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***Reverse-Engineering Tyre Analysis of Sport 2000 MCR #40***

***Race car***

***Master's project (ACHA7003)***

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# Abstract

This dissertation presents a staged reverse-engineering methodology developed and validated in MATLAB using telemetry and vehicle-setup data from the MCR #40 Sports 2000 race car, a prototype developed and operated at UWTSD by academic staff, technicians and students. The primary objective was to determine whether vehicle telemetry could be used to reconstruct effective vehicle and tyre-force characteristics without direct tyre-force measurements. Starting from the vehicle configuration and telemetry recorded during FP1 at Silverstone National Circuit, the methodology reconstructs suspension behaviour, individual vertical tyre loads, vehicle path, vehicle sideslip angle, axle lateral forces, individual-wheel slip angles and corresponding lateral tyre-force relationships. These reconstructed quantities are subsequently used to identify reduced Pacejka Magic Formula representations for the front and rear axles.

The methodology was then applied to the remaining Silverstone National race-weekend sessions—FP2, FP3, FP4, Qualifying, Race 1, Race 2 and Race 3. Leave-one-session-out validation was used to assess whether shared front and rear models could generalise between complete Silverstone sessions. Pooled LOSO performance produced an RMSE of 962.0 N and  $R^2 = 0.7598$  for the front axle, and an RMSE of 981.2 N and  $R^2 = 0.8911$  for the rear axle. The final Silverstone model was then frozen and evaluated without refitting using data from Brands Hatch and Oulton Park. Across the external datasets, the rear model retained strong predictive performance, with a pooled RMSE of 747.9 N and  $R^2 = 0.9451$ . Front-axle transfer was substantially weaker, with partial transfer at Oulton Park but poor performance at Brands Hatch, producing an overall external RMSE of 2523.3 N and  $R^2 = -0.4338$ .

The study therefore demonstrates that race-car telemetry can support useful reverse engineering of effective tyre-force characteristics, while also showing that the resulting parameters should not be interpreted automatically as uniquely identified physical tyre properties or assumed to remain valid across all operating conditions. The principal contribution is a hierarchical reconstruction and validation methodology that distinguishes directly measured quantities, reconstructed vehicle states and identified effective tyre-model parameters, while explicitly testing same-circuit generalisation, cross-circuit transfer and parameter identifiability. The results also identify where additional measurements, including tyre thermal state, direct yaw inertia, signed static camber and event-specific damper settings, would improve physical interpretation and external validity.

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# 1. Introduction

## 1.1 Background

Tyres generate the contact forces required for vehicle acceleration, braking and cornering, making the relationships between slip angle, vertical load and lateral force fundamental to vehicle dynamics. At small slip angles, lateral tyre response is approximately linear; as the operating limit is approached, the relationship becomes increasingly nonlinear and load-sensitive (Gillespie, 1992; Pacejka, 2012).

Vehicle simulation commonly represents this behaviour using empirical tyre models derived from controlled testing, most notably Pacejka's Magic Formula (Bakker et al., 1987; Pacejka and Bakker, 1992). Race telemetry provides a less controlled but practically valuable alternative because channels such as acceleration, yaw rate, steering angle, vehicle speed and suspension motion repeatedly sample the complete vehicle under realistic and often substantial lateral excitation.

The engineering question is therefore whether these measured vehicle responses can support a defensible reconstruction of the tyre force–slip relationship when direct tyre-rig measurements are unavailable. Track-based tyre identification has demonstrated this potential, but its credibility depends on reconstructing quantities such as tyre force, vertical load and slip angle with sufficient accuracy before fitting the tyre model (Farroni et al., 2018).

## 1.2 An engineering problem

The central difficulty is separating tyre behaviour from the vehicle states used to infer it. Lateral acceleration constrains total lateral force, yaw dynamics influence the front-to-rear force distribution, suspension motion influences vertical tyre load, and tyre slip angle depends on steering input, yaw rate and vehicle sideslip. Errors in any upstream reconstructed state can therefore propagate through the workflow and appear falsely as tyre-model behaviour.

Vehicle sideslip is rarely measured directly in race telemetry. The present workflow therefore avoids using a tyre-model-based observer to generate the states subsequently used to identify that same tyre model. Instead, sideslip is reconstructed using path-based kinematics and checked using beta-independent geometric consistency relations, reducing the risk of circular identification.

Tyre load sensitivity is physically well established, but an additional fitted load-sensitivity coefficient is only meaningful if the available data can identify it independently. During a lap, vertical load, slip angle, speed, camber, temperature and pressure may vary together, allowing different model parameters to compensate

for one another. Parameter compensation and optimisation-bound activity are therefore treated as identifiability warnings rather than direct physical measurements (Pacejka, 2012; Raue et al., 2009).

Independent validation is therefore central to the study. Predictive performance is separated from physical parameter interpretation using holdout validation, leave-one-session-out (LOSO) validation and cross-circuit no-refit testing (Ljung, 1999; Stone, 1974).

### 1.3 Project

The study uses telemetry from the MCR #40 Sports 2000 Duratec prototype, including vehicle speed, acceleration, yaw rate, steering, suspension and lap-related channels, together with suspension-geometry information. Silverstone National Circuit provides the development and same-circuit validation datasets, while Brands Hatch and Oulton Park are reserved for external cross-circuit transfer testing.

The work was commissioned by UWTSD academic staff to support the analysis and improvement of tyre performance. Championship regulations specify Hoosier tyres supplied through BMTR Motorsport and prohibit tyre-softening treatments, while detailed manufacturer performance data remain limited (Sport Racing Car Club [SRCC], 2026). This makes telemetry-based tyre characterisation practically relevant.

The preliminary research dissertation proposed a broader programme covering lateral and longitudinal force, self-aligning torque, tyre-pressure monitoring, driver feedback and controlled rig testing. The final dissertation narrows the primary investigation to quasi-pure-lateral behaviour because these additional outputs and tyre-state measurements were not independently available within the final telemetry dataset.

The computational work is therefore organised as a staged MATLAB inverse problem: telemetry is first audited and conditioned; suspension loads, vehicle path and sideslip are then reconstructed; axle lateral force and individual wheel slip angles are subsequently estimated; and a reduced tyre model is finally fitted and validated.

This staged structure keeps the analysis auditable and separates numerical closure, same-session model fit, same-circuit generalisation and true cross-circuit transfer.

Throughout the dissertation, an effective tyre characteristic is defined as a track-derived force–slip relationship that is valid over the sampled operating domain, rather than as a tyre-rig-derived material property. Fitted coefficients are interpreted as physically identified only where the available data and validation evidence justify that interpretation.

## 1.4 Research rationale

The practical rationale for the study is that race telemetry was available, whereas controlled tyre-rig data were not. A validated track-derived model can nevertheless support vehicle simulation and setup analysis, provided that its operating domain, assumptions and limitations are stated explicitly (Costa and Bortolussi, 2016; Doyle et al., 2019).

Academically, this work is an inverse-engineering case study conducted under imperfect measurement conditions. Its contribution therefore lies not only in the fitted tyre relationships, but also in the validation discipline, reconstruction strategy and assessment of practical parameter identifiability.

The validation hierarchy progresses from development-lap model-complexity testing, through same-circuit Silverstone transfer and leave-one-session-out validation, to frozen no-refit evaluation using the external Brands Hatch and Oulton Park datasets (Hastie et al., 2009; Stone, 1974).

## 1.5 Research aim

The aim of this dissertation is to develop and critically validate a staged MATLAB-based methodology for reverse engineering effective lateral tyre-force characteristics from MCR #40 Sports 2000 race-car telemetry, while explicitly assessing the identifiability and cross-session and cross-circuit transferability of the resulting reduced tyre model.

## 1.6 Research objectives

O1 – Audit and condition the 500 Hz telemetry while preserving timing, units and sign conventions.

O2 – Reconstruct suspension states and individual vertical tyre loads using measured vehicle, setup and suspension-geometry information.

O3 – Reconstruct vehicle path, sideslip, axle lateral forces and individual wheel slip angles.

O4 – Identify a reduced pure-lateral Magic Formula surrogate and determine whether explicit vertical-load sensitivity is independently supported by the available data.

O5 – Assess parameter stability and predictive transfer using complete Silverstone sessions, then freeze the validated model and evaluate it without refitting using the Brands Hatch and Oulton Park datasets.

## 1.7 Research questions

RQ1 – Can the required vehicle and tyre states be reconstructed consistently from race telemetry?

RQ2 – Can a reduced lateral tyre model reproduce independent Silverstone session data?

RQ3 – Does explicit vertical-load sensitivity improve independent predictive performance?

RQ4 – Are the fitted tyre-model parameters stable when complete Silverstone sessions are removed from the identification dataset?

RQ5 – Does a Silverstone-derived tyre model transfer to unseen circuits without refitting?

## 1.8 Working hypothesis

The project did not require formal statistical null-hypothesis testing, but several engineering hypotheses guided the staged validation. These hypotheses were deliberately framed so that the analysis could reject them rather than simply demonstrate a preferred outcome.

**H1** – The upstream reconstructed states will satisfy independent numerical and cross-signal consistency checks.

**H2** – Load sensitivity will only be included in the final tyre model if it improves prediction on unseen data and the fitted load-sensitivity parameter is not constrained by its optimisation bounds.

**H3** – Cross-circuit transfer will be evaluated separately by axle and operating domain rather than assumed to be universal.

## 1.9 Scope and delimitations

The dissertation focuses on lateral tyre-force behaviour under quasi-pure-lateral operating conditions. Strong braking and traction events are excluded from the primary identification population so that combined-slip behaviour is not incorporated into the initial model. The final reduced model is therefore not intended to represent the complete combined-slip operating envelope.

This scope is intentionally narrower than the preliminary research plan, which also considered longitudinal force, self-aligning torque and combined-slip behaviour. These effects were not independently constrained by the final dataset, so the dissertation prioritises a smaller model that can be validated more rigorously.

The study therefore identifies an effective track-derived lateral tyre-force relationship rather than a tyre-rig-equivalent Magic Formula. Event-specific toe settings and measured baseline damper-force characteristics are included, while continuous tyre pressure, temperature and wear state, confirmed signed static camber, measured yaw inertia and event-specific damper click mapping remain explicit limitations.

Vertical tyre load is reconstructed using suspension displacement, suspension geometry, measured vehicle parameters and nominal aerodynamic assumptions. Logger tyre-load channels are used only for dynamic

differential validation because their absolute calibration is not considered sufficiently reliable for direct use as absolute vertical-load measurements.

Vehicle sideslip is estimated rather than measured directly. Beta-independent geometric consistency checks are therefore used to constrain and validate the slip-angle reconstruction. External validation uses one objectively selected complete lap per session; extension of the analysis to all valid laps remains future work.

### 1.10 Expected contribution and originality

The contribution is not a complete laboratory tyre database but a transparent, staged method for determining which track-derived relationships are supported and where interpretation must remain limited (Farroni et al., 2018; Ljung, 1999).

The workflow separates vehicle-state reconstruction from tyre-model fitting, avoids dividing axle forces between tyres in proportion to  $F_z$ , and checks sideslip/slip geometry with path-based and beta-independent relations. These choices reduce circularity in both load-sensitivity and slip-angle identification.

Model complexity is treated as hypothesis: added load sensitivity is retained only if it improves independent prediction without problematic boundary dependence. The final model is also frozen before cross-circuit testing, separating internal fit from external transfer.

The principal engineering output is therefore both the fitted surrogate and its demonstrated limits: which parameters remain stable, which saturate at bounds and which axle relationships survive change in circuit and setup (Burnham and Anderson, 2002; Raue et al., 2009).

### 1.11 Dissertation structure

Chapter 2 reviews the literature relating to tyre modelling, vehicle-state reconstruction and model validation. Chapter 3 defines the staged methodology used for telemetry conditioning, state reconstruction, tyre-force identification and validation. Chapter 4 presents the final setup-aware results, including the Silverstone development and validation datasets, batch-processing outcomes, tyre-model identification and external transfer results. Chapter 5 discusses parameter identifiability, model limitations and cross-circuit transferability. Chapter 6 answers the research questions, summarises the principal findings and limitations, and identifies priorities for future work.

## 2. Literature review

### 2.1 Introduction

Tyres transmit the forces required for vehicle acceleration, braking and cornering, so controlled tyre-force measurements remain the strongest basis for detailed tyre-model parameterisation. Race telemetry is less controlled, but it can still be informative because the vehicle repeatedly excites the tyres across realistic operating conditions (Gillespie, 1992; Pacejka, 2012).

The principal difficulty is that vehicle sideslip, individual tyre slip angle, lateral tyre force and reliable absolute vertical tyre load are not always measured directly. Telemetry-based tyre identification is therefore an inverse problem in which vehicle-state reconstruction and its validation must precede tyre-model fitting.

### 2.2 Tyre force generation and the role of slip angle

Lateral tyre force develops when the direction of travel of a rolling tyre differs from the orientation of the wheel plane. This angular difference is defined as the tyre slip angle,  $\alpha$ .

Contact-patch and carcass deformation produce a lateral-force response that is approximately linear near zero slip angle and becomes increasingly nonlinear as the tyre approaches saturation. The local slope of the lateral-force curve near the origin is conventionally termed the cornering stiffness,  $C_\alpha$  (Gillespie, 1992; Pacejka, 2012). In the small-slip region, it may be expressed as (Eq. 2.2.1)

$$C_\alpha = \frac{dF_y}{d\alpha} \quad (\text{Eq 2.2.1})$$

where  $F_y$  is the lateral tyre force [N] and  $\alpha$  is the tyre slip angle [rad].

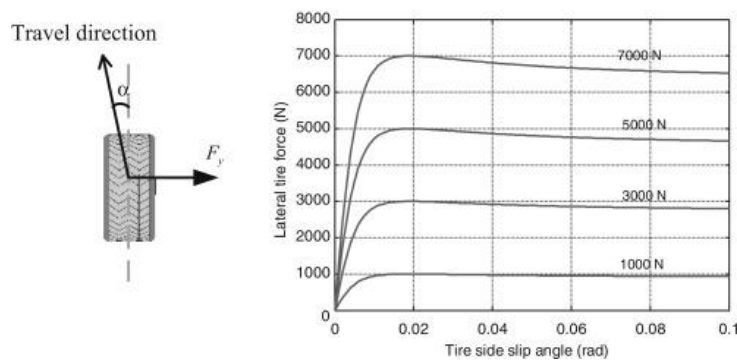


Figure 2.1: Tyre slip-angle definition and corresponding nonlinear lateral-force response for different vertical loads

The approximately linear region is useful for low-to-moderate lateral acceleration and underpins classical bicycle-model vehicle dynamics. Race vehicles, however, frequently operate closer to the nonlinear region, where a constant cornering-stiffness representation becomes insufficient. The peak lateral force, the curvature of the force-slip relationship approaching the peak, and the post-peak behaviour all influence

handling balance and the usable grip envelope. This motivates the use of nonlinear empirical tyre models (Dugoff et al., 1970; Rajamani, 2012).

Vertical load is also important because lateral force does not increase exactly in proportion to normal load. Consequently, lateral load transfer can reduce the total lateral-force capacity of an axle (Milliken and Milliken, 1995; Pacejka, 2012). Tyre pressure, temperature and camber introduce additional dependencies, while friction-estimation research shows that inferred tyre properties depend strongly on the available excitation and the accuracy of the reconstructed vehicle states (Guo et al., 2019; Gustafsson, 1997).

## 2.3 Mathematical tyre models

### 2.3.1 Linear and low-order representations

The simplest lateral tyre representation assumes the validity of equation (2.3.1.1).

$$F_y = C_\alpha \cdot \alpha \quad (\text{Eq 2.3.1.1})$$

where  $F_y$  is lateral tyre force [N],  $C_\alpha$  is cornering stiffness [N/rad], and  $\alpha$  is tyre slip angle [rad]. The sign of the relationship depends on the coordinate and tyre-force convention adopted.

This representation is attractive because it produces transparent vehicle equations and permits direct interpretation of quantities such as understeer gradient and yaw response. Linear tyre models remain valuable for small-slip vehicle observers and control applications (Rajamani, 2012). Their limitation is equally clear: a linear relationship cannot represent force saturation, peak friction, or changes in effective tyre response with operating condition (Abe, 2015; Guiggiani, 2018).

Reduced empirical models can retain nonlinear saturation while requiring fewer fitted parameters, which is advantageous when the available data do not support identification of a full parameter set. The wider literature includes analytical traction models, dynamic-friction formulations, unified empirical models and higher-fidelity tyre representations; these provide useful contrasts with the reduced Magic Formula adopted in the present study (Canudas de Wit et al., 1995; Dugoff et al., 1970).

### 2.3.2 Pacejka's Magic Formula

Bakker et al. (1987) introduced a mathematical representation designed to reproduce measured tyre behaviour in pure cornering and braking using coefficients linked to characteristic tyre quantities. This work formed the basis of the Magic Formula (Eq. 2.3.2.1). Pacejka and Bakker (1992) subsequently extended the formulation to represent tyre forces and moments under longitudinal slip, lateral slip and camber conditions (Bakker et al., 1987; Pacejka and Bakker, 1992; Pacejka, 2012).

$$Y = D \cdot \sin\{C \cdot \arctan[Bx - E(Bx - \arctan(Bx))]\} \quad \text{Eq (2.3.2.1)}$$

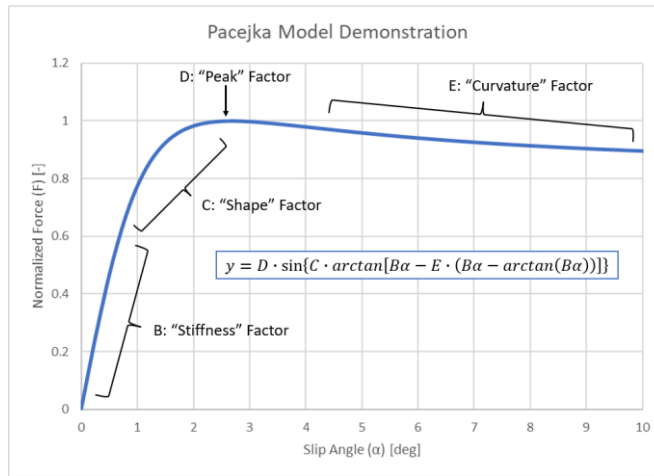


Figure 2.2: Influence of the Pacejka Magic Formula parameters  $B$ ,  $C$ ,  $D$  and  $E$  on the lateral force-slip-angle characteristic.

where  $Y$  represents the modelled tyre force or moment and  $x$  represents the relevant slip variable. In the standard interpretation,  $B$  primarily influences stiffness,  $C$  controls the overall curve shape,  $D$  governs peak magnitude, and  $E$  modifies the curvature of the transition towards saturation.

Horizontal and vertical shift terms may also be introduced to move the characteristic relative to the origin. More advanced parameterisations include dependencies on vertical load, camber, inflation pressure and combined-slip conditions (Pacejka, 2012). The Magic Formula is widely used because it can reproduce measured steady-state tyre characteristics accurately while remaining computationally convenient for vehicle simulation (Sharp and Bettella, 2003).

The same flexibility that makes the Magic Formula useful also creates an identification challenge. When only a limited region of the force-slip surface is observed, several parameters may compensate for one another. For example, stiffness, amplitude and horizontal shift can all influence the apparent slope or force level over a narrow slip-angle range. Fitting many coefficients to track data can therefore produce low residual errors

without guaranteeing that the individual parameters correspond uniquely to physical tyre properties (Farroni et al., 2018; Raue et al., 2009).

The reduced formulation adopted in this dissertation is an engineering simplification of the wider Magic Formula family. Higher-fidelity models may include inflation-pressure effects, road unevenness, transient behaviour and additional tyre-state dependencies, but the available telemetry does not independently excite or measure all these effects sufficiently for reliable identification (Besselink et al., 2010; Pacejka, 2012).

The preliminary research phase considered a broader range of tyre outputs and dependencies, including lateral and longitudinal force, self-aligning torque, vertical load and camber. This remains a useful framework for future testing, while reinforcing the importance of controlled measurements as tyre-model dimensionality and parameter count increase (Lugner, Pacejka and Plöchl, 2005).

## 2.4 Identification of Magic Formula parameters from experimental and track data

Controlled tyre-rig identification is the most direct approach because force and state variables can be prescribed independently. Track data embed tyre behaviour within the complete vehicle, so inferred properties depend on state accuracy and operating conditions (Khaleghian et al., 2017; Pasterkamp and Pacejka, 1997).

Track-based Magic Formula identification is nevertheless feasible. Tyre Road Interaction Parameters Identification (TRIP-ID) emphasises data diagnostics, operating-range coverage and guided parameter estimation rather than blind optimisation, which is directly relevant to this dissertation (Farroni et al., 2018; Li et al., 2014).

Related cornering-stiffness and friction studies likewise show that inferred tyre properties depend strongly on excitation and state estimation, reinforcing the need for independent validation (Gustafsson, 1997; Rajamani et al., 2012).

A further issue is pure versus combined slip. Magic Formula identification is commonly structured by identifying pure-slip behaviour before combined-slip terms are introduced (Pacejka, 2012). The present study therefore restricts initial identification to quasi-pure-lateral conditions by excluding strong longitudinal acceleration and braking. Studies based on braking or longitudinal excitation demonstrate the distinct information available in those regimes, but do not themselves justify the pure-lateral sequence adopted here (Alvarez et al., 2005; Matuško et al., 2008)

## 2.5 Vertical load, lateral load transfer and tyre load sensitivity

### 2.5.1 Vehicle-level load transfer

A tyre model cannot be interpreted as load-sensitive unless individual vertical tyre loads are reconstructed with sufficient credibility. Classical vehicle dynamics separates lateral load transfer into sprung-mass, geometric and unsprung contributions. The total lateral load transfer may be approximated as (Eq. 2.5.1.1)

$$\Delta F_{z,total} \approx \frac{m \cdot a_y \cdot h_{CG}}{t} \quad \text{Eq (2.5.1.1)}$$

where  $m$  is vehicle mass,  $a_y$  lateral acceleration,  $h_{CG}$  centre-of-gravity height and  $t$  track width (Gillespie, 1992; Milliken and Milliken, 1995).

During cornering, lateral acceleration acting through the CG produces a roll moment (Eq. 2.5.1.2)

$$M_{roll} = m \cdot a_y \cdot h_{CG} \quad \text{Eq (2.5.1.2)}$$

The tyres counteract that moment through a difference in vertical forces across the track. If an amount of load  $\Delta F_z$  is transferred from the inside to the outside, then:

$$M_{resisting} = \Delta F_z \cdot t \quad \text{Eq (2.5.1.3)}$$

Equating the two moments gives the lateral load-transfer relationship Eq (2.5.1.1).

In a race car, springs and anti-roll bars strongly influence how total load transfer is distributed between the front and rear axles. Suspension motion ratios also affect the relationship between measured damper displacement and wheel load. A telemetry-based reconstruction must therefore account for suspension geometry rather than applying spring rate directly to damper travel (Dixon, 2009; Milliken and Milliken, 1995).

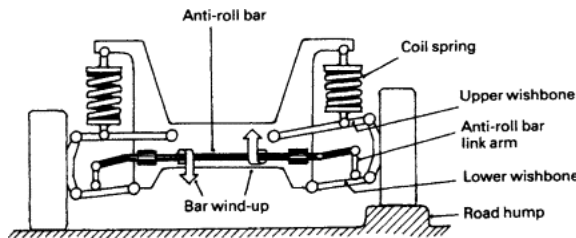


Figure 2.3.: Schematic of an anti-roll bar suspension system showing the coil springs, wishbones and anti-roll bar linkage used to resist differential wheel movement during cornering.

Aerodynamic downforce and longitudinal acceleration also alter axle vertical load. For tyre identification, individual  $F_z$  must therefore be reconstructed from static load, aerodynamic and longitudinal effects, and measured suspension motion rather than inferred from lateral acceleration alone (Doumiati et al., 2009; Gillespie, 1992).

### 2.5.2 Damper force and transient vertical load

Spring and anti-roll-bar forces are not the only suspension forces. Dampers generate forces that depend primarily on damper velocity and valving, and these can influence instantaneous wheel load during transient events.

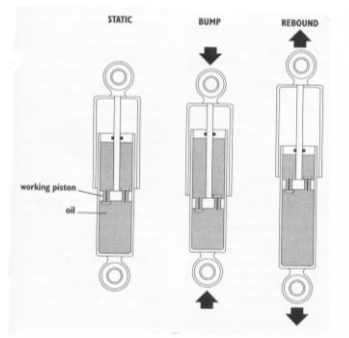


*Figure 2.4: Schematic of a hydraulic damper showing static, bump/compression and rebound operation through piston motion and oil displacement.*

Their effect should be reduced in carefully selected quasi-steady data but not necessarily eliminated. Takahashi (2018) showed that transient tyre side-force behaviour under time-varying vertical load can influence vehicle lateral response, illustrating that tyre force need not respond instantaneously to changes in  $F_z$  (Maurice and Pacejka, 1997; Talbott and Starkey, 2002).

A steady-state tyre model should therefore prioritise quasi-steady conditions and treat damper force as a secondary reconstruction term. The present method includes measured damper force but evaluates its influence separately so that transient suspension effects are not mistaken for steady tyre load sensitivity (Dixon, 2007; Takahashi, 2018).

## 2.6 Vehicle-state estimation from onboard sensors

### 2.6.1 Why sideslip is difficult to measure

Vehicle sideslip angle is the angle between the vehicle longitudinal axis and the direction of travel at the vehicle reference point. Direct measurement generally requires specialised optical or navigation equipment,

so practical studies commonly estimate it using combinations of steering angle, wheel speed, acceleration, yaw rate and GPS/INS data (Chindamo et al., 2018; Singh et al., 2019).

Leung et al. (2011) similarly review GPS/INS-based ground-vehicle state estimation and emphasise the complementary nature of inertial and satellite measurements. GPS can provide drift-resistant trajectory information but may suffer from limited sampling rate, multipath effects and signal availability, while inertial sensors provide high-rate motion information but accumulate drift when integrated. This trade-off is directly relevant to track telemetry (Bevly et al., 2006; Doumiati et al., 2013).

### 2.6.2 Dynamic observers

Sideslip estimation commonly uses vehicle models and observers. Extended Kalman, sliding-mode and nonlinear friction-adaptive methods can reconstruct sideslip and tyre forces, but their accuracy depends on modelling assumptions (Baffet et al., 2009; Best et al., 2000).

Observer-based methods can perform well when vehicle parameters and tyre assumptions are appropriate, but they may embed the same tyre behaviour that is later being identified. To reduce this circularity, the present study uses a path-based, tyre-model-independent sideslip estimate (Chindamo et al., 2018; Simon, 2006).

### 2.6.3 Kinematic and path-based approaches

Kinematic sideslip approaches avoid some dependence on tyre-force parameters by exploiting the relationship between vehicle body heading  $\psi$  and path/course direction  $\chi$ . In principle, vehicle body sideslip angle  $\beta$  (Eq. 2.6.3.1) equals the difference between course heading  $\chi$  and yaw heading  $\psi$ , while its rate  $\dot{\beta}$  (Eq. 2.6.3.3) is the difference between course-rate  $\dot{\chi}$  and yaw rate  $\dot{\psi}$ . This is attractive for inverse tyre identification because the calculation can be kept independent of the tyre model being identified (Cheli et al., 2007; Leung et al., 2011).

$$\beta = \chi - \psi \quad (\text{Eq. 2.6.3.1})$$

$$\dot{\psi} = r \quad (\text{Eq. 2.6.3.2})$$

$$\dot{\beta} = \dot{\chi} - r \quad (\text{Eq. 2.6.3.3})$$

The main disadvantage is sensitivity to bias and drift when the rate difference is integrated. Straight-running, near-zero-sideslip segments can be used to estimate bias and constrain drift, provided that sufficient reference segments are available. The present study adopts this path-rate approach and then checks slip-

angle geometry using a beta-independent front-to-rear relation. This prevents the absolute sideslip estimate,  $\beta$ , from being the sole validation of tyre slip-angle reconstruction (Cheli et al., 2007; Chindamo et al., 2018).

## 2.7 Tyre-force estimation from vehicle dynamics

Tyre forces can be estimated indirectly from measured vehicle motion. In planar vehicle dynamics, lateral acceleration constrains total lateral force, while yaw dynamics influence the front-to-rear force distribution. Observer-based methods extend these principles by incorporating additional vehicle states and sensor models (Baffet et al., 2009; Gillespie, 1992).

$$F_{y,F} + F_{y,R} = ma_y \quad (\text{Eq. 2.7.1})$$

Where the sum of total lateral force generated by the front axle  $F_{y,F}$  [N] and the total lateral force generated by the rear axle  $F_{y,R}$  [N] are equal to the vehicle mass  $m$  [kg] times the lateral acceleration of the vehicle CG  $a_y$  [m/s<sup>2</sup>].

$$I_z \cdot \dot{r} = aF_{y,F} - bF_{y,R} \quad (\text{Eq. 2.7.2})$$

Where the vehicle yaw moment of inertia  $I_z$  [kg · m<sup>2</sup>] times yaw acceleration  $\left[\frac{rad}{s^2}\right]$  corresponds to the difference between the front-axle lateral force  $F_{y,F}$  [N] multiplied by the distance from front axle to CG  $a$  [m] and the rear-axle lateral force  $F_{y,R}$  [N] multiplied by the distance from CG to rear axle  $b$  [m].

In practice, the front/rear split requires knowledge of yaw inertia and a reliable yaw-acceleration signal. If the identification population is restricted to quasi-steady cornering, the yaw-acceleration term becomes small, and the axle-force distribution can be approximated from static longitudinal mass distribution. This sacrifices transient information but reduces dependence on an unknown yaw inertia.

Baffet et al. (2009) and related observer literature show that tyre forces can be reconstructed from standard vehicle sensors when model assumptions are treated carefully. The present study follows the same inverse dynamics principle while retaining axle force as the independent target rather than imposing an individual left/right force distribution (Baffet et al., 2008; Doumiati et al., 2011).

This last point is critical for studying load sensitivity. Dividing an axle's  $F_z$  between the two tyres in direct proportion to  $F_z$  would impose equal utilisation and build a force-load relationship into the data before the tyre model is fitted. By instead allowing the tyre model to predict individual left/right forces and requiring only that their sum match the reconstructed axle force, the later test of load sensitivity remains less circular.

## 2.8 Suspension kinematics, alignment and definition of slip angle

Tyre slip angle depends not only on vehicle sideslip and yaw rate but also on the actual orientation of each wheel. Steering geometry, compliance, static toe and suspension-induced steer all affect wheel heading. Lateral force is also influenced by camber, so static camber and kinematic camber change should be treated separately (Abe, 2015; Dixon, 2009).

ISO 8855:2011 provides standard terminology for road-vehicle dynamic quantities and helps avoid ambiguity in sign conventions. A consistent coordinate system is essential because sign errors in steering, yaw, lateral acceleration or slip angle can produce numerically smooth results while reversing the physical interpretation (International Organization for Standardization, 2011; SAE International, 2022).

Race-car suspension geometry also causes motion ratio, camber and steer to vary with ride and roll. For telemetry-based identification, constant steering and motion ratios may therefore be inadequate when higher accuracy is required. Geometry maps or measured kinematic data provide a stronger basis for reconstructing wheel states (Milliken and Milliken, 1995; Dixon, 1996; Dixon, 2009).

Static alignment is especially important when the fitted tyre model includes horizontal shift terms. A fitted  $\alpha_0$  offset may represent tyre asymmetry, static toe, steering bias, compliance or a combination of these effects. Without independent setup data, such an offset should be treated as a nuisance/reference parameter rather than a physical tyre property (Besselink et al., 2010; Pacejka, 2012).

## 2.9 Signal conditioning for high-rate telemetry

Race telemetry is typically sampled much faster than dominant rigid-body vehicle motions. High sampling rates preserve transient information but also capture sensor noise, vibration and differentiation noise. Filtering is therefore often necessary when deriving yaw acceleration, longitudinal acceleration from speed, curvature rate or damper velocity (Bendat and Piersol, 2010; Butterworth, 1930).

For offline analysis, zero-phase forward-backward filtering is useful because it suppresses high-frequency content without introducing a net phase shift. Gustafsson (1996) describes methods for setting initial conditions and reducing start-up transients.

Avoiding phase delay is important when comparing steering, yaw rate and acceleration because even a small delay can reduce correlation or create false apparent dynamics.

Filter choice should be justified by signal bandwidth rather than visual smoothness. Power spectral density, raw-versus-filtered comparisons and sensitivity to candidate cut-off frequencies provide a defensible basis. The present methodology therefore treats signal conditioning as an explicit preprocessing stage (Bendat and Piersol, 2010; Welch, 1967).

## 2.10 Parameter identification, overfitting and identifiability

### 2.10.1 Fitting error versus physical information

Nonlinear parameter estimation seeks coefficients that minimise the difference between model predictions and observed data. A low residual does not prove that each coefficient uniquely represents a physical property. System-identification literature therefore distinguishes predictive fit from parameter identifiability (Bates and Watts, 1988; Ljung, 1999).

Practical non-identifiability occurs when different parameter combinations produce similar outputs over the measured range. In tyre models, amplitude, stiffness, curvature and horizontal-shift parameters may compensate for one another. Parameters that repeatedly settle on imposed bounds are therefore an identifiability warning (Conn et al., 2000; Raue et al., 2009).

Track telemetry does not excite one variable at a time: load, slip and thermal state vary together. A fitted load-sensitivity coefficient may therefore compensate for an unmeasured state, making held-out validation essential (Farroni and Sakhnevych, 2022; Guo et al., 2019).

### 2.10.2 Cross-validation and domain transfer

Cross-validation evaluates predictive generalisation using data excluded from fitting. For correlated telemetry, holding out complete sessions is more demanding than randomly splitting nearby samples (Hastie et al., 2009; Stone, 1974).

Leave-one-session-out (LOSO) validation tests transfer between complete Silverstone sessions, while cross-circuit validation is stronger because it evaluates a frozen model on unseen layouts and operating conditions.

The present hierarchy uses both: model complexity is tested on independent Silverstone sessions, then the final Silverstone model is frozen and evaluated at Brands Hatch and Oulton Park.

## 2.11 Research gap and rationale for the present study

The literature provides the required component methods: vehicle dynamics for force and load transfer, Magic Formula models for tyre representation, state estimation for sideslip and force inference, and track-based identification for parameter estimation. The gap addressed here is their staged combination for motorsport telemetry with incomplete tyre-state measurement and no direct tyre-force measurement (Costa and Bortolussi, 2016; Gadola et al., 1996).

Many estimators embed a tyre model, while identification methods often assume slip angle and force are known. To reduce circularity, the present workflow reconstructs and validates vehicle states and axle force before fitting the tyre model (Baffet et al., 2009; Singh et al., 2019).

Physical plausibility does not establish identifiability. Added load sensitivity is retained only if it improves independent prediction without problematic boundary activity; otherwise, the conclusion is insufficient evidence for that coefficient, not absence of physical load sensitivity (Burnham and Anderson, 2002; Raue et al., 2009).

A further gap concerns transferability. This study progresses from development-lap fitting to Silverstone session validation, LOSO identification and no-refit testing at Brands Hatch and Oulton Park. Racing simulation and optimal-control literature shows that circuit and formulation can materially change operating states, supporting the need for cross-domain validation (Perantoni and Limebeer, 2014; Veneri and Massaro, 2020).

## 2.12 Conceptual framework for the dissertation

The literature supports a framework in which the tyre model is the final stage of a validated inverse chain: measured vehicle motion informs vertical load, axle force and tyre slip states before the reduced tyre relationship is fitted.

Telemetry  $\rightarrow$  Vehicle state  $\rightarrow F_z$  and axle  $F_y \rightarrow$  tyre  $\alpha \rightarrow$  reduced tyre model  $\rightarrow$  held out validation

This ordering minimises circularity and keeps uncertainty explicit: beta is estimated, absolute load is model-assisted, alignment requires setup records and fitted coefficients are valid only over the sampled domain.

A track-derived model may be a useful predictive surrogate without every coefficient being uniquely physical; coefficients that fail independent validation should not be promoted to measured tyre properties (Ljung, 1999; Raue et al., 2009).

## 3 Methodology

### 3.1 Methodological overview

The methodology follows the inverse sequence established in Chapters 1 and 2: telemetry is conditioned first, vehicle and tyre states are reconstructed second, and the tyre model is fitted only after those states satisfy defined validation checks. Silverstone FP1 Lap 5 is used as the development dataset, while independent Silverstone sessions and external circuits are reserved for later validation.

Silverstone was selected for two main reasons. First, weather conditions were suitable for slick tyres, unlike some earlier Pembrey testing. Second, Silverstone was the opening event of the 2026 Sports 2000 Duratec calendar, making it a useful baseline for initial setup and performance comparison across later events.

The methodology evolved from the earlier research plan but was narrowed to the measurements available in the final dataset. MoTeC remained the inspection tool and MATLAB the analysis environment, while controlled rig testing and richer combined-slip identification were deferred to future work.

The final workflow incorporated spring assignments, circuit-specific anti-roll-bar (ARB) settings, event-specific static toe, measured tyre vertical-stiffness bookkeeping and baseline damper force-velocity data. All dependent states and validation outputs were regenerated after these updates.

### 3.2 Vehicle, circuit and event provenance

The vehicle was MCR Sports 2000 car #40, an MCR S2n Duratec prototype. The principal Silverstone FP1 development session was logged at 500 Hz. Silverstone National was used for model development and same-circuit validation, while Brands Hatch and Oulton Park were retained for external validation.

Official timing records were cross-checked against the telemetry to confirm event provenance, circuit configuration and car-entry context. Silverstone and Oulton Park details were confirmed from Timing Solutions Ltd records (Timing Solutions Ltd, 2026a; Timing Solutions Ltd, 2026b), while Brands Hatch was confirmed from the official event timing document (GT World Challenge Europe, 2026). Integrated lap distance distinguished Brands Hatch Indy from the later Brands Hatch GP event, while the Oulton Park sessions were consistent with the international layout.

Table 3.1. Official event provenance used to contextualise the telemetry datasets.

| Circuit / event | Date         | Official layout                                                 | Car #40 entry            | Use in dissertation                   |
|-----------------|--------------|-----------------------------------------------------------------|--------------------------|---------------------------------------|
| Silverstone     | 4-5 Apr 2026 | National, 2.64 km                                               | Tim Tudor – MCR S2n      | Development + same-circuit validation |
| Brands Hatch    | 2-3 May 2026 | 3.916 km event layout; telemetry separated into Indy/GP domains | Tim Tudor – MCR S2n      | External no-refit validation          |
| Oulton Park     | 13 Jun 2026  | International, 4.33 km                                          | Tudor / Gibson – MCR S2n | External no-refit validation          |

The official records are used only for provenance and environmental context; they are not inputs to the tyre-



Figure 3.1(a): Silverstone National Circuit, (b) Brands Hatch GP Circuit, (c) Oulton Park International Circuit

force reconstruction and do not independently validate reconstructed  $F_y$ ,  $F_z$ , slip angle or sideslip. At Oulton Park, the official entry is shared between Tudor and Gibson, so the timing record alone does not identify which driver produced each telemetry session (Timing Solutions Ltd, 2026b).

The principal signals used across the workflow were ground speed, GPS speed, longitudinal and lateral acceleration, yaw rate, steering-wheel angle, lap number, lap distance, four damper positions and four damper-velocity channels. Logger tyre-load channels were retained only for dynamic left-right validation because their absolute zero and gain were not considered sufficiently reliable for direct use as tyre normal force.

### 3.3 Coordinate system and sign convention

A single vehicle-fixed convention was applied throughout:  $+X$  forward,  $+Y$  to the vehicle left and  $+Z$  upward. Positive yaw rate represented a left turn, and positive road-wheel steer pointed the wheel left. The same convention was applied to acceleration, steering, suspension steer and tyre-slip calculations. This follows standard vehicle-dynamics terminology and reduces the risk of smooth but physically inverted relationships (International Organization for Standardization, 2011; SAE International, 2022).

### 3.4 Stage 0 – Import, time alignment and lap audit

Stage 0 imported the raw logger data, established a common 500 Hz timebase, converted channels to consistent SI units, and audited sample counts, missing values and lap boundaries. Physical laps were reconstructed from lap number, lap distance and time because the source sessions included out-laps, red-flag periods and numbering discontinuities.

Silverstone FP1 Lap 5, containing approximately 34,000 samples over 68 s, was selected as the development lap because it lay within the normal operating speed range and neighbouring laps 4 and 6 were uninterrupted.

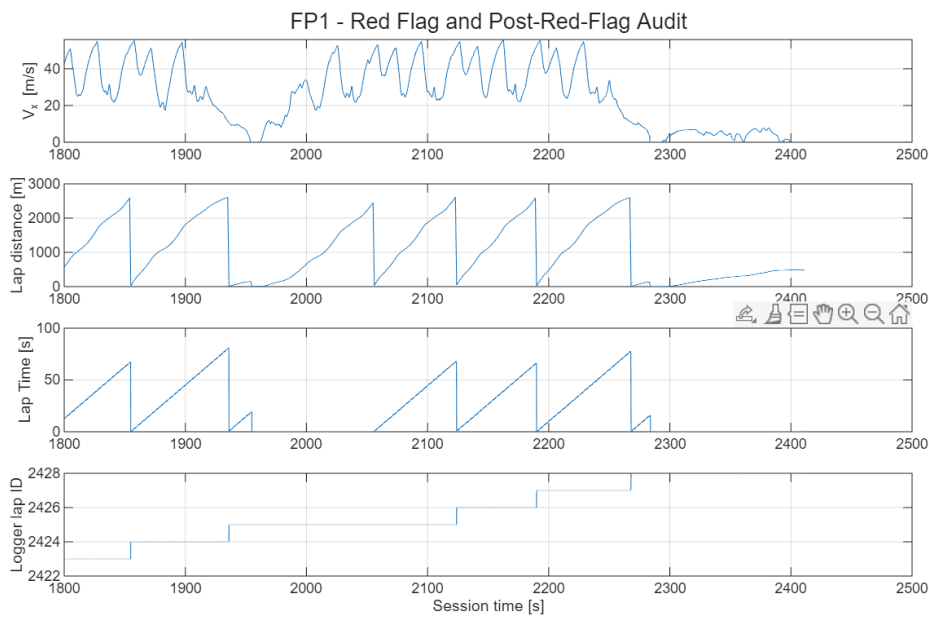


Figure 3.2: Silverstone FP1 red-flag and post-red-flag audit showing vehicle speed, lap distance, lap time and logger lap ID against session time. The discontinuities confirm the red-flag interruption and subsequent lap-numbering changes used to reconstruct the physical lap sequence.

### 3.5 Stage 1 – Signal conditioning

Stage 1 used stationary data to estimate accelerometer offsets and compared candidate low-pass filters. A fourth-order Butterworth low-pass filter with a 15 Hz half-power frequency was applied using forward-backward zero-phase filtering. The Butterworth filter provides a maximally flat passband, while forward-backward application avoids net phase delay in offline analysis. Spectral checks were used to justify the selected bandwidth (Butterworth, 1930; Gustafsson, 1996).

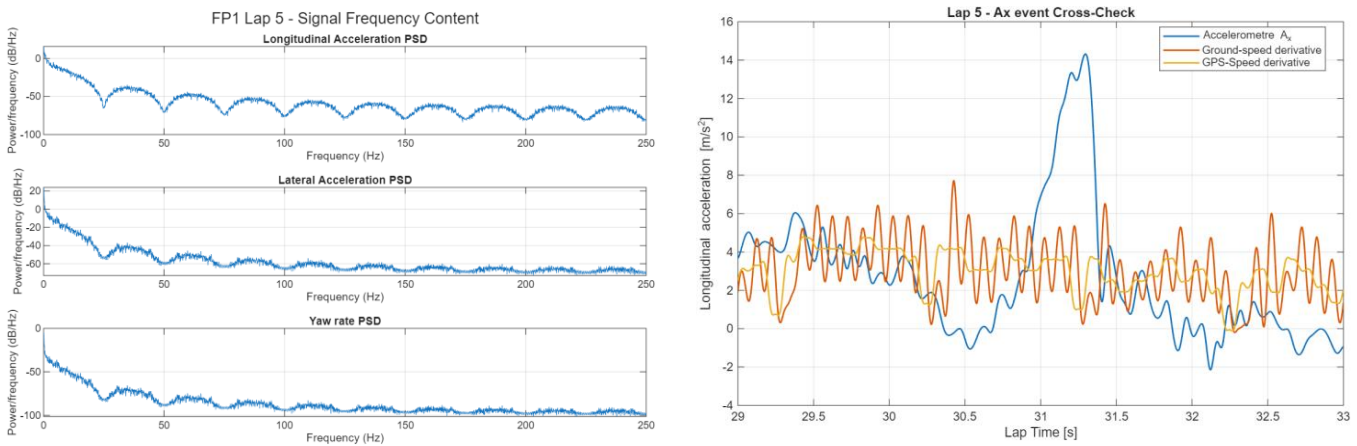


Figure 3.3: Silverstone FPI Lap 5 signal-conditioning diagnostics. Left (a): power spectral density of longitudinal acceleration, lateral acceleration and yaw rate used to assess signal bandwidth. Right (b): cross-check of the longitudinal-acceleration event against derivatives of Ground Speed and GPS Speed, showing the accelerometer spike was not supported by the independent speed-derived signals.

Ground/GPS speed and their derivative channels were compared to ensure transient sensor anomalies were not propagated into the force reconstruction (Butterworth, 1930; Gustafsson, 1996).

### 3.6 Stage 2 – Vehicle parameters, suspension kinematics and vertical-load reconstruction

3.2 Table: Vehicle parameters

| Car specs | Values           | Front           | Rear              |
|-----------|------------------|-----------------|-------------------|
| Mass      | 684 kg           | 272 kg          | 412 kg            |
| Wheelbase | 2.580 m          | Track = 1.554 m | Track = 1.48442 m |
| CG        | Height = 0.320 m | a = 1.554 m     | b = 1.026 m       |

The measured/project parameters entered standard static-load and load-transfer relationships (Gillespie, 1992; Milliken and Milliken, 1995).

Stage 2 encoded mass distribution, CG location, wheelbase, track widths, spring and ARB rates, suspension geometry, setup data and measured damper characteristics. Individual wheel vertical load combined axle-total equilibrium with left-right differential load transfer from springs, anti-roll bars, suspension geometry and damping. Suspension ride and roll behaviour was obtained from WinGeo 3 geometry data.

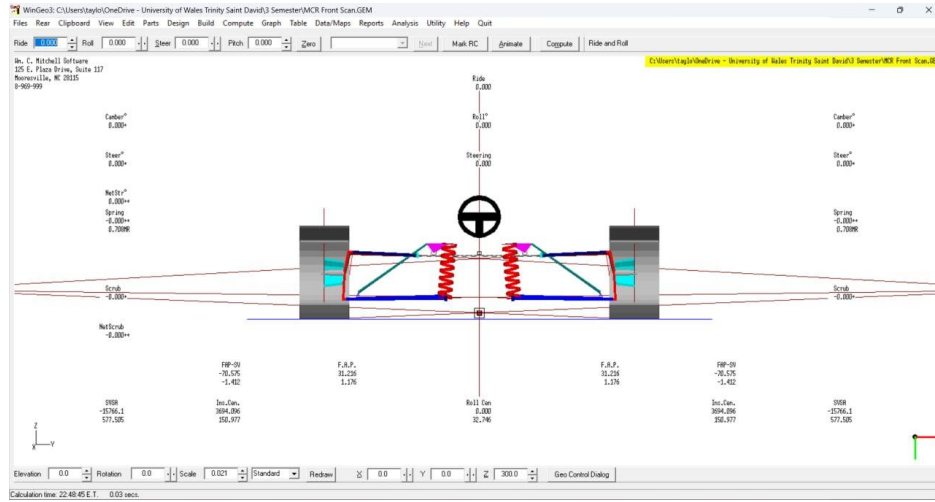


Figure 3.4: WinGeo 3 front-suspension model of the MCR #40, used to obtain suspension geometry, ride, roll and kinematic relationships for the vertical-load reconstruction.

Damper velocity was aligned to the master telemetry timebase, delayed by the validated 20 samples (0.040 s), clipped only where finite values exceeded the measured dyno envelope, and interpolated through the measured baseline force–velocity lookup. Edge NaNs remained invalid rather than being extrapolated.

The four-wheel loads were formed as half the axle total plus or minus half the reconstructed differential. Algebraic closure, positive-load checks, suspension-geometry closure and comparison with logger load differences were required before Stage 2 was accepted.

The nominal reconstruction used measured corner weights, longitudinal acceleration and nominal aerodynamic terms for axle totals. Because direct aerodynamic load measurement was unavailable, aerodynamic load remained an explicit model-assisted contribution rather than an independently measured tyre load.

Wheel stiffness and steering-wheel-angle-to-road-wheel-angle tests were conducted in the university workshop.



Figure 3.5: Left (a): Workshop measurement of tyre/wheel vertical stiffness and Right (b): steering-wheel-to-road-wheel angle calibration for the MCR #40.

### 3.7 Stage 3 – Ground speed and path reconstruction

Stage 3 compared ground speed and GPS speed, filtered the selected forward-speed signal and integrated it to reconstruct travelled distance. Vehicle sideslip was estimated without using the tyre model to be identified. The governing kinematic relations were:

$$\beta = \chi - \psi \quad (\text{Eq. 3.7.1})$$

$$\Rightarrow \dot{\beta} = \dot{\chi} - r \approx V_x \kappa - r \quad (\text{Eq. 3.7.2})$$

where  $\chi$  is the path/course heading,  $\psi$  is vehicle yaw heading,  $r$  is measured yaw rate,  $V_x$  is longitudinal speed and  $\kappa$  is path curvature. Small integration bias can produce drift, so straight-running anchor segments constrained the integrated estimate. The choice reduces tyre-model circularity and is consistent with kinematic/path-based sideslip literature, while the remaining uncertainty was checked by sideslip-independent geometry (Cheli et al., 2007; Chindamo et al., 2018).

### 3.8 Stage 4 – Sideslip $\beta$ estimation

Stage 4 estimated sideslip from the path-rate relation (Eq. 3.7.2). Straight-running segments provided drift/bias anchors, and the final sideslip was re-centred without using a tyre model. The acceleration-

integrated route was retained only as a secondary diagnostic where its sign disagreement made it unsuitable as an independent estimator.

### 3.9 Stage 5 – Event classification

Stage 5 produced strict steady-state and broader quasi-steady-state masks. Thresholds were applied to speed, lateral acceleration, steering, yaw consistency, derivatives and longitudinal acceleration. The quasi-steady population retained sufficient data for identification while excluding the strongest braking, traction and rapid transient events. This prevented the reduced pure-lateral model from absorbing combined-slip and transient behaviour it was not designed to represent (Gillespie, 1992; Milliken and Milliken, 1995).

### 3.10 Stage 6 – Axle lateral-force reconstruction

Stage 6 reconstructed axle lateral force from planar equilibrium. Because yaw inertia was not directly measured by a sensor and was only available as a calculated channel, the nominal quasi-steady split used longitudinal CG position. An approximate inertia sensitivity was then used to quantify the possible yaw-acceleration correction rather than treating yaw inertia as measured truth (Rajamani, 2012; Abe, 2015).

Axle force was deliberately not divided between the left and right tyres in proportion to  $F_z$ . Instead, the lateral tyre model predicted each tyre contribution, and their sum was compared with the independently reconstructed axle force. This avoided imposing equal utilisation or load sensitivity before model fitting (Pacejka, 2012; Farroni et al., 2018).

### 3.11 Stage 7 – Individual tyre-state reconstruction

Stage 7 reconstructed wheel-centre velocity from  $V_x$ ,  $V_y$  (Eq. 3.11.1), yaw rate and wheel coordinates, then formed slip angle from local velocity direction and actual wheel heading.

$$V_y = V_x \tan(\beta) \quad (\text{Eq. 3.11.1})$$

Front wheel heading included measured steering angle and suspension steer (Abe, 2015; Pacejka, 2012).

Rear wheel heading included suspension steer. Event-specific static toe was applied before slip angle. Because signed static camber was not independently confirmed, only kinematic camber change was retained (Dixon, 2009; Besselink et al., 2010).

Validation used numerical closure between two equivalent slip-angle formulations and a sideslip-independent front-minus-rear relation. These checks tested steering and yaw geometry without relying on the absolute  $\beta$  estimate (Baffet et al., 2009; Cheli et al., 2007).

### 3.12 Stage 8 – Reduced Magic Formula identification

Stage 8 fitted a reduced pure-lateral Magic Formula (Eq.3.12.1) with shape factor  $C = 1.30$  and vertical-load exponent applied to cornering stiffness  $p_{C_\alpha} = 1$  fixed.

|  |                                                      |              |
|--|------------------------------------------------------|--------------|
|  | $F_y = D \cdot \sin[C \cdot \arctan(B\alpha_{eff})]$ | (Eq. 3.12.1) |
|  | $\alpha_{eff} = \alpha - \alpha_0$                   | (Eq. 3.12.2) |

Cornering stiffness  $C_\alpha$  was related to vertical load  $F_z$  through (Eq. 3.12.3)

$$C_\alpha(F_z) = C_{\alpha,ref} \left( \frac{F_z}{F_{z,ref}} \right)^{p_{C_\alpha}} \quad (\text{Eq. 3.12.3})$$

So, the model assumes that cornering stiffness changes approximately proportionally to vertical load.

The baseline fixed peak friction  $p_\mu = 0$ , (Eq. 3.12.5) while the alternative added a peak-friction load exponent.

$$\mu(F_z) = \mu_{ref} \left( \frac{F_z}{F_{z,ref}} \right)^{p_\mu} \quad (\text{Eq. 3.12.4})$$

$$\mu(F_z) = \mu_{ref} \quad (\text{Eq.3.12.5})$$

Separate left/right alpha-zero terms were retained as nuisance reference offsets because measured toe had already been applied upstream (Bakker et al., 1987; Pacejka, 2012).

Only samples with  $|A_x| \leq 2.0 \frac{m}{s^2}$  were retained. Continuous segments were split into training and holdout populations, optimisation was decimated to approximately 50 Hz, and residual weights balanced left- and right-turn directions to reduce serial and directional bias (Ljung, 1999; Stone, 1974).

Stage 8 selection was provisional: a more complex model could proceed only if holdout performance justified it and boundary activity remained acceptable (Burnham and Anderson, 2002; Raue et al., 2009).

### 3.13 Stage 9 – Multi-session no-refit validation of load sensitivity

Stage 9 froze the Stage 8 baseline and load-sensitive parameter vectors and evaluated them on seven additional Silverstone datasets with no refit. Support for  $p_\mu$  required transferable error reduction and acceptable parameter identifiability; a boundary-dependent gain was treated as predictive evidence, not a physically identified coefficient (Ljung, 1999; Burnham and Anderson, 2002).

### 3.14 Stage 10 – Global Silverstone baseline and LOSO

Stage 10 fixed  $p_\mu = 0$  and fitted shared front and rear models across eight authoritative Silverstone datasets. Eight leave-one-session-out folds held out one complete dataset at a time, with session- and direction-balanced fitting on the remaining seven. A final all-session model was fitted only after LOSO validation (Hastie et al., 2009; Stone, 1974).

### 3.15 Cross-circuit reconstruction and Stage 11

The cross-circuit batch repeated Stages 0–7 for Brands Hatch and Oulton Park using the same algorithms and event-specific setup database. Any fallback relaxed only state-reference criteria; tyre parameters were not tuned. Damper-velocity channels were aligned to each session timebase before reconstruction (Ljung, 1999; Farroni et al., 2018).

Stage 11 applied the frozen Stage 10 parameters to ten external datasets: four Brands Hatch Indy, two Brands Hatch GP and four Oulton Park International sessions. Predictions were pooled by layout and overall, with no optimisation. Batch01 regenerated all seven Silverstone validation datasets, while Batch02 regenerated all ten external datasets before the final analysis (Farroni et al., 2018; Sierra et al., 2006).

### 3.16 Validation, uncertainty and reproducibility

Validation was hierarchical: numerical closure, cross-signal consistency, beta-independent geometry, segment holdout, complete session no-refit validation, LOSO and unseen-circuit transfer. Root mean square error (RMSE) (Eq. 3.16.1) provided an absolute force-error scale and coefficient of determination  $R^2$  (Eq. 3.16.2) assessed performance relative to a constant-mean benchmark (Bates and Watts, 1988; Ljung, 1999).

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (\text{Eq. 3.16.1})$$

$$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2} \quad (\text{Eq. 3.16.2})$$

Where  $y_i$  corresponds to the actual or reconstructed value,  $\hat{y}_i$  represents the model predictions,  $n$  is the number of samples and  $\bar{y}$  is the mean of the actual values.

A simple interpretation is:

- $R^2 = 1 \rightarrow$  perfect prediction
- $R^2 = 0.9 \rightarrow$  about 90% of the variation is captured by the model
- $R^2 = 0 \rightarrow$  the model is no better than just predicting the mean
- $R^2 < 0 \rightarrow$  the model is worse than simply predicting the mean

Key limitations remained explicit: event-specific damper click mapping, signed static camber, direct yaw inertia, continuous tyre temperature/pressure/wear state and fully measured aerodynamic load were unavailable. The model is therefore interpreted only over its validated operating domain (Pacejka, 2012; Farroni and Sakhnevych, 2022).

Each stage saved versioned MAT results, audit tables and selected figures; batch summaries identified authoritative datasets and excluded stale files, making the final chain reproducible from telemetry to the frozen transfer test (Ljung, 1999; Conn et al., 2000).

### 3.17 Mathematical implementation and audit logic

The implementation separated axle-total vertical load (Eq. 3.17.1, Eq. 3.17.2) from left-right differential load transfer (Eq. 3.17.4, Eq. 3.17.5). Longitudinal load-transfer contribution is governed approximately by (Eq. 3.17.3)

$$F_{z,F} = F_{z,F,static} + F_{z,F,aero} \pm \Delta F_{z,long} \quad (\text{Eq. 3.17.1})$$

$$F_{z,R} = F_{z,R,static} + F_{z,R,aero} \pm \Delta F_{z,long} \quad (\text{Eq. 3.17.2})$$

$$\Delta F_{z,long} = \frac{ma_x h_{CG}}{L} \quad (\text{Eq. 3.17.3})$$

With  $m$  the vehicle mass,  $a_x$  longitudinal acceleration,  $h_{CG}$  CG height and  $L$  wheelbase. When turning left:

$$F_{z,FL} = \frac{F_{z,F} - \Delta F_{z,F}}{2} \quad (\text{Eq. 3.17.4a})$$

$$F_{z,FR} = \frac{F_{z,F} + \Delta F_{z,F}}{2} \quad (\text{Eq. 3.17.4b})$$

$$F_{z,RL} = \frac{F_{z,R} - \Delta F_{z,R}}{2} \quad (\text{Eq. 3.17.5a})$$

$$F_{z,RR} = \frac{F_{z,R} + \Delta F_{z,R}}{2} \quad (\text{Eq. 3.17.5b})$$

Individual wheel loads were subsequently obtained from the axle total and axle differential with equivalent expressions at the rear, subject to the adopted sign convention (Dixon, 2009; Milliken and Milliken, 1995).

Static (Eq. 3.17.6, Eq. 3.17.7), aerodynamic (Eq. 3.17.8, Eq. 3.17.9, Eq. 3.17.10) and longitudinal terms set axle totals;

$$F_{z,F,static} = mg \frac{b}{L} \quad (\text{Eq. 3.17.6})$$

$$F_{z,R,static} = mg \frac{a}{L} \quad (\text{Eq. 3.17.7})$$

$$F_{z,F,aero} = D_F = \lambda_F D \quad (\text{Eq. 3.17.8})$$

$$F_{z,R,aero} = D_R(1 - \lambda_F)D \quad (\text{Eq. 3.17.9})$$

$$D = \frac{1}{2} \rho V^2 S C_L \quad (\text{Eq. 3.17.10})$$

Where  $\rho$  is air density,  $V$  is vehicle speed,  $S$  is reference area and  $C_L$  represents the coefficient of lift.

For positive  $a_x$  :

$$F_{z,F} = F_{z,F,static} + F_{z,F,aero} - \Delta F_{z,long} \quad (\text{Eq. 3.17.11})$$

$$F_{z,R} = F_{z,R,static} + F_{z,R,aero} + \Delta F_{z,long} \quad (\text{Eq. 3.17.12})$$

Whereas during braking the signs naturally reverse as load moves towards the front.

For an axle:

$$\Delta F_{z,axle} = \Delta F_{z,spring} + \Delta F_{z,ARB} + \Delta F_{z,geom} + \Delta F_{z,damper} \quad (\text{Eq. 3.17.13})$$

This is the quantity that determines how much more load is carried by the outside tyre than the inside tyre during cornering.

Spring (Eq. 3.17.14, Eq. 3.17.15), ARB (Eq.3.17.16), geometric and damper terms set the differential, from which individual wheel loads were formed (Dixon, 2009; Milliken and Milliken, 1995).

$$F_{spring} = k_{\omega} \Delta z \quad (\text{Eq. 3.17.14})$$

$$\Delta F_{z,spring} = k_{\omega} (\Delta z_R - \Delta z_L) \quad (\text{Eq. 3.17.15})$$

Where the linear spring/wheel rate approximation is the wheel rate  $k_{\omega}$  times the wheel displacement  $\Delta z$ . The left-right contribution can therefore be represented as (Eq. 3.17.17) with the precise sign depending on the selected coordinate convention.

$$F_{ARB} = k_{ARB} (\Delta z_R - \Delta z_L) \quad (\text{Eq. 3.17.16})$$

$$\Delta F_{z,ARB} = K_{ARB} \Delta z_{roll} \quad (\text{Eq. 3.17.17})$$

Where  $k_{ARB}$  is the effective anti-roll stiffness at the wheels and  $\Delta z_{roll}$  is the difference in suspension/wheel displacement between the left and right side

Damper force (Eq. 3.17.18) was zero-centred and interpolated from the measured baseline curve before motion-ratio conversion.

$$F_d = f(v_d) \quad (\text{Eq. 3.17.18})$$

Where  $v_d$  is the damper velocity after removal of the static dyno force offset. The resulting damper force was then converted through the measured suspension motion ratio before inclusion in the dynamic wheel-load

reconstruction. Removing the static offset prevented the damper model from altering the measured static corner weights and therefore avoided double-counting preload (Dixon, 2007; Talbott and Starkey, 2002).

Tyre-state reconstruction began from vehicle sideslip  $V_y$  and the planar rigid-body velocity relationships at each wheel location (Eq. 3.17.19, Eq. 3.17-20)

$$V_{x,i} = V_x - ry_i \quad (\text{Eq. 3.17.19})$$

$$V_{y,i} = V_y + rx_i \quad (\text{Eq. 3.17.20})$$

Where  $x_i, y_i$  describe the wheel position relative to the vehicle CG. Tyre slip angle was then obtained from the difference between wheel heading and local velocity direction (Eq. 3.17.21),

$$\alpha_i = \delta_i - \arctan\left(\frac{V_{y,i}}{V_{x,i}}\right) \quad (\text{Eq. 3.17.21})$$

Using the sign convention defined earlier in the methodology. Event-specific toe was incorporated into wheel heading before slip angle was calculated. An additional audit, the front-rear slip-angle relationship was checked independently of the absolute sideslip estimate using yaw-rate and path-curvature geometry (Abe, 2015; Baffet et al., 2009).

For small angles, this consistency relationship can be represented as

$$\alpha_F - \alpha_R \approx \delta - \frac{Lr}{V_x} \quad (\text{Eq. 3.17.22})$$

$$\Rightarrow k \approx \frac{r}{V_x} \Rightarrow \alpha_F - \alpha_R \approx \delta - L\kappa \quad (\text{Eq. 3.17.23})$$

With  $\delta$  as the front road-wheel steer angle.

Agreement between these equivalent forms provided a useful geometry check without depending directly on the reconstructed value of  $\beta$ .

For quasi-steady cornering:

$$r \approx \frac{a_y}{V_x} \quad (\text{Eq. 3.17.24})$$

With  $a_y$  as lateral acceleration and  $V_x$  as longitudinal speed. Lateral acceleration is approximately:

$$a_y \approx V_x r \quad (\text{Eq. 3.17.25})$$

The reduced Magic Formula (Eq. 3.17.26, Eq. 3.17.27, Eq.3.17.28) fixed poorly identifiable shape/load exponents and fitted only a small parameter set plus residual left/right angle offsets, limiting parameter trade-offs when telemetry did not excite all tyre effects (Farroni et al., 2018; Raue et al., 2009).

$$F_y = D \cdot \sin[C \cdot \arctan(B\alpha)] \quad (\text{Eq. 3.17.26})$$

$$D = \mu(F_z)F_z \quad (\text{Eq. 3.17.27})$$

$$B = \frac{C_\alpha}{CD} \quad (\text{Eq. 3.17.28})$$

State reconstruction retained the original 500 Hz signals, while optimisation used a decimated subset for computational efficiency. Validation was performed using all eligible samples (Ljung, 1999; Bendat and Piersol, 2010).

All fallbacks and assumptions were recorded in stage metadata and batch summaries so that relaxed reference criteria could not become implicit tyre-model tuning (Conn et al., 2000; Raue et al., 2009).

### 3.18 Sensitivity philosophy and prevention of retrospective tuning

Sensitivity analysis varied only measured or explicitly uncertain physical inputs, while tyre parameters remained frozen during validation. The final setup update therefore regenerated upstream states before repeating model identification and external transfer (Conn et al., 2000; Raue et al., 2009).

Parameter bounds were not widened simply because an optimum reached them. Boundary activity remained an identifiability warning, and added complexity was accepted only when independent prediction improved without relying on saturated parameters.

## 4 Results

### 4.1 Overview and batch-validation sequence

The results follow the staged reconstruction and validation sequence established in Chapter 3. Stages 0–7 establish signal quality, suspension and path reconstruction, quasi-steady selection, axle-force reconstruction and individual tyre states. Stage 8 tests model complexity on the development data; Stage 9 tests whether the additional load-sensitivity term transfers to independent Silverstone sessions; Stage 10 evaluates the final Silverstone model using leave-one-session-out (LOSO) validation; and Stage 11 applies the frozen model to Brands Hatch and Oulton Park without refitting.

The batch scripts separate pipeline reproducibility from tyre-model fitting. Batch00 audited the additional Silverstone inputs before automated processing, Batch01 repeated Stages 0–7 for seven additional Silverstone sessions, and Batch02 repeated the reconstruction for ten external Brands Hatch and Oulton Park datasets. Selected code excerpts are provided in the Appendix.

### 4.2 Stage 0-1: data quality and signal conditioning

The Silverstone FP1 data were sampled at 500 Hz. Stationary references supported accelerometer offset removal, giving final offsets of  $0.01698 \text{ m/s}^2$  in  $A_x$  and  $0.60616 \text{ m/s}^2$  in  $A_y$ . A 15 Hz zero-phase low-pass filter was retained after the signal-quality audit to reduce high-frequency noise without introducing a net timing shift between steering, yaw-rate and acceleration channels.

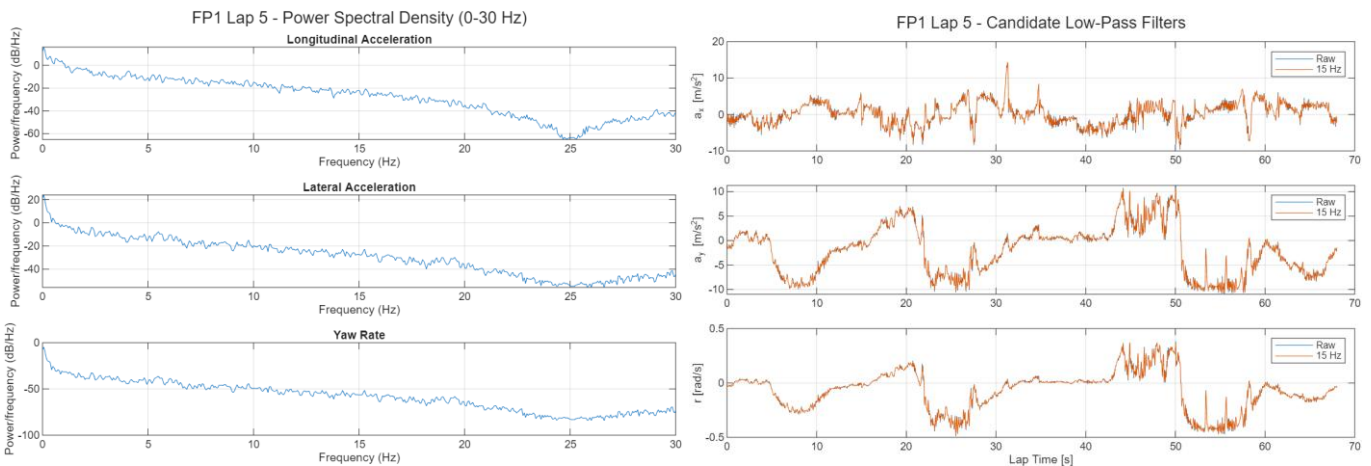


Figure 4.1: Silverstone FP1 Lap 5 signal-frequency analysis and filter validation. Left: power spectral density of longitudinal acceleration, lateral acceleration and yaw rate. Right: raw and 15 Hz zero-phase filtered signals, showing noise reduction while preserving the main vehicle-response trends.

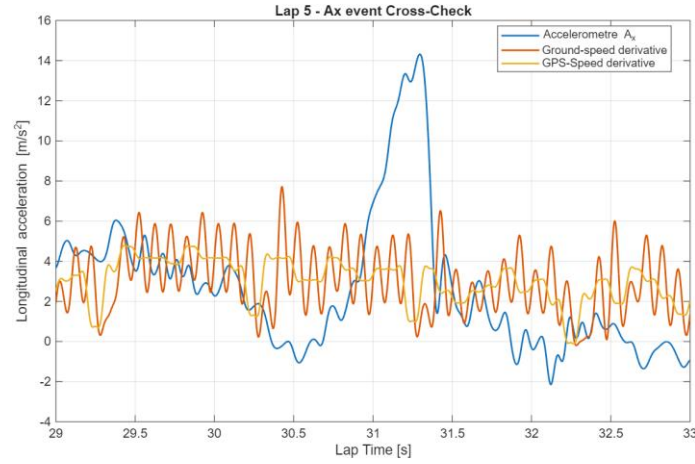


Figure 4.2: Silverstone FP1 Lap 5 longitudinal-acceleration cross-check. The accelerometer  $A_x$  spike is compared with Ground Speed- and GPS Speed-derived acceleration, showing that the event was not supported by the independent speed-based signals.

The isolated positive  $A_x$  excursion on the development lap was not treated as genuine vehicle acceleration because it was inconsistent with speed-derived acceleration. Stage 2 therefore used the filtered Ground/GPS derivative consensus for longitudinal acceleration. Agreement between the independent speed channels was quantified in Stage 3 rather than assumed from visual similarity.

Ground- and GPS-speed derivatives remained highly consistent, with longitudinal-acceleration correlation  $r = 0.9886$  and RMS disagreement of  $0.463 \text{ m/s}^2$ . Here,  $r$  is the Pearson correlation coefficient, where:

- $r = +1 \rightarrow$  perfect positive linear relationship
- $r = 0 \rightarrow$  no linear relationship
- $r = -1 \rightarrow$  perfect negative linear relationship

Therefore,  $r = 0.9886$  indicates a very strong positive correlation between the two quantities.

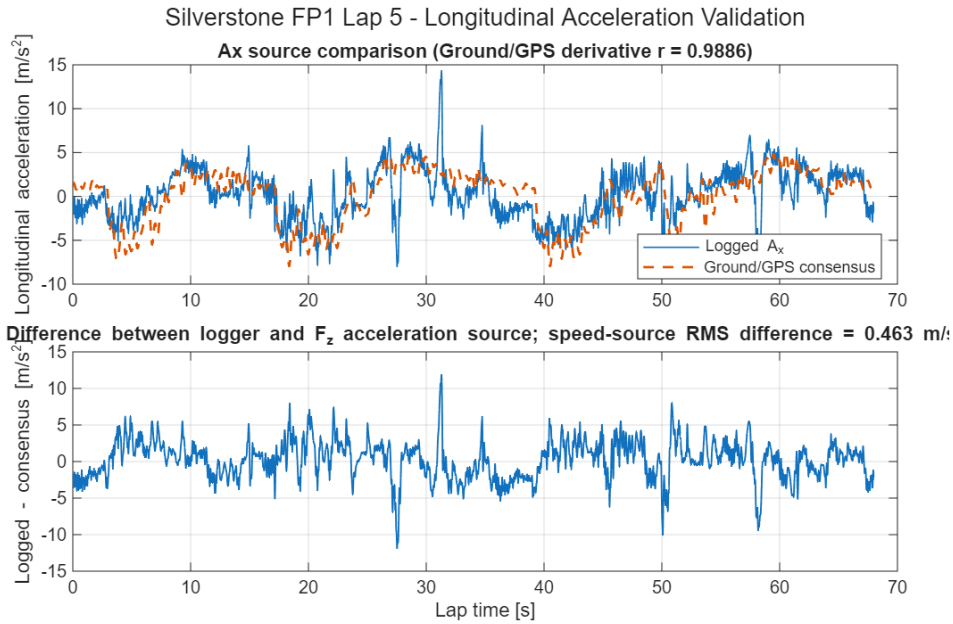


Figure 4.3: Silverstone FP1 Lap 5 longitudinal-acceleration validation. Logged  $A_x$  is compared with the Ground/GPS speed-derivative consensus; the lower panel shows their difference. The speed-derived channels remained strongly correlated ( $r = 0.9886$ ) with an RMS disagreement of  $0.463 \text{ m/s}^2$ .

### 4.3 Stage 2: suspension reconstruction, measured damping and $F_z$

The 2026 suspension inputs were  $131.35 \text{ kN/m}$  front and  $140.10 \text{ kN/m}$  rear springs. Silverstone rear ARB used the derived soft wheel rate of  $40.25 \text{ kN/m}$ ; the front retained the  $63.0 \text{ kN/m}$  mid assumption because the supplied Silverstone setup sheet did not record a front setting.

WinGeo closure remained very small:  $0.000599 \text{ mm}$  RMS at both front corners and approximately  $0.00016 \text{ mm}$  at the rear. Roll versus lateral acceleration remained  $r = 0.8724$  front and  $r = 0.8758$  rear.

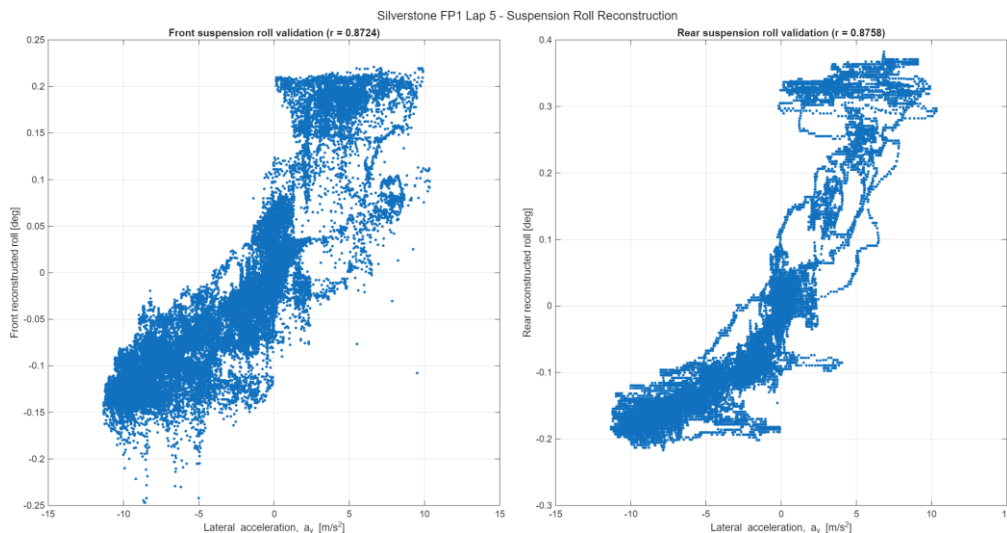


Figure 4.4: Silverstone FP1 Lap 5 suspension-roll validation. Reconstructed front and rear roll responses are plotted against lateral acceleration, showing strong positive correlations of  $r = 0.8724$  and  $r = 0.8758$ , respectively.

The measured baseline damper dyno was applied after the validated 20-sample (0.040 s) velocity alignment. No development-lap samples exceeded the measured dyno envelope. Damper force altered the left-right load difference by 76.9 N RMS at the front and 60.2 N RMS at the rear, with 99.5th-percentile absolute effects of 362.6 N and 283.3 N, respectively (Dixon, 2007; Takahashi, 2018).

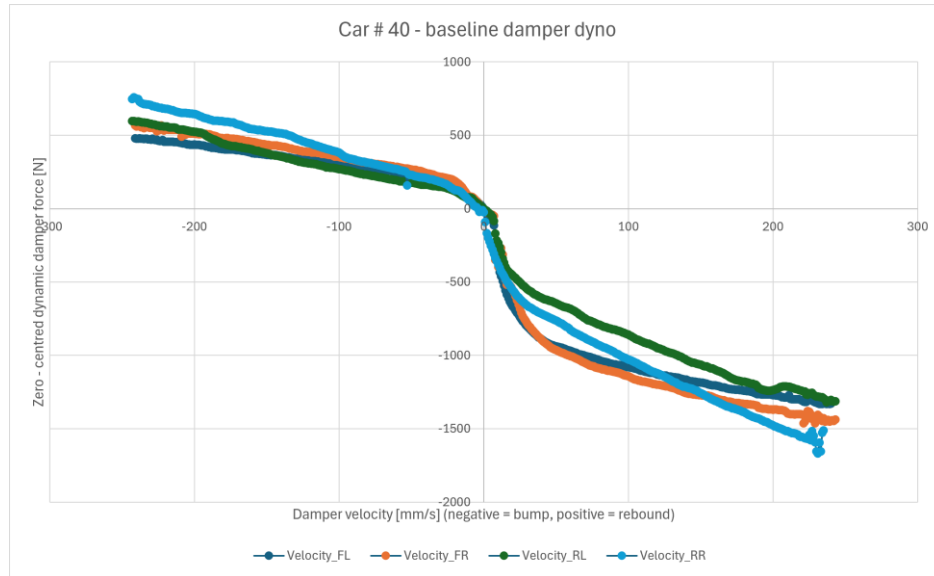


*Figure 4.5: Baseline damper dyno force–velocity characteristics for all four corners of MCR #40. Negative damper velocity represents bump/compression and positive velocity represents rebound/extension.*

Measured baseline damper force–velocity characteristics (Figure 4.5) were used in the Stage 2 wheel-load reconstruction. The curves were zero-centred before interpolation so that static dyno offsets could not alter the measured corner weights.

With spring, ARB, geometric and damper terms combined, load-difference correlation with  $A_y$  was  $r = 0.9205$  at the front and  $r = 0.9337$  at the rear. Independent logger load-difference validation gave  $r = 0.9449$  at the front and  $r = 0.9605$  at the rear.

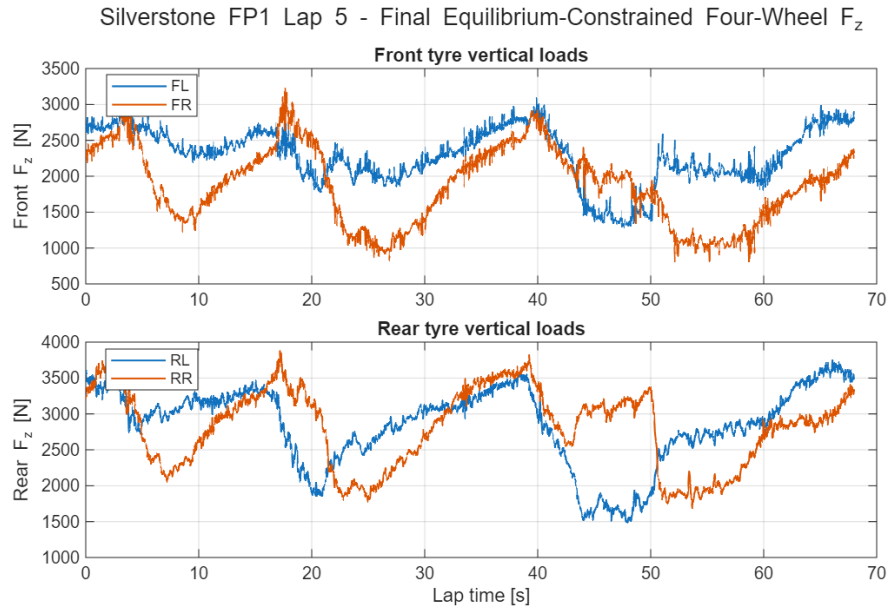


Figure 4.6: Silverstone FP1 Lap 5 reconstructed individual tyre vertical loads,  $F_z$ , for the front and rear axes. The traces show the equilibrium-constrained left-right load variation throughout the lap.

Final  $F_z$  ranges were:

- FL: 1282.0 – 3284.8 N
- FR: 809.0 – 3224.2 N
- RL: 1479.1 – 3747.0 N
- RR: 1685.1 – 3878.1 N

#### 4.4 Stages 3-4: path and sideslip consistency

Ground speed and GPS speed remained almost identical in shape, while curvature and yaw-rate checks continued to provide strong path consistency.

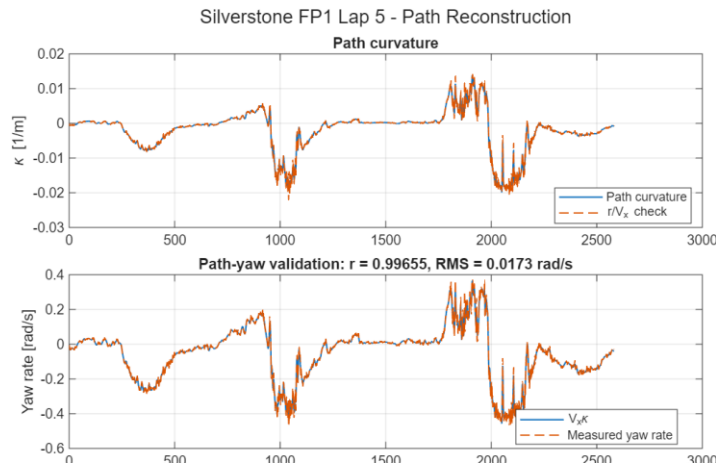


Figure 4.7: Silverstone FP1 Lap 5 path-reconstruction validation. Reconstructed path curvature is compared with  $r/V_x$ , while path-predicted yaw rate  $V_x k$  is compared with measured yaw rate, showing strong agreement ( $r = 0.99655$ ,  $RMS = 0.0173$  rad/s).

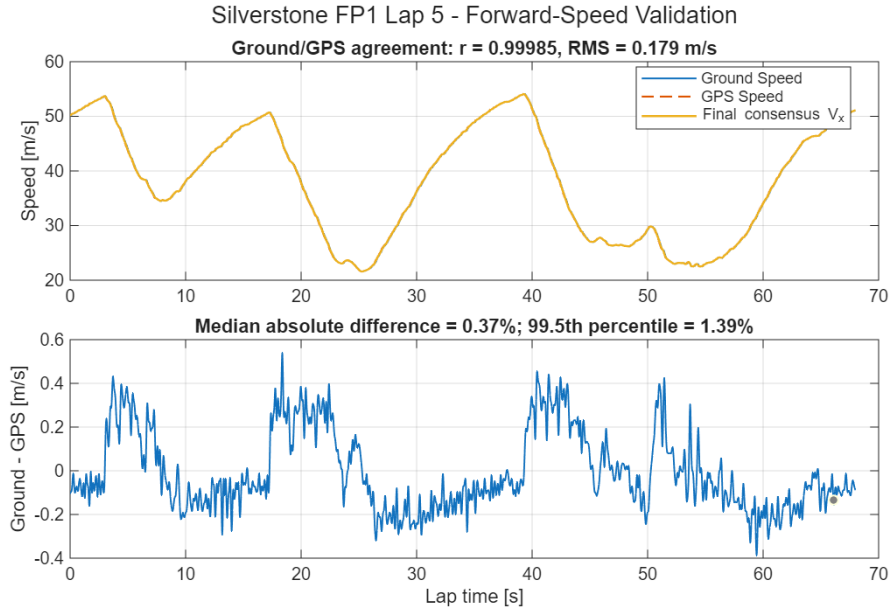


Figure 4.8: Silverstone FP1 Lap 5 forward-speed validation. Ground Speed and GPS Speed showed near-perfect agreement ( $r=0.99985$ ,  $RMSE = 0.179$  m/s), supporting the final consensus  $V_x$  used for subsequent path and tyre-state reconstruction.

The path-based sideslip method retained small  $\beta$  magnitudes and used straight-line segments to constrain drift. Because  $\beta$  was estimated rather than directly measured, the Stage 7 beta-independent checks were retained as the stronger independent structural validation.

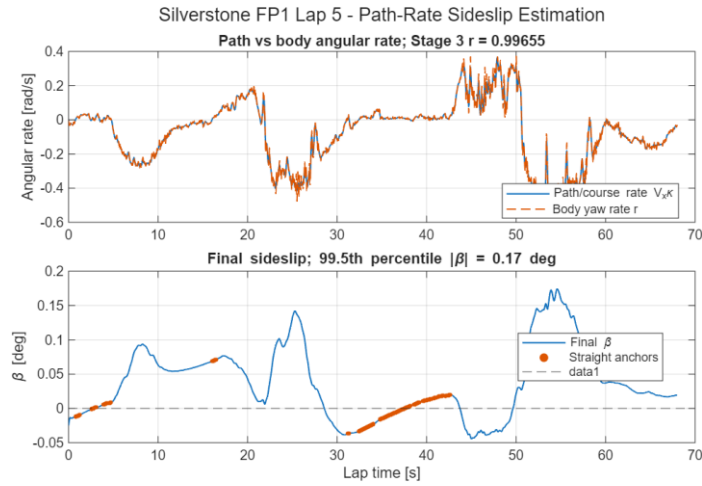


Figure 4.9: Path-rate sideslip estimation for Silverstone FP1 Lap 5.

The upper panel compares path/course rate  $V_{xk}$  with measured yaw rate; the lower panel shows the drift-constrained final sideslip estimate and straight-running anchors.

The final sideslip remained small, with the 99.5th percentile of  $|\beta|$  approximately  $0.17^\circ$ . The secondary  $A_y$ -integrated diagnostic exhibited strong opposite-sign correlation ( $r=-0.9811$ ) and was therefore retained only as a diagnostic rather than used to reverse the sign of the primary estimate. This disagreement is reported

as an uncertainty rather than hidden by retrospective sign selection. The secondary sideslip cross-check and the straight-anchor drift-correction audit are provided in Appendix Figures A.9 and A.10, respectively.

## 4.5 Stages 5-7: identification population and tyre states

Stage 5 retained 6570 quasi-steady samples, 19.32% of Silverstone FP1 Lap 5, with 152 strict steady-state samples.

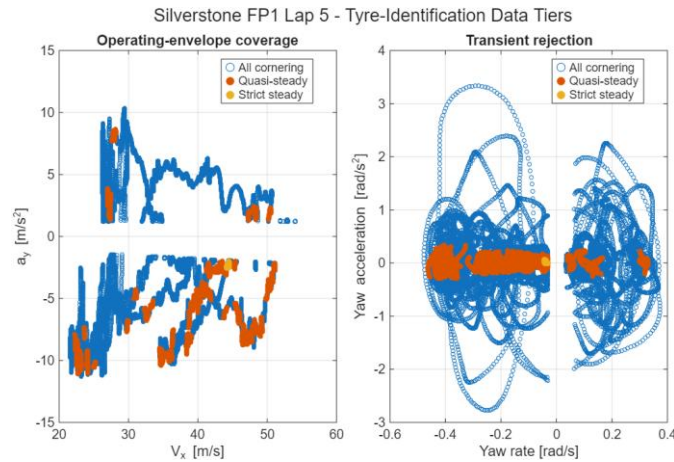


Figure 4.10: Silverstone FP1 Lap 5 tyre-identification data tiers. The quasi-steady and strict-steady masks retain representative lateral-acceleration and speed coverage while excluding regions of high yaw acceleration associated with strongly transient manoeuvres.

The full lap-level manoeuvre classification used to generate these masks is shown in Appendix Figure A.11.

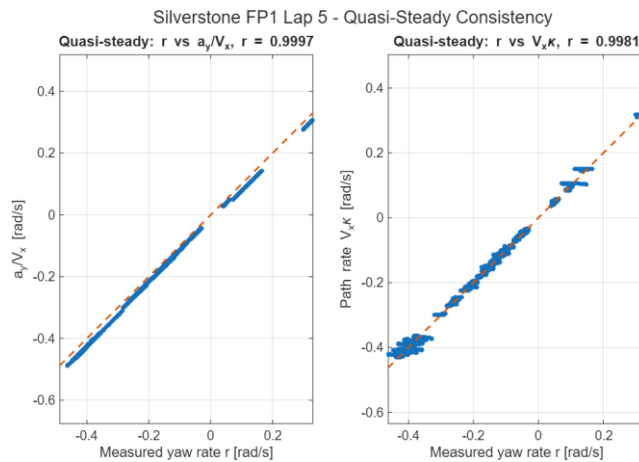


Figure 4.11: Quasi-steady consistency checks for Silverstone FP1 Lap 5. Samples retained by the quasi-steady classifier exhibited near-unity agreement between measured yaw rate and both  $a_y/V_x$  ( $r=0.9997$ ) and path-derived  $V_x \kappa$  ( $r=0.9981$ ), supporting their suitability for subsequent force reconstruction and tyre identification.

Stage 6 reconstructed front and rear axle lateral force from planar equilibrium without dividing axle force between left and right tyres in proportion to  $F_z$ . The approximate two-lumped-mass yaw-inertia sensitivity used  $I_z=1090.6 \text{ kg m}^2$ ; within the identification mask, the resulting yaw-inertia force correction was only 41.9 N RMS, supporting the quasi-steady axle-force formulation. The complete yaw-inertia sensitivity diagnostic is provided in Appendix Figure A.12.

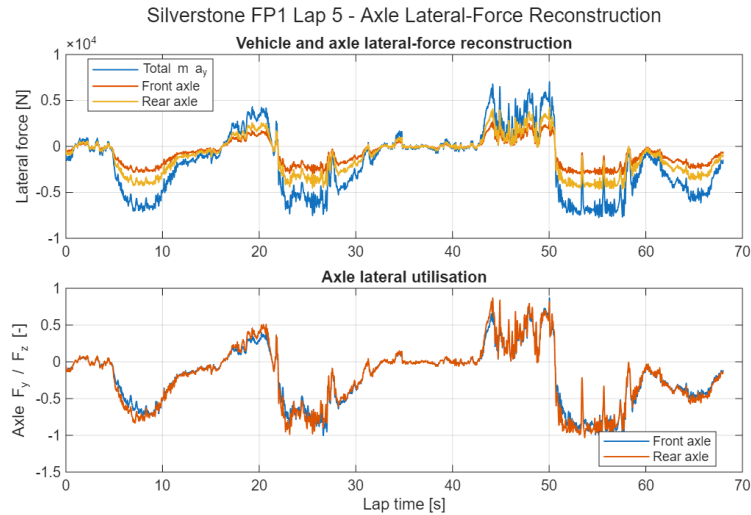


**Figure 4.12: Silverstone FP1 Lap 5 axle lateral-force reconstruction.** Total vehicle lateral force was resolved into front and rear axle contributions using planar equilibrium. The lower panel shows axle lateral utilisation  $F_y/F_z$ , illustrating the normalised lateral-force demand throughout the lap.

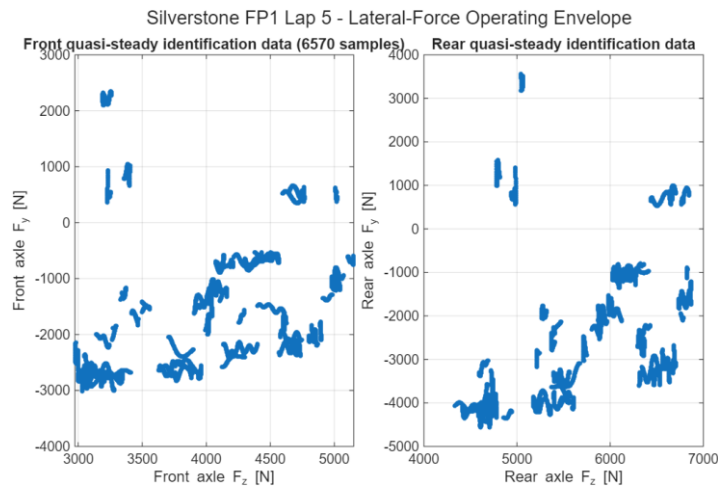


**Figure 4.13: Silverstone FP1 Lap 5 lateral-force operating envelope for the quasi-steady identification data, showing front and rear axle lateral force  $F_y$  against axle vertical load  $F_z$ .**

Stage 7 applied the measured Silverstone static toe of  $+0.30^\circ/-0.30^\circ$  at FL/FR and  $-0.15^\circ/+0.15^\circ$  at RL/RR. Slip-angle numerical closure was zero to the printed precision. The beta-independent  $r/V_x$  relation gave  $r = 1.00000$  with effectively zero RMS residual; the path-curvature form gave  $r = 0.99941$  and  $0.0522^\circ$  RMS residual. The individual FL, FR, RL and RR slip-angle time histories are shown in Appendix Figure A.13.

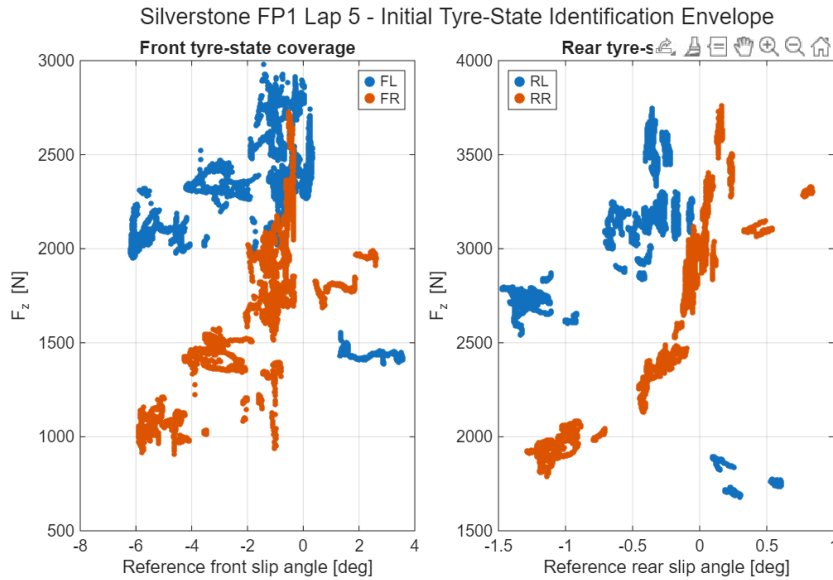


Figure 4.14: Individual tyre-state identification envelope for Silverstone FP1 Lap 5. Reconstructed front and rear tyre slip angles are shown against individual vertical tyre load for the quasi-steady identification data. The distributions demonstrate that the telemetry excites a range of slip-angle and vertical-load conditions at each tyre, defining the operating domain available for subsequent tyre-model identification.

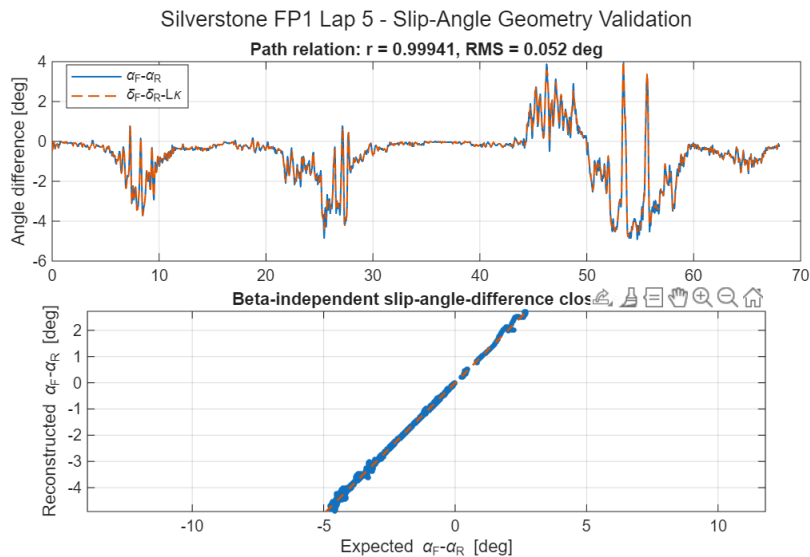


Figure 4.15: Silverstone FP1 Lap 5 beta-independent slip-angle geometry validation. Reconstructed front-to-rear slip-angle difference is compared with the path-based geometric relation, showing strong agreement ( $r = 0.99941$ ,  $RMS = 0.052^\circ$ ).

Front mean slip angle versus axle lateral-force correlation was  $r = 0.78741$ , while the rear correlation was  $r = 0.80866$ . All 34,000 development-lap samples passed the core validity mask.

These results show that the tyre-identification states were not accepted solely on physical plausibility; they also passed kinematic and beta-independent consistency checks before model fitting.

## 4.6 Stage 8: development-lap model complexity test

After the  $|A_x| \leq 2.0 \text{ m/s}^2$  restriction, 4138 samples remained. Thirty-eight contiguous segments produced 3313 training samples and 825 holdout samples.

Table 4.1: Stage 8 holdout performance for baseline and load-sensitive front and rear tyre models, showing RMSE,  $R^2$  and fitted  $p_\mu$ .

| Axle                 | RMSE    | $R^2$                                  | $p_\mu$                                        |
|----------------------|---------|----------------------------------------|------------------------------------------------|
| Front baseline       | 556.2 N | 0.7702                                 | 0                                              |
| Front load-sensitive | 561.2 N | 0.7660<br>(improvement of -<br>0.91 %) | -0.50                                          |
| Rear baseline        | 544.6 N | 0.9040                                 | 0                                              |
| Rear load-sensitive  | 526.1 N | 0.9104<br>(improvement of<br>+3.38 %)  | +0.25 with $C_{a,ref} = 350,000 \text{ N/rad}$ |

The Stage 8 results therefore rejected front load sensitivity and retained rear load sensitivity only as a provisional predictive candidate (Burnham and Anderson, 2002; Raue et al., 2009).

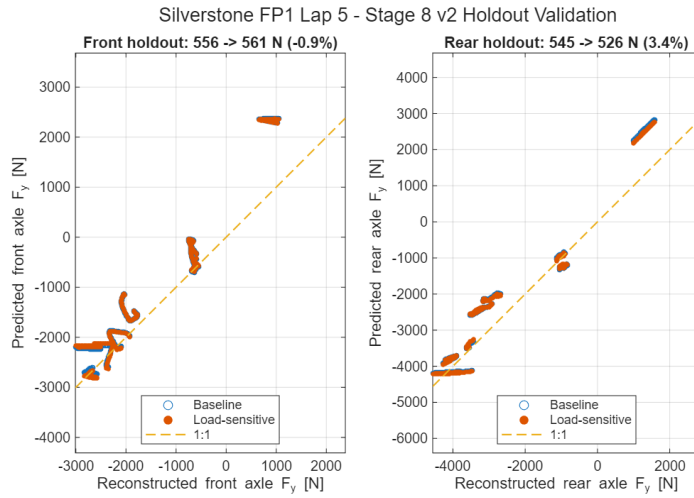


Figure 4.16: Stage 8 holdout validation for the reduced lateral tyre models on Silverstone FP1 Lap 5. Predicted axle lateral force is compared with independently reconstructed axle force for the baseline  $p_{\mu} = 0$  model and the load-sensitive alternative. The additional load-sensitivity term slightly worsened front holdout RMSE from 556 to 561 N (-0.9%), while rear holdout RMSE improved from 545 to 526 N (+3.4%).

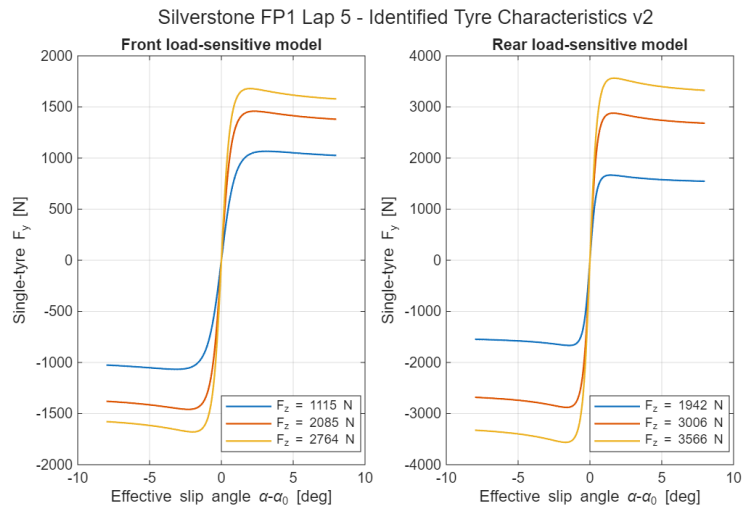


Figure 4.17: Provisional Stage 8 reduced Magic Formula characteristics for Silverstone FP1 Lap 5. Predicted single-tyre lateral force is shown against effective slip angle  $\alpha - \alpha_0$  at representative vertical loads. The curves illustrate the fitted load-dependent force response prior to the multi-session model-selection tests of Stage 9; they should therefore be interpreted as provisional predictive characteristics rather than final physically identified tyre properties.

## 4.7 Batch00 – Batch 01 and Stage 9: independent Silverstone load-sensitivity decision

Batch00 first audited the seven additional Silverstone files for session identification and required Stage 0–7 channels without loading full arrays. Batch01 then applied the validated reconstruction workflow to FP2, FP3, FP4, Qualifying, Race 1, Race 2 and Race 3 without changing the physics equations or thresholds.

One validation lap per session completed successfully: FP2 Lap 1, FP3 Lap 14, FP4 Lap 15, Qualifying Lap 17, Race 1 Lap 4, Race 2 Lap 9 and Race 3 Lap 15. The final Batch01 outcome was therefore 7/7.

Stage 9 froze the Stage 8 baseline and load-sensitive parameter sets and evaluated them on these seven Silverstone datasets without refitting.

Front load sensitivity failed in all seven sessions. Median improvement was  $-23.68\%$ , session win fraction was 0.00, and pooled RMSE increased from 1217.4 N to 1540.4 N.

The rear candidate improved all seven sessions. Median improvement was  $+5.92\%$ , session win fraction was 1.00, and pooled RMSE decreased from 1267.4 N to 1194.6 N.

Despite this rear improvement, the load-sensitive parameters remained boundary-active. Under the predefined identifiability criterion, explicit  $p_\mu$  support was therefore rejected on both axles, and Stage 10 retained  $p_\mu = 0$  (Ljung, 1999; Raue et al., 2009). The corresponding Stage 9 comparison plots are shown in Figure 4.18.

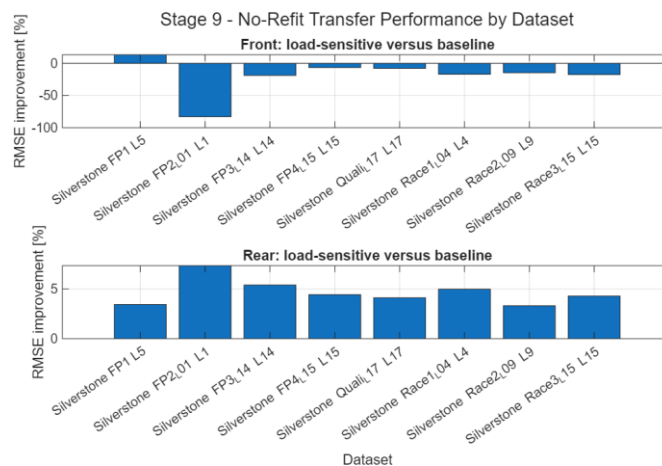


Figure 4.18: Session-by-session RMSE improvement of the Stage 8 load-sensitive candidate relative to the baseline model during independent Silverstone no-refit validation (Stage 9).

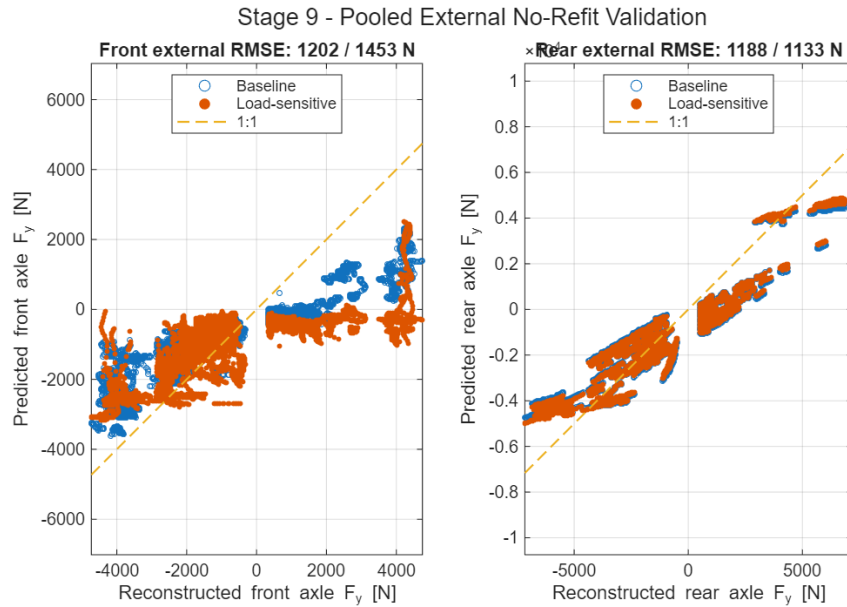


Figure 4.19: Pooled external no-refit validation at Silverstone comparing the baseline and load-sensitive models against reconstructed front and rear axle lateral force (Stage 9).

## 4.8 Stage 10: global Silverstone baseline and LOSO

Eight Silverstone datasets entered the final LOSO analysis.

Table 4.2: Stage 10 Silverstone leave-one-session-out (LOSO) validation performance for the front and rear axle tyre models, showing pooled RMSE,  $R^2$  and median fold RMSE.

| Axle  | RMSE    | $R^2$  | Median fold RMSE |
|-------|---------|--------|------------------|
| Front | 962.0 N | 0.7598 | 872.6 N          |
| Rear  | 981.2 N | 0.8911 | 880.3 N          |

The front fold in which FP1 was held out performed poorly, with  $\text{RMSE} = 1329.5 \text{ N}$  and  $R^2 = -0.4431$ . By contrast, FP3, FP4, Qualifying and Race 1 achieved  $R^2$  values of 0.9179, 0.8332, 0.8968 and 0.8783, respectively. FP1 was therefore less representative of the wider Silverstone front-axle behaviour.

Across LOSO folds, front  $\mu_{\text{ref}} = 1.0558 \pm 0.0452$  and  $C_{\alpha, \text{ref}} = 291,384 \pm 26,504 \text{ N/rad}$ . The final all-session front parameters were  $\mu_{\text{ref}} = 1.05549$ ,  $C_{\alpha, \text{ref}} = 289,996.6 \text{ N/rad}$ ,  $\alpha_{0,L} = +0.59729^\circ$  and  $\alpha_{0,R} = -1.08847^\circ$ . No final front parameter was near an optimisation bound.

The identified front reference cornering stiffness was therefore approximately 290 kN/rad, with moderate fold-to-fold variation and no boundary activity. It is treated as a stable effective model parameter rather than a direct tyre-rig measurement.

Rear LOSO prediction remained strong, but  $\mu_{\text{ref}} = 2.2000$  in every fold, reaching the upper optimisation bound. Final rear parameters were  $\mu_{\text{ref}} = 2.2000$  and  $C_{\alpha,\text{ref}} = 338,571.7 \text{ N/rad}$ , with  $\alpha_{0,L} = +1.53779^\circ$  and  $\alpha_{0,R} = -1.17480^\circ$ . These coefficients are therefore treated as boundary-limited surrogate parameters rather than direct physical tyre constants.

Final all-session performance was  $\text{RMSE} = 881.1 \text{ N}$  and  $R^2 = 0.7985$  at the front, and  $\text{RMSE} = 918.6 \text{ N}$  and  $R^2 = 0.9046$  at the rear. The corresponding LOSO summaries, parameter-stability plots and final characteristics are shown in Figures 4.20–4.23.

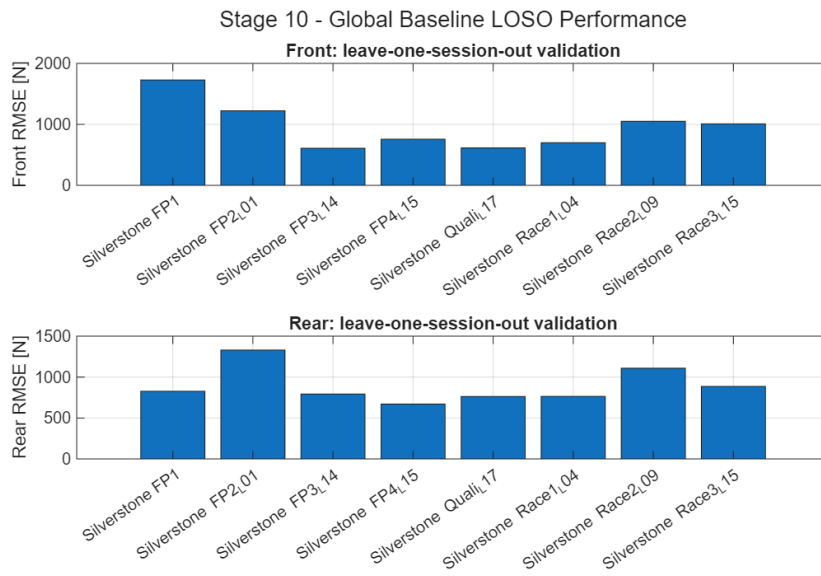


Figure 4.20: Leave-one-session-out RMSE by held-out Silverstone dataset for the final baseline front and rear models

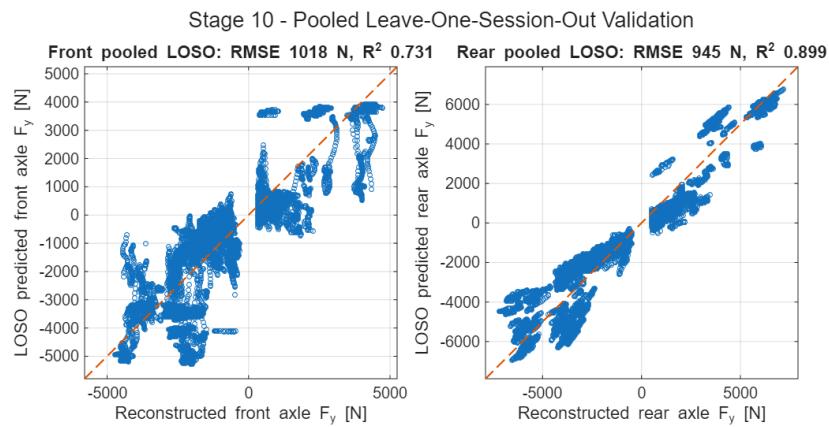


Figure 4.21: pooled leave-one-session-out validation comparing predicted and reconstructed axle lateral force for the final baseline model.

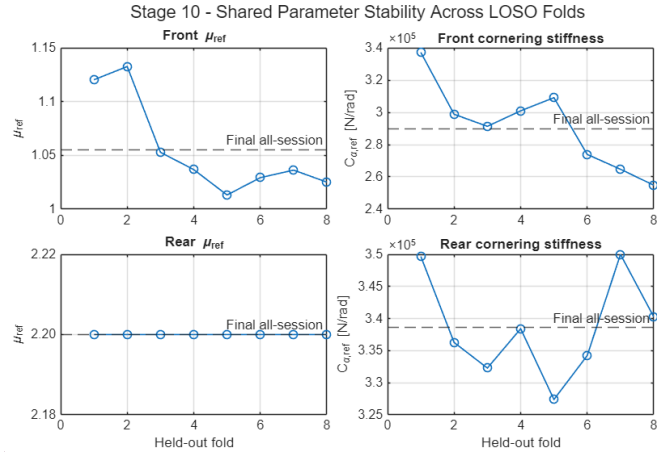


Figure 4.22: stability of the fitted front and rear baseline parameters across the eight leave-one-session-out folds.

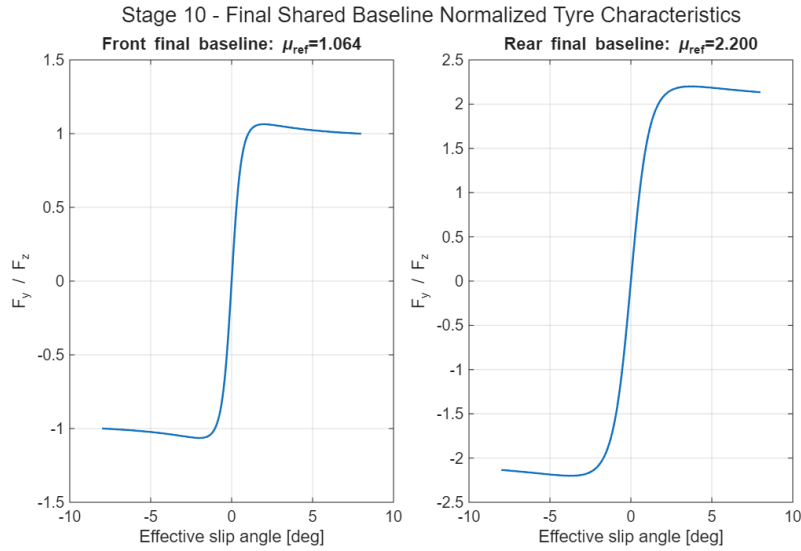


Figure 4.23: Final shared baseline normalised lateral tyre characteristics identified across the authoritative Silverstone datasets.

## 4.9 Batch02 and Stage 11: cross-circuit no-refit validation

Batch02 extended the frozen reconstruction workflow to Brands Hatch and Oulton Park, processing six Brands Hatch and four Oulton Park datasets through Stages 0–7 while leaving the Stage 10 Silverstone tyre parameters unchanged. All ten external datasets completed successfully. This ordering ensured that external state reconstruction was completed before Stage 11, preventing circuit-specific refitting from being introduced during preprocessing.

Stage 11 then applied the frozen Stage 10 model with no optimisation. Ten external datasets were evaluated with the tyre parameters unchanged. Front performance was poor at Brands Hatch Indy and GP, while rear performance remained strong. Oulton Park showed partial front transfer and strong rear transfer.

Table 4.3: Stage 11 cross-circuit validation performance by circuit layout and axle, showing pooled RMSE and  $R^2$  for the frozen Silverstone-derived tyre model.

| Circuit/axle            | RMSE     | $R^2$   |
|-------------------------|----------|---------|
| Brands Hatch Indy/Front | 3469.2 N | -1.8912 |
| Brands Hatch Indy/Rear  | 742.6 N  | 0.9423  |
| Brands Hatch GP/Front   | 3934.8 N | -1.6482 |
| Brands Hatch GP/Rear    | 754.7 N  | 0.9575  |
| Oulton Park/Front       | 1443.5 N | 0.3984  |
| Oulton Park/Rear        | 747.7 N  | 0.9296  |

Oulton Park front transfer was materially better than Brands Hatch. FP1, FP2 and FP3 retained positive  $R^2$ , while Qualifying was approximately neutral at  $R^2 = -0.0297$ . This demonstrates useful no-refit transfer over part, but not all, of the unseen Oulton Park operating domain.

Rear prediction transferred strongly across every layout.

Across all external samples, front RMSE was 2523.3 N with  $R^2 = -0.4338$ , whereas rear RMSE was 747.9 N with  $R^2 = 0.9451$ . Relative to Silverstone LOSO, front RMSE increased by 162.31%, while rear RMSE decreased by 23.78%. No Stage 11 optimisation was performed (Hastie et al., 2009; Stone, 1974). The corresponding Stage 11 per-dataset, pooled and layout-based transfer plots are shown in Figure 4.24-4.27.

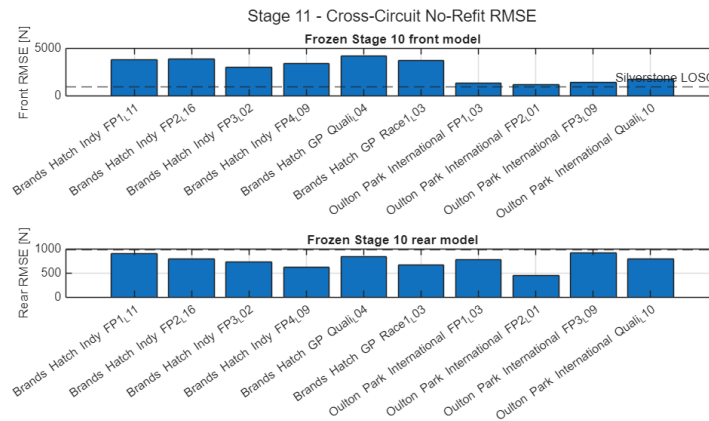


Figure 4.24: Cross-circuit no-refit RMSE for each external dataset using the frozen Stage 10 front and rear parameter sets.

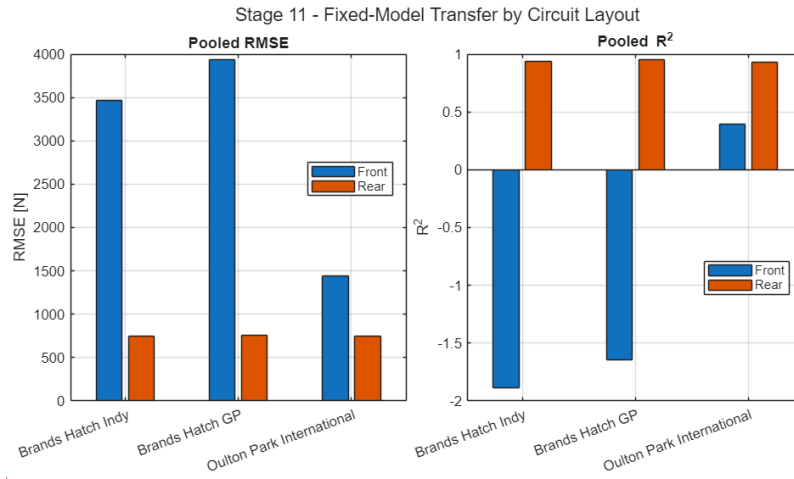


Figure 4.25: Fixed-model transfer performance grouped by circuit layout for the frozen Stage 10 baseline model

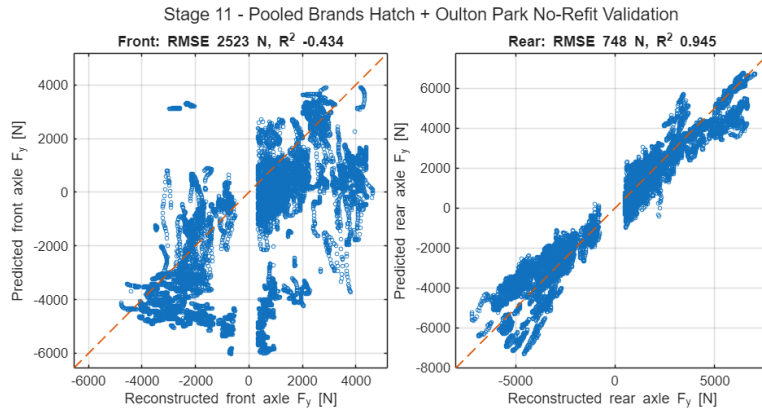


Figure 4.26: Pooled Brands Hatch and Oulton Park no-refit validation comparing predicted and reconstructed front and rear axle lateral force.

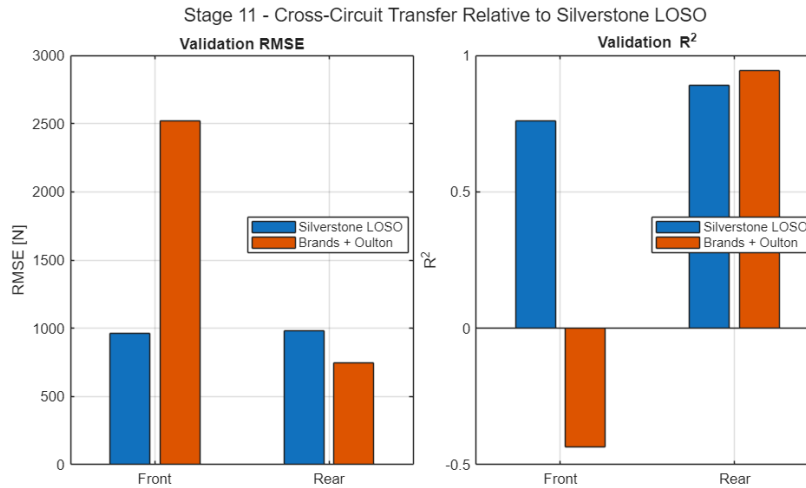


Figure 4.27: Cross-circuit transfer relative to the Silverstone leave-one-session-out benchmark, expressed through pooled RMSE and  $R^2$ .

## 4.10 Final result synthesis

The final data support an axle-dependent conclusion. The rear reduced surrogate is strongly predictive within Silverstone and across the evaluated external circuits, but its boundary-limited coefficients should not be interpreted as uniquely identified physical tyre constants.

The front model is useful within Silverstone and partially transferable to Oulton Park but does not transfer to Brands Hatch. This discrepancy remains after incorporating measured static toe, corrected spring/ARB settings and measured baseline damper force, so it cannot be attributed solely to the earlier zero-toe or suspension assumptions.

The load-sensitivity conclusion remains conservative. Although the Stage 8 rear candidate improved holdout prediction and the Stage 9 rear candidate improved all seven independent Silverstone sessions,  $p_\mu$  remained boundary-active. The final transferable model therefore retains  $p_\mu = 0$  on both axes, separating predictive usefulness from physical identifiability.

## 5 Discussion

### 5.1 Overview

The principal contribution is not a single tyre coefficient, but evidence that useful tyre-force relationships can be reconstructed while preserving a hierarchy between numerical closure, physical state reconstruction, local fit, parameter identifiability, same-circuit generalisation and unseen-circuit transfer. This distinction is important because nonlinear models can predict well even when some fitted coefficients are not uniquely identifiable (Bates and Watts, 1988; Ljung, 1999).

### 5.2 Credibility of the reconstructed states

Ground/GPS speed derivatives were reconstructed, WinGeo 3 closure remained extremely small, and suspension load difference retained strong correlation with  $A_y$  and the independent logger load-difference signal (Baffet et al., 2009; Doumiati et al., 2009).

Stage 7 provided another strong structural check. The slip-angle definition closed numerically, and the beta-independent  $r/V_x$  relation gave  $r = 1.00000$ . The path-curvature relation also remained very strong at  $r = 0.99941$ . These results do not make estimated  $\beta$  a direct measurement, but they make a gross sign or geometry error unlikely (Cheli et al., 2007; Leung et al., 2011).

Measured static toe was applied rather than absorbed into the tyre fit. This improves interpretation of  $\alpha_{0,L/R}$ : they are residual reference/model offsets after alignment correction, not estimates of physical toe. Static camber remains a limitation because only magnitudes were available without an independently confirmed sign convention (Abe, 2015; Pacejka, 2012).

### 5.3 What the load-sensitivity experiment established

Real pneumatic tyres exhibit vertical-load sensitivity, and pressure can influence that behaviour (Guo et al., 2019; Pacejka, 2012). This dissertation asked a narrower question: could an additional  $p_\mu$  parameter be identified robustly from the telemetry? (Gustafsson, 1997; Wang and Wang, 2013).

At the front, the answer was no. Stage 8 holdout error worsened by 0.91% and  $p_\mu$  reached its lower bound; Stage 9 then showed poorer performance in every external Silverstone session, with median improvement of  $-23.68\%$  and pooled RMSE increasing from 1217.4 N to 1540.4 N. This suggests the added parameter

captured development-specific structure rather than transferable load sensitivity (Burnham and Anderson, 2002; Raue et al., 2009).

Rear load sensitivity improved Stage 8 holdout RMSE by 3.38% and all seven Stage 9 Silverstone sessions, with median improvement of 5.92%. However,  $p_\mu$  remained upper-bound active, so a unique physical load-sensitivity coefficient was not identified. The conservative  $p_\mu = 0$  model was therefore retained (Bates and Watts, 1988; Raue et al., 2009). Figures 4.1–4.2 show this contrast.

## 5.4 What Stage 10 identified at Silverstone

Front pooled LOSO performance ( $R^2 = 0.7598$ ) demonstrates useful within-Silverstone prediction, while the all-session  $R^2 = 0.7985$  describes the final shared fit. The final front parameters,  $\mu_{\text{ref}} = 1.05549$  and  $C_{\alpha,\text{ref}} = 289,997$  N/rad, were not boundary-limited but remain effective rather than directly measured tyre parameters (Ljung, 1999; Sierra et al., 2006). Figures 4.3–4.6 summarise LOSO performance, parameter stability and final characteristics.

Front LOSO performance was nevertheless heterogeneous. Holding out FP1 produced  $R^2 = -0.4431$ , whereas several later sessions exceeded  $R^2 = 0.83$ . This suggests that FP1, although useful for developing the reconstruction workflow, was not the most representative front tyre-state dataset. This supports the transition from single-lap fitting to a shared multi-session model.

The rear model achieved  $R^2 = 0.8911$  in LOSO and  $R^2 = 0.9046$  in the all-session fit. However,  $\mu_{\text{ref}}$  remained exactly 2.20 across every fold and the final  $C_{\alpha,\text{ref}}$  was 338,572 N/rad, with the rear parameter set still boundary-limited. Predictive performance and parameter identifiability therefore diverged. The rear model is best treated as a strong effective surrogate rather than a direct measurement of a 2.20 friction coefficient (Bates and Watts, 1988; Raue et al., 2009).

## 5.5 Cross-circuit transfer is axle dependent

The strongest external result is the rear axle. The frozen rear model achieved  $R^2 = 0.9423$  at Brands Hatch Indy,  $R^2 = 0.9575$  at Brands Hatch GP and  $R^2 = 0.9296$  at Oulton Park International, with pooled external  $R^2 = 0.9451$  and RMSE = 747.9 N. External rear RMSE was 23.78% lower than the Silverstone LOSO benchmark. This robustness remained after setup, damper and batch-reconstruction corrections, increasing confidence that the rear relationship captures a repeatable feature of the sampled vehicle/tyre system (Farroni et al., 2018). Figures 4.7–4.10 show the corresponding cross-circuit comparisons.

The front result was not universal. Oulton Park pooled  $R^2 = 0.3984$  demonstrates partial transfer, with FP1, FP2 and FP3 remaining positively predictive and qualifying approximately neutral. Brands Hatch front  $R^2$  remained strongly negative on both Indy and GP layouts. Front transfer is therefore best described as circuit- and operating-domain-dependent rather than simply successful or unsuccessful.

## 5.6 Why the Brands Hatch front discrepancy remains

Even after applying measured static toe, corrected spring/ARB settings, baseline damper forces and time-aligned damper velocity, Brands Hatch front  $R^2$  remained strongly negative.

These corrections reduced known model uncertainty but did not explain the Brands Hatch discrepancy. Because they were measured independently and applied upstream, the remaining error cannot simply be attributed to tyre-model parameters requiring retuning.

Directional residual asymmetry also remained. For example, Brands Hatch Indy FP2 front RMSE was approximately 5230/2502 N left/right, and FP4 approximately 5099/2395 N. This is unlikely to result from a uniform friction change alone; possible missing states include camber, tyre temperature/pressure, tyre history, compliance and transient effects.

These mechanisms remain hypotheses. Pressure, temperature and transient vertical load are known to affect tyre behaviour, but the present data do not identify which omitted state caused the Brands Hatch front discrepancy (Besselink et al., 2010; Takahashi, 2018; Farroni and Sakhnevych, 2022).

Official timing records provide environmental context only. Brands Hatch qualifying was dry at approximately 28–29 °C track temperature, while Race 1 reached approximately 31–34 °C; Silverstone and Oulton Park were also reported as dry (GT World Challenge Europe, 2026; Timing Solutions Ltd, 2026a; Timing Solutions Ltd, 2026b). Because continuous tyre temperature and hot-pressure data were unavailable, thermal variation remains a plausible uncontrolled contributor rather than an identified cause.

## 5.7 Why Oulton Park matters

Oulton Park prevents an overly pessimistic interpretation of the front model. The pooled  $R^2 = 0.3984$ , together with positive results in FP1, FP2 and FP3, shows that a Silverstone-derived front relationship can predict part of an unseen circuit without refitting. This suggests that the poor Brands Hatch result is not a universal failure of the front reconstruction or model family.

The contrast between Brands Hatch and Oulton Park is consistent with, but does not prove, greater operating-domain overlap between Oulton Park and the Silverstone identification data. Transfer would be expected to improve where slip, load, alignment and unmeasured tyre/transient states are more similar to the training domain. Testing more than one external circuit was therefore important: Brands Hatch alone would understate the front model, while Oulton Park FP2 alone could overstate its generality (Perantoni and Limebeer, 2014; Veneri and Massaro, 2020).

## 5.8 Measured damper force and quasi-steady modelling

The measured damper sensitivity provides a useful scale for a previously omitted transient contribution. RMS changes in left–right  $F_z$  difference were 76.9 N at the front and 60.2 N at the rear, with 99.5th-percentile effects of 362.6 N and 283.3 N. Relative to multi-kilonewton axle forces, the RMS contribution is secondary, supporting the focus on quasi-steady identification (Dixon, 2007; Takahashi, 2018).

The larger tail values remain relevant. They justify retaining damping in the final reconstruction and excluding strongly transient events from the steady pure-lateral tyre fit. Dyno hysteresis was also retained as an uncertainty rather than reduced to a single idealised curve (Dixon, 2007; Talbott and Starkey, 2002).

## 5.9 Methodological contribution

The most important methodological decision was to avoid validating the tyre model using quantities generated by the same tyre assumptions.  $\beta$  was estimated independently of the Magic Formula, axle force was reconstructed without a proportional  $-F_z$  left/right split, and external tyre parameters remained frozen. These choices reduce circularity (Baffet et al., 2009; Cheli et al., 2007).

The second contribution is the validation hierarchy. Stage 8 identified locally attractive models; Stage 9 rejected unsupported complexity; Stage 10 distinguished session-specific fit from shared Silverstone behaviour; and Stage 11 tested whether that behaviour transferred across circuits. This progressive validation makes the conclusions more defensible than a single fitted force–slip relationship (Burnham and Anderson, 2002; Stone, 1974).

The third contribution is the separation of predictive utility from parameter identifiability. The rear model is the clearest example: it predicts strongly across circuits while some fitted parameters remain boundary limited. This is not a failure, but an important identification result (Bates and Watts, 1988; Raue et al., 2009).

## 5.10 Limitations

Individual tyre lateral force was not measured directly. Stage 6 reconstructs axle force, while the reduced tyre model predicts the two individual tyre contributions whose sum is compared with that target. This avoids a circular  $F_z$ -based force split, but individual  $F_y$  remains model-dependent (Baffet et al., 2009; Doumiati et al., 2011).

Sideslip is estimated rather than directly measured. The beta-independent Stage 7 checks provide strong structural validation but cannot remove all absolute- $\beta$  uncertainty. Direct dual-antenna GNSS or optical sideslip measurement would provide a valuable future reference (Chindamo et al., 2018; Singh et al., 2019).

Measured yaw inertia was unavailable, although the quasi-steady reconstruction reduces dependence on yaw acceleration. Signed static camber was not independently confirmed, aerodynamic coefficients remain nominal rather than map-based, and tyre temperature, hot pressure, wear and tyre-set history were not continuous model states (Casanova et al., 2000; Katz, 2006).

Damper data were measured, but the numerical mapping between the DY6 Baseline/Max/Mid/Min traces and event click positions was unavailable. The final damper term is therefore best described as a measured baseline sensitivity model rather than fully event-click-specific damping.

## 5.11 Future work

The highest-value next step would be controlled tyre-rig lateral-force testing using the same tyre specification, loads and pressures. This would allow direct comparison between the telemetry-derived surrogate and a controlled tyre reference (Farroni et al., 2018; Tsinias and Mavros, 2019).

The preliminary research dissertation already provides a practical architecture for this work. The proposed constrained rig comprises five functional subsystems: a structural load frame, force/moment measuring hub, steering mechanism for slip-angle control, vertical-loading mechanism, and data-acquisition/control system. Such a rig would allow slip angle and vertical load to be prescribed independently and provide the direct force measurements unavailable from telemetry. Its first role should be validation of the present lateral-force surrogate rather than immediate identification of a large combined-slip parameter set.

Measured yaw inertia confirmed signed static camber and documented damper-click calibration would remove important vehicle-model uncertainties. If camber can be established reliably, a limited camber-sensitivity term could then be tested using the same independent-validation approach.

Continuous tyre temperature and hot-pressure measurement would be particularly valuable for investigating the Brands Hatch front discrepancy, as thermal and pressure states can materially affect tyre-force behaviour (Besselink et al., 2010; Farroni et al., 2019).

The original race-weekend plan also remains useful future work. CAN TPMS could record pressure and temperature continuously, while tyre-set/run history and structured driver feedback could identify degradation, surface or interruption effects. These variables should be tested against residual structure rather than used retrospectively to explain poor fits.

Finally, the batch framework could be expanded from one representative lap per session to all valid laps using hierarchical session/lap holdout. Combined-slip and transient models should be considered only after the current quasi-steady baseline remains validated, as relaxation-length and higher-fidelity models require richer measurement and excitation (Gipser, 2007; Schmeitz, 2004).

## 5.12 Overall interpretation

The final study supports a staged inverse-reconstruction methodology rather than a universal tyre-parameter claim. The reconstructed vehicle states remained coherent after measured setup and damper corrections. Explicit  $p_{\mu}$ load sensitivity was not independently identifiable under the predefined criteria, so the conservative baseline was retained.

Within Silverstone, the shared model achieved pooled LOSO  $R^2 = 0.7598$  at the front and  $R^2 = 0.8911$  at the rear. Across unseen circuits, the rear surrogate retained strong predictive performance, with pooled  $R^2 = 0.9451$  and RMSE = 747.9 N. Front transfer was domain-dependent: Oulton Park retained pooled  $R^2 = 0.3984$ , while poor Brands Hatch performance reduced the all-external front result to  $R^2 = -0.4338$ .

The persistence of the Brands Hatch front discrepancy after the principal measured setup corrections narrows the interpretation but does not establish a unique cause. The strength of the dissertation is therefore in defining that boundary: what the telemetry supports, what remains uncertain, and where additional measurement is required before stronger physical claims can be made.

## 5.13 Practical interpretation and use of the identified characteristics

For Silverstone simulation or comparative setup work, the shared front and rear curves are better supported than the development-lap parameters because they were fitted across eight sessions and evaluated by LOSO.

The LOSO error is therefore the appropriate empirical within-Silverstone transfer scale (Hastie et al., 2009; Stone, 1974).

The rear surrogate has the strongest external evidence:  $R^2$  remained above 0.92 across both Brands Hatch layouts and Oulton Park without refitting. Boundary-limited coefficients still prohibit interpreting  $\mu_{ref}$  as a universal physical friction coefficient or extrapolating beyond the validated load/slip range (Bates and Watts, 1988; Ljung, 1999).

The front curve should be treated as a Silverstone-referenced effective characteristic. It transfers usefully over part of Oulton Park but not Brands Hatch, so it should not be used as a universal front tyre law without additional state information.

Because measured static toe is already included, the fitted left/right nuisance offsets may represent sensor datum, compliance, camber or other residual asymmetry. They are computational offsets and should not be interpreted as physical alignment settings.

Rear boundary activity indicates that the data constrain the observed force mapping more strongly than its decomposition into  $\mu_{ref}$  and  $C\alpha_{ref}$ . Prediction metrics and boundary flags must therefore be reported together (Ljung, 1999).

When an additional coefficient is physically plausible but poorly constrained, it should be retained only if held-out prediction improves without problematic boundary activity. Otherwise, the simpler model is the more defensible choice (Akaike, 1974; Burnham and Anderson, 2002).

The Oulton/Brands contrast shows why circuit transfer should not be summarised by one pooled metric. Layout-, session- and direction-level results reveal where the model is useful and where it fails as the distribution of speed, steering, load and transients changes.

## 5.14 Relationship of the final findings to prior literature

Published observer studies show that sideslip, normal force and lateral tyre force can be inferred from onboard sensors and vehicle-dynamic models. This study extends that principle into a staged tyre-identification and circuit-transfer framework (Baffet et al., 2009; Doumiati et al., 2011).

Controlled tests vary operating states more independently than race telemetry. On track, load and slip covary, so flexible coefficients may fit numerically without being uniquely physical. The boundary-active  $p_{\mu}$  result therefore indicates limited identifiability rather than evidence for a specific load-sensitivity law (Farroni et al., 2018; Pacejka, 2012).

Pressure, temperature and transient loading remain plausible contributors to the circuit-dependent front residuals, but the present data do not establish causation. Additional measurement is therefore preferable to retrospective coefficient tuning (Besselink et al., 2010; Takahashi, 2018).

Complete-session and cross-domain holdouts are more demanding than randomly interleaved correlated telemetry samples. The progression from segment holdout to LOSO and unseen-circuit validation therefore applies established validation principles to this inverse tyre-identification problem (Hastie et al., 2009; Stone, 1974).

## 6 Conclusion

### 6.1 Conclusions against the research questions

RQ1 – Can the required vehicle and tyre states be reconstructed consistently from telemetry? Yes, within the stated uncertainty. Independent speed derivatives, suspension closure, load-difference validation, path/yaw agreement and beta-independent slip geometry supported the upstream reconstruction. The measured-damper update did not destabilise these checks.

RQ2 – Can a reduced lateral tyre model reproduce independent Silverstone data? Yes. The final conservative baseline achieved pooled LOSO  $R^2 = 0.7598$  front and  $R^2 = 0.8911$  rear across eight complete-session folds.

RQ3 – Does explicit vertical-load sensitivity improve true predictive performance? Not sufficiently to support a physical  $p_\mu$  parameter. The front candidate deteriorated on every external Silverstone session. The rear candidate produced a modest improvement, but  $p_\mu$  remained boundary-active. The final model therefore retained  $p_\mu = 0$  on both axles.

RQ4 – Are fitted parameters stable when complete sessions are removed? The front model showed reasonable but imperfect stability and no final boundary flag. The rear model retained strong predictive performance but remained boundary-limited in  $\mu_{\text{ref}}$  and  $C_{\alpha,\text{ref}}$ , showing that predictive performance does not guarantee unique physical parameter identification.

RQ5 – Does a Silverstone-derived model transfer to unseen circuits? The answer is axle- and domain-dependent. Rear transfer was very strong across Brands Hatch and Oulton Park, reaching pooled external  $R^2 = 0.9451$  and RMSE = 747.9 N. Front transfer remained useful at Oulton Park ( $R^2 = 0.3984$ ) but failed at Brands Hatch, giving an all-external pooled  $R^2 = -0.4338$ . This shows that a telemetry-derived surrogate can generalise strongly on one axle while remaining circuit-sensitive on the other.

### 6.2 Engineering contribution

The project provides a reproducible route from ordinary race telemetry to a defensible effective tyre-force surrogate without requiring that every tyre quantity be directly measured. The strongest contribution is not the final parameter vector but the architecture of the validation: each reconstructed state is audited before optimisation, extra model complexity is challenged on unseen sessions, and final parameters are frozen before cross-circuit testing.

The measured 2026 setup update also demonstrates how new physical information should be incorporated. Spring rates, ARB setting, static toe and damper force were corrected upstream, and derived damper-velocity channels were time-aligned to the master telemetry time base before the complete validation chain was rerun. Batch01 regenerated seven of seven Silverstone validation datasets and Batch02 regenerated ten of ten external validation datasets across Brands Hatch and Oulton Park. External tyre parameters were not tuned to recover Brands Hatch performance. This preserves scientific separation between better measurement and better fitting.

## 6.3 Final limitations and future work

The final conclusions remain limited by the absence of direct tyre-rig lateral-force data, individual-wheel  $F_y$  measurements, measured yaw inertia, confirmed signed static camber, and continuous tyre temperature, pressure, wear and history data. The baseline damper curves were measured but were not fully mapped to event click settings.

Future work should prioritise controlled tyre-rig comparison, direct yaw-inertia measurement, signed camber confirmation, documented damper-click calibration and continuous tyre-state measurement. The preliminary research dissertation provides a practical route through its proposed constrained-rig architecture, wheel-mounted CAN TPMS programme and structured driver-feedback plan. Once direct lateral-force, tyre-state and alignment measurements are available, the model could be extended systematically to include camber sensitivity, self-aligning torque and combined-slip behaviour. These additions would be more valuable than immediately increasing Magic Formula complexity because they directly address the information limitations identified by the present validation.

## 6.4 Final statement

The dissertation shows that race telemetry can reveal useful and transferable tyre-force behaviour, if modelling claims remain consistent with the information available in the data. The rear surrogate demonstrated strong external predictive performance, while the front model showed that transfer can depend strongly on operating domain. By retaining failed tests, boundary warnings and cross-circuit discrepancies rather than tuning them away, the final workflow provides a more defensible engineering model and a clearer basis for future measurement.

## 6.5 Contribution to engineering practice and research

For engineering practice, the workflow demonstrates that existing race telemetry can be used for more than driver or setup comparison. When signal provenance, suspension kinematics and validation are treated carefully, the same data can constrain effective tyre-force behaviour and quantify where that behaviour transfers. This is particularly relevant to teams with detailed telemetry but limited access to controlled tyre-rig testing.

For research, the study provides a practical example of identifiability under imperfect measurement. The rear result is especially instructive: prediction generalises strongly while individual coefficients remain boundary limited. The front result provides the complementary case, where same-circuit prediction is useful but external transfer remains domain-dependent. Together, these findings show why parameter confidence, validation domain and prediction error should be reported separately rather than reduced to a single claim of model accuracy.

The frozen workflow therefore provides a foundation for future tyre-rig comparison, thermal-state augmentation and combined-slip modelling without losing the validated baseline. Additional complexity can be introduced as a testable hypothesis and evaluated against the same session- and circuit-level validation hierarchy rather than replacing the current model wholesale.

## 7 References

- Abe, M. (2015) *Vehicle Handling Dynamics: Theory and Application*. 2nd edn. Oxford: Butterworth-Heinemann. doi:10.1016/C2014-0-04001-4.
- Akaike, H. (1974) 'A new look at the statistical model identification', *IEEE Transactions on Automatic Control*, 19(6), pp. 716–723. doi:10.1109/TAC.1974.1100705.
- Alvarez, L., Yi, J., Horowitz, R. and Olmos, L. (2005) 'Dynamic Friction Model-Based Tire-Road Friction Estimation and Emergency Braking Control', *Journal of Dynamic Systems, Measurement, and Control*, 127(1), pp. 22–32. doi:10.1115/1.1870036.
- Baffet, G., Charara, A. and Lechner, D. (2009) 'Estimation of vehicle sideslip, tire force and wheel cornering stiffness', *Control Engineering Practice*, 17(11), pp. 1255–1264. doi:10.1016/j.conengprac.2009.05.005.
- Baffet, G., Charara, A., Lechner, D. and Thomas, D. (2008) 'Experimental evaluation of observers for tire–road forces, sideslip angle and wheel cornering stiffness', *Vehicle System Dynamics*, 46(6), pp. 501–520. doi:10.1080/00423110701493963.
- Bakker, E., Nyborg, L. and Pacejka, H.B. (1987) 'Tyre Modelling for Use in Vehicle Dynamics Studies', *SAE Technical Paper 870421*. doi:10.4271/870421.
- Bates, D.M. and Watts, D.G. (1988) *Nonlinear Regression Analysis and Its Applications*. New York: Wiley. doi:10.1002/9780470316757.
- Bendat, J.S. and Piersol, A.G. (2010) *Random Data: Analysis and Measurement Procedures*. 4th edn. Hoboken, NJ: Wiley. doi:10.1002/9781118032428.
- Besselink, I.J.M., Schmeitz, A.J.C. and Pacejka, H.B. (2010) 'An improved Magic Formula/Swift tyre model that can handle inflation pressure changes', *Vehicle System Dynamics*, 48(sup1), pp. 337–352. doi:10.1080/00423111003748088.
- Best, M.C., Gordon, T.J. and Dixon, P.J. (2000) 'An Extended Adaptive Kalman Filter for Real-time State Estimation of Vehicle Handling Dynamics', *Vehicle System Dynamics*, 34(1), pp. 57–75. doi:10.1076/0042-3114(200008)34:1;1-K;FT057.
- Bevly, D.M., Ryu, J. and Gerdes, J.C. (2006) 'Integrating INS Sensors With GPS Measurements for Continuous Estimation of Vehicle Sideslip, Roll, and Tire Cornering Stiffness', *IEEE Transactions on Intelligent Transportation Systems*, 7(4), pp. 483–493. doi:10.1109/TITS.2006.883110.

- Burnham, K.P. and Anderson, D.R. (2002) *Model Selection and Multimodel Inference: A Practical Information-Theoretic Approach*. 2nd edn. New York: Springer. doi:10.1007/b97636.
- Butterworth, S. (1930) 'On the Theory of Filter Amplifiers', *Experimental Wireless and the Wireless Engineer*, 7, pp. 536–541. Available at: [https://www.changpuak.ch/electronics/downloads/On\\_the\\_Theory\\_of\\_Filter\\_Amplifiers.pdf](https://www.changpuak.ch/electronics/downloads/On_the_Theory_of_Filter_Amplifiers.pdf).
- Canudas de Wit, C., Olsson, H., Åström, K.J. and Lischinsky, P. (1995) 'A New Model for Control of Systems with Friction', *IEEE Transactions on Automatic Control*, 40(3), pp. 419–425. doi:10.1109/9.376053.
- Casanova, D., Sharp, R.S. and Symonds, P. (2000) 'Minimum Time Manoeuvring: The Significance of Yaw Inertia', *Vehicle System Dynamics*, 34(2), pp. 77–115. Available at: <https://www.tandfonline.com/doi/abs/10.1076/0042-3114%28200008%2934%3A2%3B1-G%3BFT077>.
- Cheli, F., Sabbioni, E., Pesce, M. and Melzi, S. (2007) 'A methodology for vehicle sideslip angle identification: comparison with experimental data', *Vehicle System Dynamics*, 45(6), pp. 549–563. doi:10.1080/00423110601059112.
- Chindamo, D., Lenzo, B. and Gadola, M. (2018) 'On the Vehicle Sideslip Angle Estimation: A Literature Review of Methods, Models, and Innovations', *Applied Sciences*, 8(3), 355. doi:10.3390/app8030355.
- Conn, A.R., Gould, N.I.M. and Toint, P.L. (2000) *Trust Region Methods*. Philadelphia: SIAM. doi:10.1137/1.9780898719857.
- Costa, R.P. and Bortolussi, R. (2016) 'Lap Time Simulation of Formula SAE Vehicle With Quasi-steady State Model', *SAE Technical Paper 2016-36-0164*. doi:10.4271/2016-36-0164.
- Dixon, J.C. (1996) *Tires, Suspension and Handling*. 2nd edn. Warrendale, PA: SAE International. doi:10.4271/R-168.
- Dixon, J.C. (2007) *The Shock Absorber Handbook*. 2nd edn. Chichester: Wiley. doi:10.1002/9780470516430.
- Dixon, J.C. (2009) *Suspension Geometry and Computation*. Chichester: Wiley. doi:10.1002/9780470682906.
- Doumiati, M., Victorino, A., Charara, A. and Lechner, D. (2009) 'Lateral load transfer and normal forces estimation for vehicle safety: experimental test', *Vehicle System Dynamics*, 47(12), pp. 1511–1533. doi:10.1080/00423110802673091.

- Doumiati, M., Victorino, A., Charara, A. and Lechner, D. (2011) 'Onboard Real-Time Estimation of Vehicle Lateral Tire–Road Forces and Sideslip Angle', *IEEE/ASME Transactions on Mechatronics*, 16(4), pp. 601–614. doi:10.1109/TMECH.2010.2048118.
- Doumiati, M., Victorino, A., Charara, A., Lechner, D. and Baffet, G. (2013) *Vehicle Dynamics Estimation using Kalman Filtering*. Chichester: Wiley. doi:10.1002/9781118578988.
- Doyle, D.A., Cunningham, G., White, G. and Early, J. (2019) 'Lap Time Simulation Tool for the Development of an Electric Formula Student Car', *SAE Technical Paper 2019-01-0163*. doi:10.4271/2019-01-0163.
- Dugoff, H., Fancher, P.S. and Segel, L. (1970) 'An Analysis of Tire Traction Properties and Their Influence on Vehicle Dynamic Performance', *SAE Technical Paper 700377*. doi:10.4271/700377.
- Farroni, F. and Sakhnevych, A. (2022) 'Tire multiphysical modeling for the analysis of thermal and wear sensitivity on vehicle objective dynamics and racing performances', *Simulation Modelling Practice and Theory*, 117, 102517. doi:10.1016/j.simpat.2022.102517.
- Farroni, F., Lamberti, R., Mancinelli, N. and Timpone, F. (2018) 'TRIP-ID: A tool for a smart and interactive identification of Magic Formula tyre model parameters from experimental data acquired on track or test rig', *Mechanical Systems and Signal Processing*, 102, pp. 1–22. doi:10.1016/j.ymssp.2017.07.025.
- Farroni, F., Russo, M., Sakhnevych, A. and Timpone, F. (2019) 'TRT EVO: Advances in real-time thermodynamic tire modeling for vehicle dynamics simulations', *Proceedings of the Institution of Mechanical Engineers, Part D*, 233(1), pp. 121–135. doi:10.1177/0954407018808992.
- Gadola, M., Vetturi, D., Cambiaghi, D. and Manzo, L. (1996) 'A Tool for Lap Time Simulation', *SAE Technical Paper 962529*. doi:10.4271/962529.
- GT World Challenge Europe (2026) Sports 2000, GT World Challenge Europe powered by AWS Sprint Cup Round 2, Brands Hatch, 2–3 May 2026: qualifying and race results. Official event timing document.
- Gillespie, T.D. (1992) *Fundamentals of Vehicle Dynamics*. Warrendale, PA: SAE International. doi:10.4271/R-114.
- Gipser, M. (2007) 'FTire – the tire simulation model for all applications related to vehicle dynamics', *Vehicle System Dynamics*, 45(sup1), pp. 139–151. doi:10.1080/00423110801899960.
- Guiggiani, M. (2018) *The Science of Vehicle Dynamics: Handling, Braking, and Ride of Road and Race Cars*. 2nd edn. Cham: Springer. doi:10.1007/978-3-319-73220-6.

- Guo, K., Chen, P., Xu, N., Yang, C. and Li, F. (2019) 'Tire Side Force Characteristics with the Coupling Effect of Vertical Load and Inflation Pressure', *SAE International Journal of Vehicle Dynamics, Stability, and NVH*, 3(1), pp. 19–30. doi:10.4271/10-03-01-0002.
- Gustafsson, F. (1996) 'Determining the initial states in forward-backward filtering', *IEEE Transactions on Signal Processing*, 44(4), pp. 988–992. doi:10.1109/78.492552.
- Gustafsson, F. (1997) 'Slip-based tire-road friction estimation', *Automatica*, 33(6), pp. 1087–1099. doi:10.1016/S0005-1098(97)00003-4.
- Hastie, T., Tibshirani, R. and Friedman, J. (2009) *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*. 2nd edn. New York: Springer. doi:10.1007/978-0-387-84858-7.
- International Organization for Standardization (2011) *ISO 8855:2011 Road vehicles — Vehicle dynamics and road-holding ability — Vocabulary*. Geneva: ISO. Available at: <https://www.iso.org/standard/51180.html>.
- Katz, J. (2006) 'Aerodynamics of Race Cars', *Annual Review of Fluid Mechanics*, 38, pp. 27–63. doi:10.1146/annurev.fluid.38.050304.092016.
- Khaleghian, S., Emami, A. and Taheri, S. (2017) 'A technical survey on tire-road friction estimation', *Friction*, 5(2), pp. 123–146. doi:10.1007/s40544-017-0151-0.
- Leung, K.T., Whidborne, J.F., Purdy, D. and Dunoyer, A. (2011) 'A review of ground vehicle dynamic state estimations utilising GPS/INS', *Vehicle System Dynamics*, 49(1–2), pp. 29–58. doi:10.1080/00423110903406649.
- Li, B., Yang, X. and Yang, J. (2014) 'Tire Model Application and Parameter Identification – A Literature Review', *SAE International Journal of Passenger Cars – Mechanical Systems*, 7(1), pp. 231–243. doi:10.4271/2014-01-0872.
- Ljung, L. (1999) *System Identification: Theory for the User*. 2nd edn. Upper Saddle River, NJ: Prentice Hall. Available at: <https://books.google.com/books?q=Ljung+System+Identification+Theory+for+the+User+2nd+1999>.
- Lugner, P., Pacejka, H.B. and Plöchl, M. (2005) 'Recent advances in tyre models and testing procedures', *Vehicle System Dynamics*, 43(6–7), pp. 413–436.
- Matuško, J., Petrović, I. and Perić, N. (2008) 'Neural network based tire/road friction force estimation', *Engineering Applications of Artificial Intelligence*, 21(3), pp. 442–456. doi:10.1016/j.engappai.2007.05.001.

- Maurice, J.P. and Pacejka, H.B. (1997) 'Relaxation Length Behaviour of Tyres', *Vehicle System Dynamics*, 27(sup1), pp. 339–342. doi:10.1080/00423119708969668.
- Milliken, W.F. and Milliken, D.L. (1995) *Race Car Vehicle Dynamics*. Warrendale, PA: SAE International. Available at: <https://www.sae.org/publications/books/content/r-146/>.
- Pacejka, H.B. (2012) *Tire and Vehicle Dynamics*. 3rd edn. Oxford: Butterworth-Heinemann. doi:10.1016/C2010-0-68548-8.
- Pacejka, H.B. and Bakker, E. (1992) 'The Magic Formula Tyre Model', *Vehicle System Dynamics*, 21(sup1), pp. 1–18. doi:10.1080/00423119208969994.
- Pasterkamp, W.R. and Pacejka, H.B. (1997) 'The Tyre as a Sensor to Estimate Friction', *Vehicle System Dynamics*, 27(5–6), pp. 409–422. doi:10.1080/00423119708969339.
- Perantoni, G. and Limebeer, D.J.N. (2014) 'Optimal control for a Formula One car with variable parameters', *Vehicle System Dynamics*, 52(5), pp. 653–678. doi:10.1080/00423114.2014.889315.
- Rajamani, R. (2012) *Vehicle Dynamics and Control*. 2nd edn. New York: Springer. doi:10.1007/978-1-4614-1433-9.
- Rajamani, R., Phanomchoeng, G., Piyabongkarn, D. and Lew, J.Y. (2012) 'Algorithms for real-time estimation of individual wheel tire-road friction coefficients', *IEEE/ASME Transactions on Mechatronics*, 17(6), pp. 1183–1195. doi:10.1109/TMECH.2011.2159240.
- Raue, A., Kreutz, C., Maiwald, T., Bachmann, J., Schilling, M., Klingmüller, U. and Timmer, J. (2009) 'Structural and practical identifiability analysis of partially observed dynamical models by exploiting the profile likelihood', *Bioinformatics*, 25(15), pp. 1923–1929. doi:10.1093/bioinformatics/btp358.
- SAE International (2022) *SAE J670\_202206 Vehicle Dynamics Terminology*. SAE International. doi:10.4271/J670\_202206.
- Schmeitz, A.J.C. (2004) *A semi-empirical three-dimensional model of the pneumatic tyre rolling over arbitrarily uneven road surfaces*. Doctoral thesis. Delft University of Technology. Available at: <https://repository.tudelft.nl/record/uuid:eba07a1d-d69e-4fd5-b03c-91ff47ef5a90>.
- Sharp, R.S. and Bettella, M. (2003) 'Shear Force and Moment Descriptions by Normalisation of Parameters and the 'Magic Formula'', *Vehicle System Dynamics*, 39(1), pp. 27–56. doi:10.1076/vesd.39.1.27.8244.

Sport Racing Car Club (SRCC) (2026) The SRCC Sports 2000 Championship 2026: Sporting and Technical Regulations. Published Copy.

Sierra, C., Tseng, E., Jain, A. and Peng, H. (2006) 'Cornering stiffness estimation based on vehicle lateral dynamics', *Vehicle System Dynamics*, 44(sup1), pp. 24–38. doi:10.1080/00423110600867259.

Simon, D. (2006) *Optimal State Estimation: Kalman,  $H^\infty$ , and Nonlinear Approaches*. Hoboken, NJ: Wiley. doi:10.1002/0470045345.

Singh, K.B., Arat, M.A. and Taheri, S. (2019) 'Literature review and fundamental approaches for vehicle and tire state estimation', *Vehicle System Dynamics*, 57(11), pp. 1643–1665. doi:10.1080/00423114.2018.1544373.

Stone, M. (1974) 'Cross-Validatory Choice and Assessment of Statistical Predictions', *Journal of the Royal Statistical Society: Series B*, 36(2), pp. 111–133. doi:10.1111/j.2517-6161.1974.tb00994.x.

Takahashi, T. (2018) 'The influence of tyre transient side force properties on vehicle lateral acceleration for a time-varying vertical force', *Vehicle System Dynamics*, 56(5), pp. 734–752. doi:10.1080/00423114.2017.1367016.

Talbott, M.S. and Starkey, J. (2002) 'An Experimentally Validated Physical Model of a High-Performance Mono-Tube Damper', *SAE Technical Paper 2002-01-3337*. doi:10.4271/2002-01-3337.

Timing Solutions Ltd (2026a) Sports 2000 Championship: MSVR Club Car Championships, Silverstone National, 4–5 April 2026. Timing and results.

Timing Solutions Ltd (2026b) Sports 2000 Championship: MSVR Club Car Championships, Oulton Park International, 13 June 2026. Timing and results.

Tsinias, V. and Mavros, G. (2019) 'Efficient experimental identification of three-dimensional tyre structural properties', *Proceedings of the Institution of Mechanical Engineers, Part D*. doi:10.1177/0954407018773561.

Veneri, M. and Massaro, M. (2020) 'A free-trajectory quasi-steady-state optimal-control method for minimum lap-time of race vehicles', *Vehicle System Dynamics*, 58(6), pp. 933–954. doi:10.1080/00423114.2019.1608364.

Wang, R. and Wang, J. (2013) 'Tire–road friction coefficient and tire cornering stiffness estimation based on longitudinal tire force difference generation', *Control Engineering Practice*, 21(1), pp. 65–75. doi:10.1016/j.conengprac.2012.09.009.

Welch, P.D. (1967) 'The use of fast Fourier transform for the estimation of power spectra: A method based on time averaging over short, modified periodograms', *IEEE Transactions on Audio and Electroacoustics*, 15(2), pp. 70–73. doi:10.1109/TAU.1967.1161901.

## 8 Appendix

### Appendix A – Key MATLAB Implementation Extracts

This appendix reproduces concise extracts from the final validated MATLAB workflow. The extracts are included to make the computational implementation auditable without reproducing the complete Stage 0–11 codebase in the dissertation. They preserve the key numerical and methodological decisions discussed in Chapter 3; the full scripts remain part of the project submission files.

#### A.1 Signal conditioning and zero-phase filtering

Source: Stage01\_Signal\_Conditioning.m

```
fc = 15;

dynFilter = designfilt("lowpassiir", ...
    "FilterOrder", 4, ...
    "HalfPowerFrequency", fc, ...
    "SampleRate", data.Fs);

data.Ax_Filt = filtfilt(dynFilter, data.Ax_Zeroed);
data.Ay_Filt = filtfilt(dynFilter, data.Ay_Zeroed);
data.YawRate_Filt = filtfilt(dynFilter, data.YawRate);
```

#### A.2 Damper-channel alignment and measured-dyno lookup

Sources: Batch01\_Silverstone\_Stage0\_to\_Stage7\_v11\_DAMPER2026\_FIXED\_v2.m and  
Stage02\_Vehicle\_Parameters\_DAMPER2026\_BASELINE\_v2\_CORRECTED.m

```
x = interp1( ...
    tUnique, xUnique, tMaster, ...
    "linear", NaN);

vClip = v;
finiteV = isfinite(v);
vClip(finiteV) = min(max(v(finiteV),vMin),vMax);
vClip(~finiteV) = NaN;

damperForceCell{iDamper} = interp1( ...
    DamperDyno2026.(c).Velocity_mmpps, ...
    DamperDyno2026.(c).ForceMid_N, ...
    vClip, "linear", NaN);
```

#### A.3 Four-wheel vertical-load reconstruction and closure

Source: Stage02\_Vehicle\_Parameters\_DAMPER2026\_BASELINE\_v2\_CORRECTED.m

```
FzFL_N = 0.5*(FzFrontEq_N - FzDiffFront_N);
FzFR_N = 0.5*(FzFrontEq_N + FzDiffFront_N);
FzRL_N = 0.5*(FzRearEq_N - FzDiffRear_N);
FzRR_N = 0.5*(FzRearEq_N + FzDiffRear_N);

frontClosure_N = (FzFL_N + FzFR_N) - FzFrontEq_N;
```

```
frontClosureValid = isfinite(frontClosure_N);

assert(max(abs(frontClosure_N(frontClosureValid))) < 1e-8, ...
    "Front Fz algebraic closure failed.");
```

## A.4 Path-rate sideslip estimation

Source: Stage04\_Sideslip\_PathRate.m

```
betaDotRaw_rads = rPath - rBody;

betaDotFilt_rads = zeroPhaseLowPassContinuous( ...
    betaDotRaw_rads, betaRateFilter_Hz, Fs);

betaDotBias_rads = ...
    median(betaDotFilt_rads(anchorMask),'omitnan');

betaDotCorrected_rads = ...
    betaDotFilt_rads - betaDotBias_rads;

betaRaw_rad = cumtrapz(t,betaDotCorrected_rads);
```

## A.5 Axle lateral-force reconstruction

Source: Stage06\_Lateral\_Force\_Reconstruction.m

```
FyFront_QS_N = ...
    frontForceFraction_QS .* FyTotal_N;

FyRear_QS_N = ...
    rearForceFraction_QS .* FyTotal_N;

forceBalanceResidual_QS_N = ...
    FyFront_QS_N + FyRear_QS_N - FyTotal_N;

yawMomentResidual_QS_Nm = ...
    a.*FyFront_QS_N - b.*FyRear_QS_N;
```

## A.6 Individual tyre slip-angle reconstruction and closure

Source: Stage07\_Tyre\_State\_Reconstruction\_SETUP2026.m

```
velocityAngle_rad.FL = atan2( ...
    VyWheelBody.FL, VxWheelBody.FL);

alphaRef_rad.FL = angleWrapPi( ...
    deltaRef_rad.FL - velocityAngle_rad.FL);

alphaFromTyreVelocity_rad.FL = angleWrapPi( ...
    -atan2(VyTyre.FL,VxTyre.FL));

alphaClosure_deg.FL = rad2deg(angleWrapPi( ...
    alphaRef_rad.FL-alphaFromTyreVelocity_rad.FL));
```

## A.7 Reduced Magic Formula and load-sensitivity term

Source: Stage08\_Reduced\_MF\_Identification\_v2.m

```
lambda = max(Fz./FzRef,0.05);

mu = muRef.*lambda.^pMu;
D = mu.*Fz;
Ca = CaRef.*lambda.^pCaFixed;
B = Ca./max(Cshape.*D,1.0);

alphaEff = alpha-deg2rad(alpha0_deg);
```

```
Fy = D.*sin(Cshape.*atan(B.*alphaEff));
```

## A.8 LOSO and strict external no-refit validation

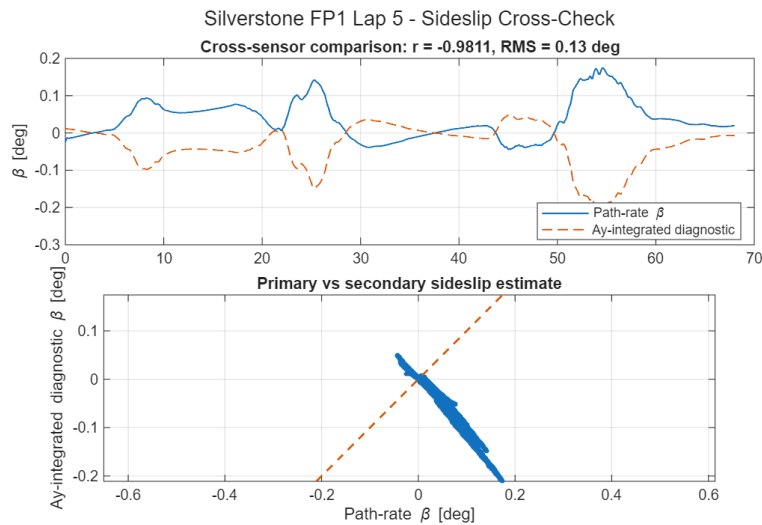
Sources: Stage10\_Global\_Baseline\_LOSO.m and Stage11\_CrossCircuit\_NoRefit\_Validation\_v2.m

```
for iHold = 1:nDatasets
    trainIdx = setdiff(1:nDatasets,iHold);
    frontFit = fitGlobalBaseline( ...
        Datasets(trainIdx), "Front", bounds, ...
        initialFront, nStartsLOSO, rngSeed+100*iHold, ...
        FzRefFront, Cshape, pCaFixed);
end

% Stage 11: Stage 10 front/rear parameters remain fixed.
% No Stage 11 optimisation is performed.
```

## A.9 Secondary sideslip cross-check

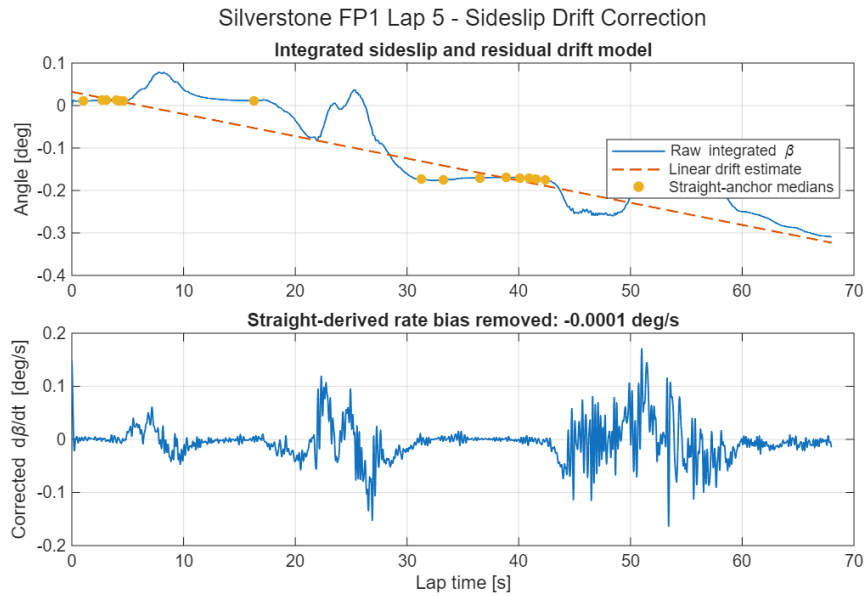
The secondary acceleration-integrated sideslip estimate was retained as an audit rather than used as the primary state estimate. It showed strong magnitude correspondence but opposite sign relative to the path-rate estimate ( $r = -0.9811$ ;  $\text{RMS} = 0.13^\circ$ ). Because this sign discrepancy could not be resolved independently, the diagnostic was not used to reverse or tune the primary path-rate sideslip estimate.



Appendix Figure A.9 Silverstone FP1 Lap 5 sideslip cross-check. The primary path-rate beta estimate is compared with the secondary Ay-integrated diagnostic. The strong opposite-sign correlation ( $r=-0.9811$ ) demonstrates that the secondary estimate was unsuitable as an absolute-sign reference and was therefore retained only as a diagnostic.

## A.10 Sideslip integration-drift correction

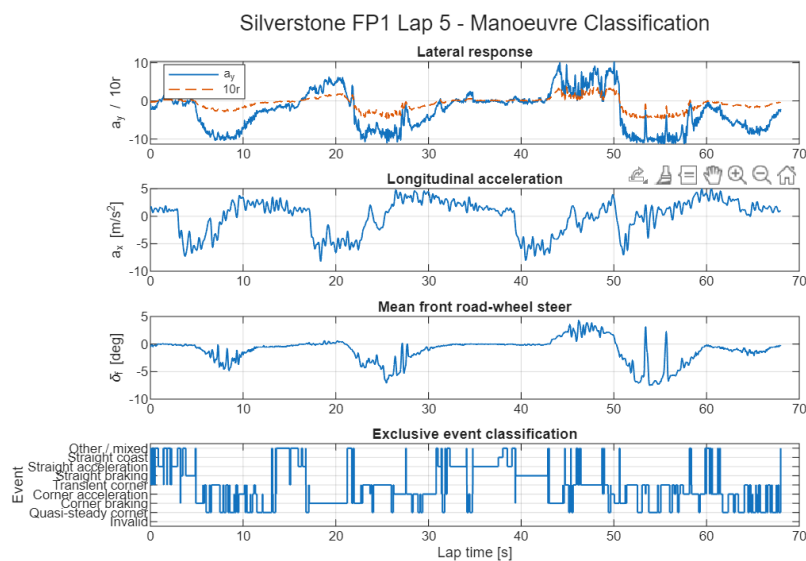
Because sideslip was obtained by integrating the path-rate relation, even a small residual rate bias could accumulate into a non-physical offset. Straight-running segments were therefore used as anchors to estimate and remove slow integration drift without forcing agreement with the tyre model.



Appendix Figure A.10. Silverstone FP1 Lap 5 sideslip drift-correction audit. The upper panel shows the raw integrated beta, the straight-anchor medians and the estimated linear drift; the lower panel shows the corrected beta-rate signal. The straight-derived rate bias was approximately  $-0.0001^\circ/\text{s}$ .

## A.11 Lap-level manoeuvre classification

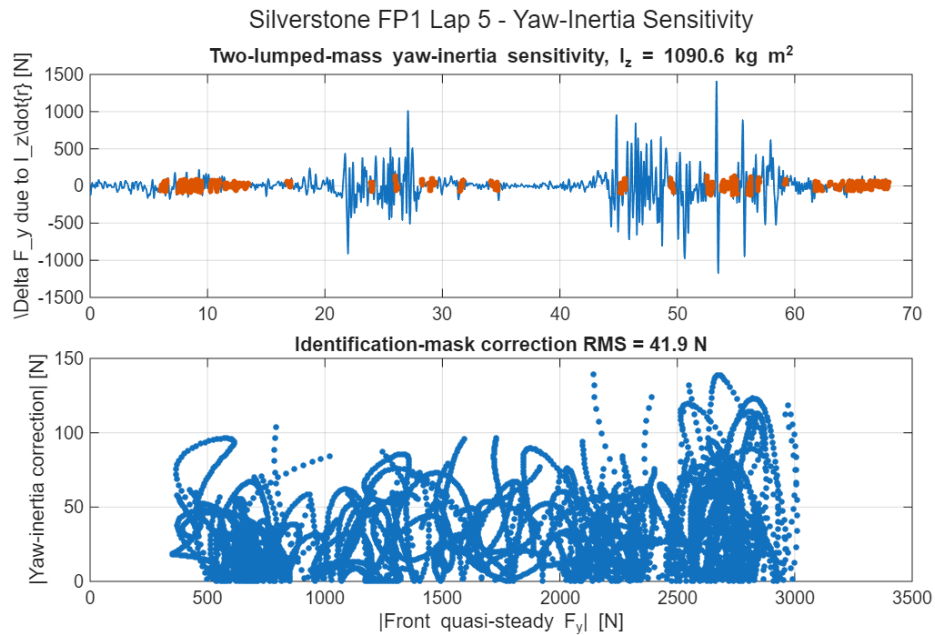
The complete manoeuvre classifier was inspected over the development lap to confirm that the quasi-steady mask resulted from the intended combination of lateral response, longitudinal acceleration, road-wheel steer and transient behaviour rather than from isolated hand-selected regions.



Appendix Figure A.11. Silverstone FP1 Lap 5 manoeuvre classification. Lateral acceleration, longitudinal acceleration and mean front road-wheel steer are shown above the exclusive event classification used to separate quasi-steady cornering from braking, acceleration, transient and straight-line operation.

## A.12 Yaw-inertia sensitivity

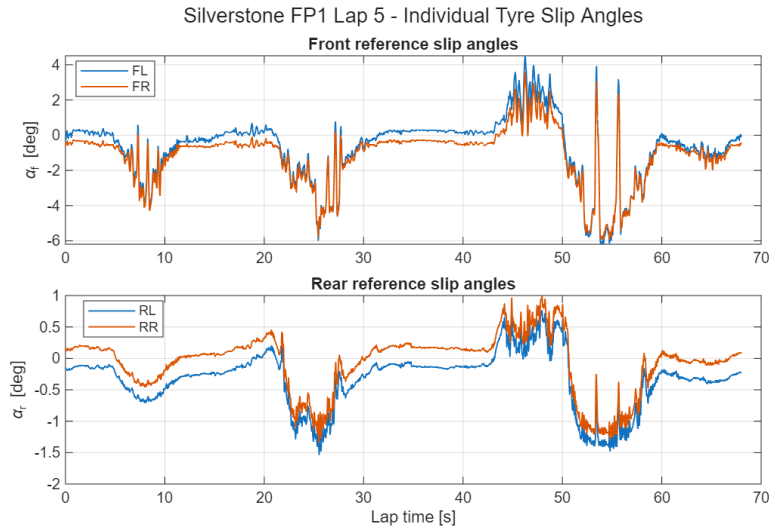
Measured yaw inertia was unavailable, so the nominal quasi-steady force split did not rely on an unverified inertia value. Instead, a two-lumped-mass estimate of  $I_z = 1090.6 \text{ kg m}^2$  was used as a sensitivity check to quantify the possible yaw-acceleration correction. Within the identification mask, this correction remained small relative to the reconstructed axle force, with an RMS magnitude of 41.9 N.



Appendix Figure A.12. Silverstone FP1 Lap 5 yaw-inertia sensitivity. The upper panel shows the axle-force correction associated with the two-lumped-mass  $I_z$  estimate over the full lap; the lower panel isolates its magnitude within the quasi-steady identification population, for which the correction RMS was 41.9 N.

## A.13 Individual tyre slip-angle histories

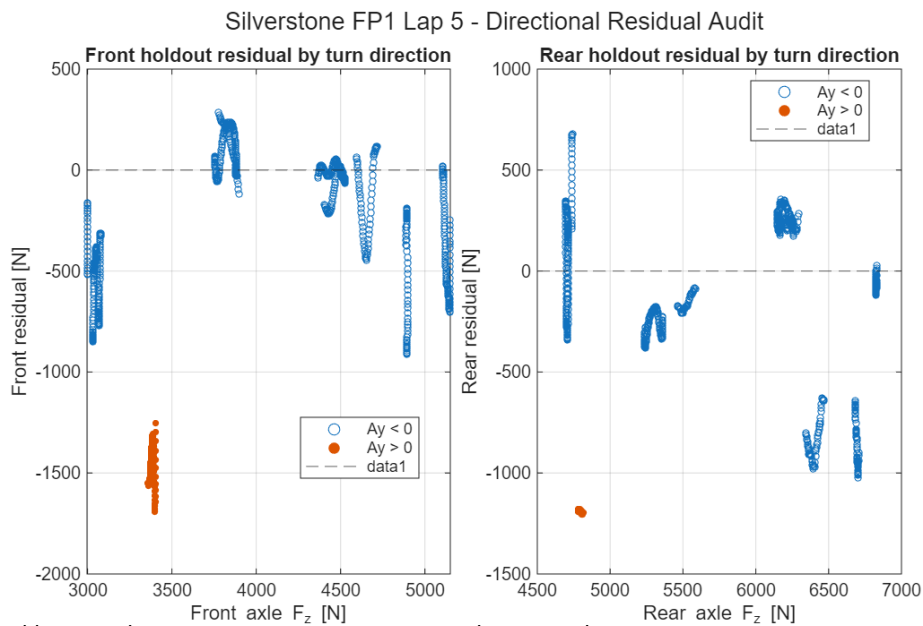
Stage 7 generated tyre-specific slip-angle histories rather than assigning a single axle-average angle to both tyres. Differences between the left and right traces result from local wheel-centre velocities, steering geometry and event-specific toe applied before slip-angle calculation.



Appendix Figure A.13. Silverstone FP1 Lap 5 individual tyre slip-angle histories. Reconstructed front-left, front-right, rear-left and rear-right reference slip angles are shown over the complete development lap, illustrating the tyre-specific states produced by the Stage 7 geometry.

## A.14 Stage 8 directional residual audit

Holdout residuals were also separated by turn direction to expose systematic mismatch that could be hidden by a single aggregate RMSE. The structured residual patterns show that the reduced model did not capture every direction-dependent effect, supporting the retention of left/right nuisance angle offsets and caution when interpreting the fitted coefficients as unique physical constants.



Front and rear lateral-force residuals are plotted against axle vertical load and separated by turn direction, revealing remaining direction-dependent structure not represented by the reduced tyre model.

The appendix is intentionally selective. Routine file I/O, plotting, table export and batch-management code are omitted because they do not materially affect interpretation of the reconstructed tyre characteristics. The additional diagnostic figures are retained because they document important audit decisions that support, but would otherwise overcrowd, the main Results chapter.