt{2\pi}} \left[\frac{7}{4-j\omega} e^{(4-j\omega)t} \right]_{-\infty}^0 + \frac{1}{\sqrt{2\pi}} \left[\frac{7}{-(4+j\omega)} e^{-(4+j\omega)t} \right]_0^{\infty}$ $= \frac{1}{\sqrt{2\pi}} \left[\frac{7}{4-j\omega} e^0 - 0 \right] + \frac{1}{\sqrt{2\pi}} \left[0 + \frac{7}{(4+j\omega)} e^0 \right]$ $= \frac{1}{\sqrt{2\pi}} \left[\frac{7}{4-j\omega} + \frac{7}{4+j\omega} \right]$ $= \frac{1}{\sqrt{2\pi}} \left[\frac{7(4+j\omega) + 7(4-j\omega)}{(4-j\omega)(4+j\omega)} \right]$ $= \frac{1}{\sqrt{2\pi}} \left[\frac{28 + 7j\omega + 28 - 7j\omega}{16 + 4j\omega - 4j\omega - j^2\omega^2} \right]$ $= \frac{1}{\sqrt{2\pi}} \left[\frac{56}{16 + \omega^2} \right]$	5	3	5	5	4	5
						35	

Test 2 Solutions/Mark Scheme

				Qu total
2(i)	$6 \sin x + 2$ $6 \sin(30^\circ) + 2 = 5$ $6 \sin(-30^\circ) + 2 = -1$	NEITHER	5	
(ii)	$x^7 - 3x$ $2^7 - 3(2) = 122$ $(-2)^7 - 3(-2) = -122$	ODD	5	
(iii)	$5 - 3 \cos x$ $5 - 3 \cos(30^\circ) = 2.402$ $5 - 3 \cos(-30^\circ) = 2.402$	EVEN	5	
(iv)	x^3 $2^3 = 8$ $(-2)^3 = -8$	ODD	5	
(v)	$\sin^2(x)$ $\sin^2(30^\circ) = 0.25$ $\sin^2(-30^\circ) = 0.25$	EVEN	5	25
3.	 Even function $\therefore b_n = 0$		4	
			1	

Test 2 Solutions/Mark Scheme

			Qu total
$a_0 = \frac{2}{\pi} \int_0^{\pi} 6x \, dx$ $= \frac{2}{\pi} \left[\frac{6x^2}{2} \right]_0^{\pi}$ $= \frac{2}{\pi} [3\pi^2 - 0]$ $a_0 = 6\pi$	3		
$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx \, dx$ $= \frac{2}{\pi} \int_0^{\pi} 6x \cos nx \, dx$	2		
Let $u = 6x$ $\frac{dv}{dx} = \cos nx$ $\frac{du}{dx} = 6$ $v = \frac{1}{n} \sin nx$	4		
By parts			
$\int 6x \cos nx \, dx = 6x \left(\frac{1}{n} \sin nx \right) -$ $\int \frac{1}{n} \sin nx \cdot 6 \, dx$	4		
$= \frac{6x}{n} \sin nx + \frac{6}{n^2} \cos nx$	4		
$\therefore a_n = \frac{2}{\pi} \left[\frac{6x}{n} \sin nx + \frac{6}{n^2} \cos nx \right]_0^{\pi}$	2		

Test 2 Solutions/Mark Scheme

			Qu total
$a_n = \frac{2}{\pi} \left[\frac{6\pi}{n} \sin n\pi + \frac{6}{n^2} \cos(n\pi) \right]$ $- \frac{2}{\pi} \left[\frac{6(0)}{n} \sin 0 + \frac{6}{n^2} \cos 0 \right]$ $= \frac{2}{\pi} \left[\frac{6}{n^2} \cos(n\pi) \right] - \frac{2}{\pi} \left[\frac{6}{n^2} \right]$ $a_n = \frac{12}{\pi n^2} [\cos n\pi - 1]$ $n=1 \quad a_1 = \frac{12}{\pi(1)^2} [\cos \pi - 1] = -\frac{24}{\pi(1)^2}$ $n=2 \quad a_2 = \frac{12}{\pi(2)^2} [\cos 2\pi - 1] = 0$ $n=3, \quad a_3 = \frac{12}{\pi(3)^2} [\cos 3\pi - 1] = -\frac{24}{\pi(3)^2}$ $a_4 = 0 \quad \text{etc.}$ $\therefore f(x) = \frac{1}{2}(6\pi) - \frac{24}{\pi(1)^2} \cos x$ $- \frac{24}{\pi(3)^2} \cos 3x - \frac{24}{\pi(5)^2} \cos 5x \dots$ $= 3\pi - \frac{24}{\pi} \cos x - \frac{24}{9\pi} \cos 3x$ $- \frac{24}{25\pi} \cos 5x \dots$	4		
	3		
	4		
	5		
			40

100

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