

TYPES OF ATTRIBUTE CONTROL CHARTS

There are FOUR principle types of attribute control charts available.

1. The np chart. The number defective chart.

This monitors the **number** of defective items found in each sample. The sample must be of **constant** size.

2. The p chart. The proportion defective chart.

This monitors the **proportion** of defective items found in each sample. The sample does **not** have to be of constant size.

3. The c chart. The number of defects chart.

This monitors the **number** of defects found, where more than one defect can be found in each unit. The sample must be of **constant** size.

4. The u chart. The number of defects per unit chart.

This monitors the **number** of defects found **per inspection unit**. The number of units inspected can vary.

All are based on the principle of determining the **mean** value of defective items or defects produced, and setting control limits at 3 standard deviations **above** and **below** this.

NOMENCLATURE:-

P	=	Percentage defective	e.g. 2%.
p	=	Proportion defective	e.g. 0.02
n	=	Sample size	
m	=	Number of samples	
c	=	np = Number of defectives or defects	
u	=	c/n = defects per unit	

The formulae for mean and standard deviation are summarised below.

DEFECTIVES

i. SAMPLE SIZE CONSTANT

np chart for number defective

$$\text{Mean} = \bar{c} = n\bar{p}, \quad \sigma = \sqrt{\bar{c} \left[1 - \frac{\bar{c}}{n} \right]}$$

ii. SAMPLE SIZE VARIES

P chart for percentage defective

$$\text{Mean} = \bar{P} \text{ in } \%, \quad \sigma = \sqrt{\frac{\bar{P}(100 - \bar{P})}{\bar{n}}}$$

p chart for proportion defective

$$\text{Mean} = \bar{p} \text{ as proportion}, \quad \sigma = \sqrt{\frac{\bar{p}(1 - \bar{p})}{\bar{n}}}$$

DEFECTS

i. SAMPLE SIZE CONSTANT

c chart for number of defects

$$\text{Mean} = \bar{c}, \quad \sigma = \sqrt{\bar{c}}$$

ii. SAMPLE SIZE VARIES

u chart for defects per item

$$\text{Mean} = \bar{u}, \quad \sigma = \sqrt{\frac{\bar{u}}{\bar{n}}}$$

The rules for process control on all the charts are the same as for variable charts:-

ACTION

On Special Causes

Any point outside the control limits.

A run of 7 points, all ABOVE the central line.

A run of 7 points, all BELOW the central line.

A run of 7 points going UP (each point equal to or greater than the point before).

A run of 7 points going DOWN (each point less than or equal to the point before).

An obviously non-random pattern.

INTERPRETATION FOR CONTROL CHARTS FOR ATTRIBUTES

These are based on process capability, that is the past history of the process. The aim is to improve the process, which means incorporating any causes that move the process data downwards and eliminating causes that move it upwards.

The ultimate objective is to produce zero defects, but it is necessary to move forward in stages. Attribute charts can be used to examine the effect of process changes and to monitor these stages.

PROCESS CAPABILITY

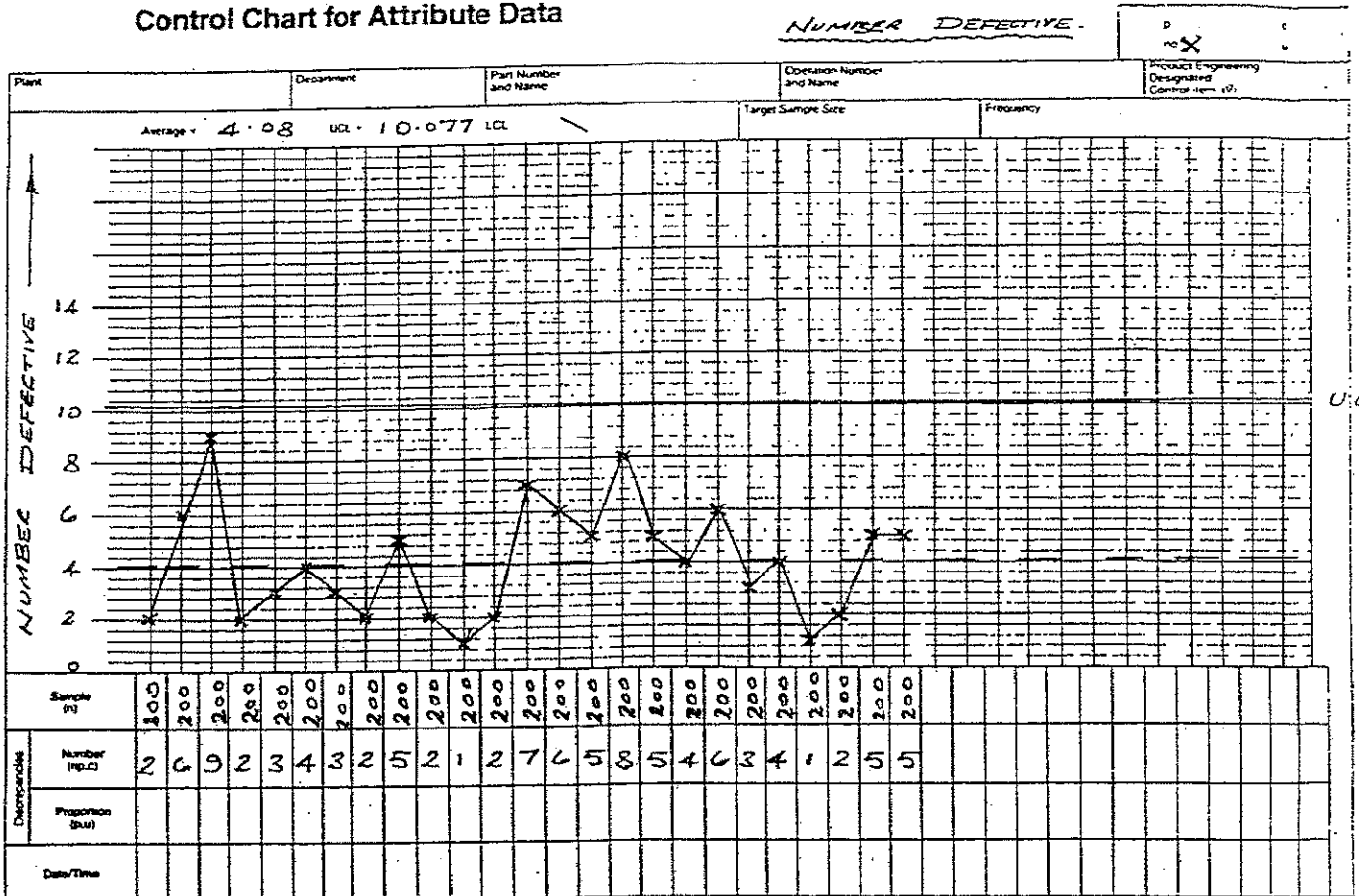
Capability for p and np charts is expressed as the proportion conforming to specification.

$$\text{Capability} = 1 - \bar{p}$$

For c and u charts $\bar{c}$ and $\bar{u}$ are used, but as measures of process performance.

Control Chart for Attribute Data

NUMBER DEFECTIVE



Any Change in people, materials, equipment, methods or environment should be noted. These notes will help you to take corrective or process improvement action when signalled by the control chart.

n p chart NUMBER DEFECTIVE

$$\text{Mean, } n\bar{p} = \bar{c} = \frac{\text{Total number of defectives}}{\text{Number of samples}}$$

$$= \frac{102}{25} = 4.08$$

$$\text{Standard deviation, } \sigma = \sqrt{\bar{c} \left(1 - \frac{\bar{c}}{n}\right)}$$

$$= \sqrt{4.08 \left(1 - \frac{4.08}{200}\right)}$$

$$= 1.999$$

Control limits, $\bar{c} \pm 3\sigma$

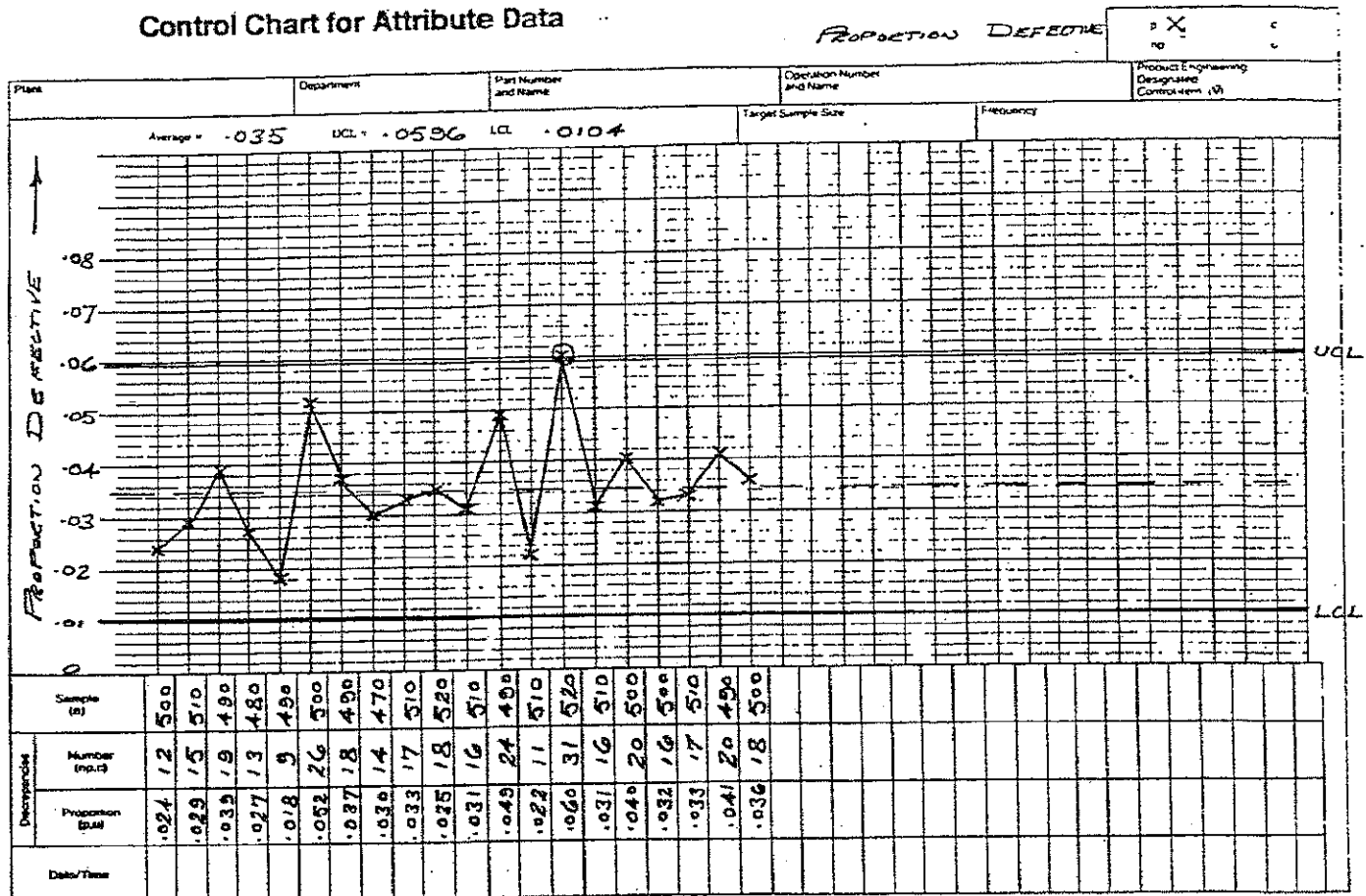
$$\text{UCL, } 4.08 + 3(1.999) = 10.077$$

$$\text{LCL, } 4.08 - 3(1.999) = -1.917$$

i.e. no lower control limit

Control Chart for Attribute Data

PROPORTION DEFECTIVE



Any Change in people, materials, equipment, methods or environment should be noted. These notes will help you to take corrective or process improvement action when signalled by the control chart.

p chart - PROPORTION DEFECTIVE

Calculations:-

For each sample, the proportion defective $p = \frac{np}{n}$

e.g. sample 1 $p = \frac{12}{500} = 0.024$

Mean $\bar{p} = \frac{\text{Total number of defectives}}{\text{Total number of samples}}$

$\bar{p} = \frac{12+15+13+\dots+18}{500+510+490+\dots+500} = \frac{350}{10,000} = 0.035$

Average sample size $\bar{n} = \frac{\text{Total number sampled}}{\text{Number of samples}}$

$\bar{n} = \frac{10,000}{20} = 500$

Standard deviation $\sigma = \sqrt{\left[\bar{p} \frac{(1-\bar{p})}{\bar{n}} \right]}$

$\sigma = \sqrt{\left[\frac{(0.035)(1-0.035)}{500} \right]} = 0.0082$

Control limits, $\bar{p} \pm 3\sigma$

UCL, $0.035 + 0.0246 = 0.0596$

LCL, $0.035 - 0.0246 = 0.0104$

P chart PERCENTAGE DEFECTIVE

Calculations:-

For each sample the percentage defective $P = \frac{n_p}{n} \times 100$

e.g. sample 1 $P = \frac{12}{500} \times 100 = 2.4\%$

$$\text{Mean } \bar{P} = \frac{\text{Total number of defectives}}{\text{Total number sampled}} \times 100$$

$$\bar{P} = \frac{12 + 15 + 19 \dots + 18}{500 + 510 + 490 \dots + 500} \times 100 = \frac{350 \times 100}{10,000} = 3.5\%$$

$$\text{Average sample size, } \bar{n} = \frac{\text{Total number sampled}}{\text{Number of samples}}$$

$$\bar{n} = \frac{10,000}{20} = 500$$

$$\begin{aligned} \text{Standard deviation } \sigma &= \sqrt{\left[\frac{\bar{P}(100 - \bar{P})}{\bar{n}} \right]} \\ &= \sqrt{\left[\frac{3.5(100 - 3.5)}{500} \right]} = 0.82\% \end{aligned}$$

Control limits, $\bar{P} \pm 3\sigma$

$$\text{UCL, } 3.5 + 2.46 = 5.96\%$$

$$\text{LCL, } 3.5 - 2.46 = 1.04\%$$

QUESTION 7

- a) Percentage defective charting (p chart) is used to monitor certain criteria.

From the results of a number of samples the chart was set up using the following data:

mean proportion of rejects ($\bar{p}$) 0.016
mean sample size ($\bar{n}$) 108

control lines are set using UWL at $\bar{p} + 2\sqrt{\bar{p}(1-\bar{p})/\bar{n}}$

and UAL at $\bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/\bar{n}}$ 9 Marks

- (i) determine the control limits and draw the chart to scale.
(ii) the next six samples taken gave the following data:

Number in sample	95	76	120	145	106	80
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Number of rejects	4	3	4	6	4	2
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Plot the proportion defective on the chart, comment on the state of the process. NOTE: the control lines can remain static provided the sample size varies from the average size ($\bar{n}$) by no more than $\pm 25\%$.

9 Marks

- b) Explain the difference between: control charts for number of defects (c) and charts for number of defects per unit (u).

2 Marks

DATA SHEET

Tables of Constants and Formulae

$\bar{X}$ and R Charts *				
Subgroup Size n	Chart for Averages ($\bar{X}$)	Chart for Ranges (R)		
	Factors for Control Limits	Divisors for Estimate of Standard Deviation	Factors for Control Limits	
	A_2	d_2	D_3	D_4
2	1.880	1.128	—	3.267
3	1.023	1.693	—	2.574
4	0.729	2.059	—	2.282
5	0.577	2.326	—	2.114
6	0.483	2.534	—	2.004
7	0.419	2.704	0.076	1.924
8	0.373	2.847	0.136	1.864
9	0.337	2.970	0.184	1.816
10	0.308	3.078	0.223	1.777

$$UCL\bar{X}, LCL\bar{X} = \bar{\bar{X}} \pm A_2\bar{R}$$

$$UCLR = D_4\bar{R}$$

$$LCLR = D_3\bar{R}$$

$$\hat{\sigma} = \bar{R}/d_2$$

Cp		Division	Department	Week no.	
Cpk					
Measuring equipment		Unit of measurement		Sampling frequency	
		Zero			
MEAN $\bar{X}$ RANGE R					
				Time	
				Date	
				X ₁	
				X ₂	
				X ₃	
				X ₄	
				X ₅	
				ΣX	
				$\bar{X}$	
				R	

