



A unified approach for concrete impact

H.M. Wen^{*}, Y.X. Xian

CAS Key Laboratory for Mechanical Behavior and Design of Materials, University of Science and Technology of China, Hefei, Anhui Province 230027, PR China



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ABSTRACT

A unified approach is presented herein for concrete impact which represents an extension and further development of the UMIST formulae. The paper consists of two parts, i.e. a critical overview and assessment of the accuracy of the UMIST formulae in the wake of the new test results and the data made available recently; derivation of a new set of empirical equations by introducing a previously suggested mean resistive pressure to predict the depth of penetration, through-thickness cone cracking, scabbing and perforation of reinforced concrete targets subjected to projectile impact. It is shown that the UMIST formulae can be used with reasonable confidence for large mass low velocity projectile impact onto reinforced concrete targets and that they considerably underestimate the scabbing of reinforced concrete targets struck by non-flat-ended missiles and significantly overpredict the perforation of very thick concrete targets as compared to the experimental data. It is also shown that the present empirical formulae are in good agreement with available experimental data over a wider range of impact conditions, being applicable not only to flat-faced projectiles but also to non-flat-ended (i.e., conical-, spherical-, ogival-nosed) missiles; not only to low strength concrete but also to high strength concrete; not only to low impact velocity but also to high impact velocity and that the modified UMIST formulae are in reasonable agreement with available test data.

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1. Introduction

Concrete is a common material which has been widely used to construct protective structures to resist impact by missiles and explosive loads. Hence, the problem of missile impact on reinforced concrete targets has long been of interest to military as well as civil engineers. In nuclear industry, potential missiles include crashing aircrafts, fragments generated by accidental explosions or resulting from pressured vessel failure, pipe break, turbine blade failure and high speed rotating equipment. These missiles may vary in shapes and sizes, impact velocities, deforming rigidities, hardness and produce a wide damage spectrum in reinforced concrete targets [1,2]. Generally speaking, missiles can be categorized as either “soft” or “hard” depending on whether the missile deformability is large or small relative to the target deformability. “Soft” missile such as aircraft, which impact a large area, affect the whole reinforced concrete structure and are studied in connection with the overall survivability of the structure. “Hard” missile impact on concrete is a difficult problem to solve and major efforts have been

invested in the development and validation of numerical modeling techniques but with very limited success. On the other hand, various empirical formulae have been developed to assess the local impact damages [2]. Though different failure modes have been identified with a hard/rigid flat-nosed projectile as it represents the most dangerous scenario in regard to nuclear power industry empirical equations for four damage modes such as penetration, through-thickness cone cracking, scabbing and perforation have been proposed [2–4].

The objective of this paper is to formulate a unified approach for concrete impact which represents an extension and further development of the UMIST formulae. An overview and assessment of the UMIST formulae is first conducted in the wake of the new test data and the results made available recently and then various empirical equations are derived by introducing a previously proposed mean resistive pressure within a unified framework. Comparisons are made and discussed.

2. An overview and assessment of the UMIST formulae

The UMIST formulae [2,4] have been proposed to predict the response and failure of reinforced concrete targets struck by rigid

^{*} Corresponding author. Tel.: +86 55163607740; fax: +86 55163606459.
E-mail address: hmwen@ustc.edu.cn (H.M. Wen).

projectiles with different nose shapes as shown schematically in Fig. 1. Various equations were derived which can be used to predict penetration, through-thickness cone cracking, scabbing and perforation of the targets as shown schematically in Fig. 2. In the following an overview of the UMIST formulae is first conducted and then the accuracy of the UMIST formulae is assessed in the wake of newly acquired test data.

2.1. An overview of the UMIST formulae

The UMIST penetration formula is given by the following equation [2,4]

$$\frac{x}{d} = \left(\frac{2}{\pi}\right) \frac{N^* M V_i^2}{0.72 \sigma_t d^3} \quad (1)$$

where x is penetration depth, the nose shape factor N^* is 0.72 for flat nose, 0.84 for a hemispherical nose, 1.0 for a blunt nose and 1.13 for a sharp nose. M is projectile mass, d projectile diameter and V_i projectile impact velocity. σ_t is defined by the following expression:

$$\sigma_t (\text{MPa}) = 4.2 f'_c (\text{MPa}) + 135.0 + (0.014 f'_c (\text{MPa}) + 0.45) V_i (\text{m/s}) \quad (2)$$

In which f'_c is the unconfined compressive strength of concrete. V_i is the impact velocity of the projectile. As can be seen from the Eq. (1), the penetration depth of projectile is relevant to the compressive strength f'_c of the concrete, the impact velocity V_i and the diameter d of the projectile. The effect of the projectile's diameter is equivalent to the slenderness ratio of the projectile.

Eq. (1) has been verified for $50 < d < 600$ (mm), $35 < M < 2500$ (kg), $0 < x/d < 2.5$ and $3 < V_i < 66.2$ (m/s).

The critical impact energies of a projectile to cause through-thickness cone cracking (E_c), scabbing (E_s) and perforation (E_p) are given as a function of the ratio of target thickness to missile diameter.

(i) $H/d < 5$

The UMIST formula for cone cracking is expressed as follows [2–4]:

$$\frac{E_c}{\eta \sigma_t d^3} = -0.00031 \left(\frac{H}{d}\right) + 0.00113 \left(\frac{H}{d}\right)^2 \quad 0 < \frac{H}{d} \leq 2 \quad (3a)$$

$$\frac{E_c}{\eta \sigma_t d^3} = -0.00325 \left(\frac{H}{d}\right) + 0.00130 \left(\frac{H}{d}\right)^3 \quad 2 < \frac{H}{d} < 5 \quad (3b)$$

where H is target thickness and the influence of the nose shape can be neglected.

The UMIST formula for scabbing is written as [2,4]

$$\frac{E_s}{\eta \sigma_t d^3} \frac{N^*}{0.72} = -0.005441 \left(\frac{H}{d}\right) + 0.01386 \left(\frac{H}{d}\right)^2 \quad 0 < \frac{H}{d} < 5 \quad (4)$$

where the nose shape factor N^* is 0.72 for flat nose, 0.84 for a hemispherical nose, 1.0 for a blunt nose and 1.13 for a sharp nose.

The UMIST formula for perforation is written in the following forms [2–4]

$$\frac{E_p}{\eta \sigma_t d^3} = -0.00506 \left(\frac{H}{d}\right) + 0.01506 \left(\frac{H}{d}\right)^2 \quad 0 < \frac{H}{d} \leq 1 \quad (5a)$$

$$\frac{E_p}{\eta \sigma_t d^3} = -0.01 \left(\frac{H}{d}\right) + 0.02 \left(\frac{H}{d}\right)^3 \quad 1 < \frac{H}{d} \leq 5 \quad (5b)$$

where in above equations, η is determined by the following expressions [2–4]

$$\eta = \begin{cases} 0.5 + \frac{3}{8} \left(\frac{d}{C_r}\right) r_t & \frac{d}{C_r} < \sqrt{\frac{d}{d_r}} \\ 0.5 + \frac{3}{8} \left(\sqrt{\frac{d}{d_r}}\right) r_t & \frac{d}{C_r} \geq \sqrt{\frac{d}{d_r}} \end{cases} \quad (6)$$

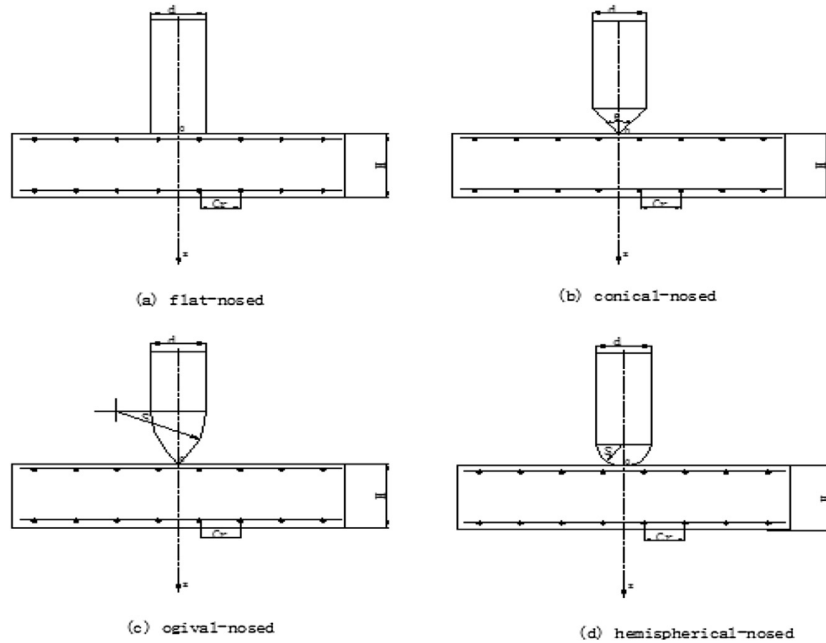


Fig. 1. Schematic diagram showing concrete targets struck by projectiles with different nose shapes.

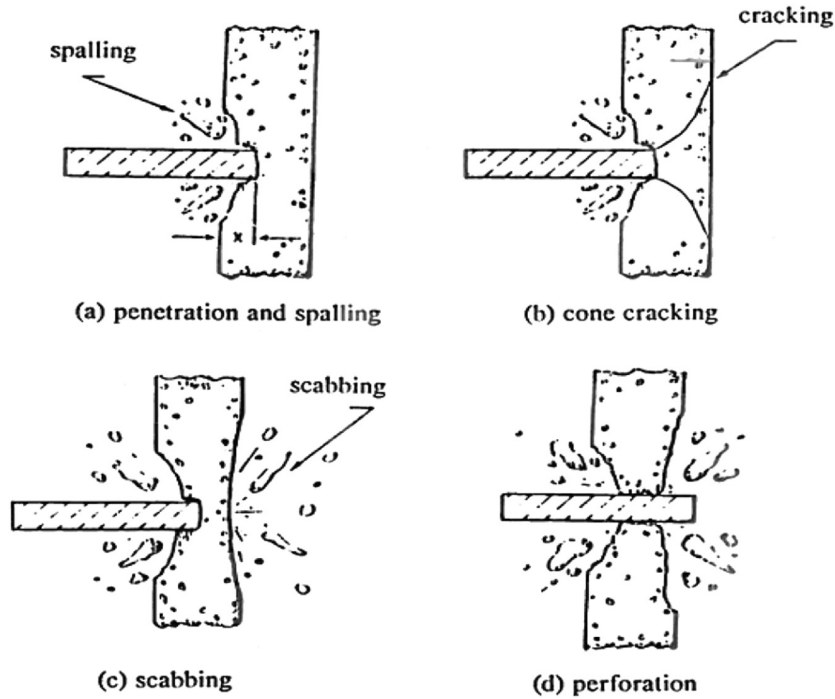


Fig. 2. Schematic diagram showing penetration, cone cracking, scabbing and perforation of concrete slabs struck by “hard” missiles.

where d_r is the rebar diameter, C_r is the rebar spacing and r_t is the total bending reinforcement ($r_t = 4r$ with r being % EWEF, defined as $r = \pi d_r^2 / 4HC_r$).

(ii) $H/d \geq 5$

For $H/d \geq 5$, the effect of the reinforcement on the response and failure of the targets may be neglected. The majority of the impact energy of the missile will be dissipated in the tunneling process. The critical energies for cone cracking, scabbing and perforation are given by the following expressions [2–4]:

$$\frac{E_c}{\sigma_t d^3} = \frac{\pi}{4} \left[\frac{H}{d} - 4.7 \right] \quad (7)$$

$$\frac{E_s}{\sigma_t d^3} \frac{N^*}{0.72} = \frac{\pi}{4} \left[\frac{H}{d} - 4.3 \right] \quad (8)$$

$$\frac{E_p}{\sigma_t d^3} = \frac{\pi}{4} \left[\frac{H}{d} - 3.0 \right] \quad (9)$$

2.2. An assessment of the UMIST formulae

In the wake of the new test data and the data made available recently, the UMIST formulae are assessed and discussed in the following section.

2.2.1. Depth of penetration

Comparisons are made between the UMIST penetration formula (Eq. (1)) and the experimental data available in the literature for flat-nosed projectiles [5–8] in Fig. 3 and for non-flat-nosed (i.e. conical-, ogival-nosed) missiles [9–20] in Fig. 4. In the legends of Figs. 3 and 4, (f) (c) and (o) indicate flat conical and ogival nosed projectiles respectively whilst (N) (H) denote normal and high

strength concrete. It is seen from Fig. 3 that the predictions of the UMIST penetration formula for flat-nosed projectiles are in good agreement with the experimental data for $E_k/\sigma_t d^3 < 1.5$, and are lower than the experimental data for $E_k/\sigma_t d^3 > 1.5$. The difference between the predictions of the UMIST penetration formula and the test results increases with increasing $E_k/\sigma_t d^3$. It is also seen from Fig. 4 that the predictions of the UMIST penetration formula for non-flat-nosed projectiles are in good agreement with the experimental data for $E_k/\sigma_t d^3 < 7.5$, and are lower than experimental data for $E_k/\sigma_t d^3 \geq 7.5$. The difference between the predictions of the UMIST penetration formula and the experiment data also increases with increasing $E_k/\sigma_t d^3$. In other words, the UMIST penetration formula (Eq. (1)) is most likely applicable to relatively low velocity

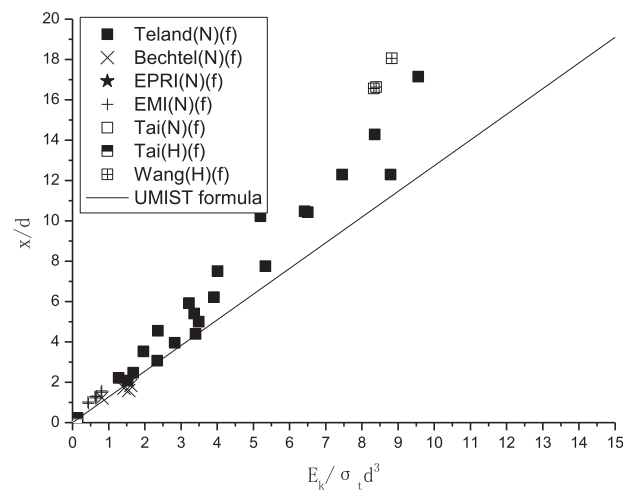


Fig. 3. Comparison of the UMIST penetration formula (Eq. (1)) with the experimental data for flat-nosed projectiles [5–8].

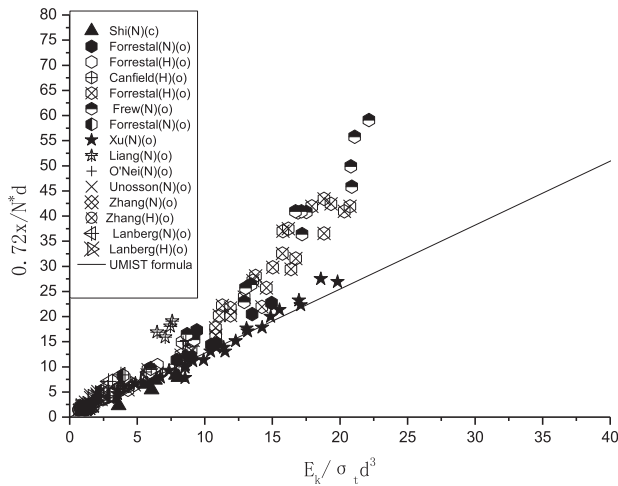


Fig. 4. Comparison of the UMIST penetration formula (Eq. (1)) with the experimental data for non-flat-nosed projectiles [9–20].

impact. This is not surprising since the original UMIST penetration formula was derived on the basis of the test results acquired by nuclear power industry for large mass low velocity impact [3].

2.2.2. Cone cracking

Comparisons are made between the UMIST formulae for cone cracking and the experimental data for $H/d < 5$ [21,22] in Fig. 5 and for $H/d \geq 5$ [21] in Fig. 6. It is seen from Figs. 5 and 6 that the predictions of the UMIST formulae for cone cracking are in good agreement with the available test data for flat-faced missiles.

2.2.3. Scabbing

Comparisons are made between the UMIST scabbing formulae and the experimental results for flat-faced missiles for $H/d < 5$ [6] in Fig. 7a and for $H/d \geq 5$ [6] in Fig. 8. Comparison is also made between the UMIST scabbing formula (Eq. (4)) and the test data for non-flat-ended projectiles for $H/d < 5$ [6,23,24] in Fig. 9. In the legends of Figs. 7–9, (f) (c) and (s) indicate flat conical and spherical nosed projectiles respectively whilst (N) (H) denote normal and high strength concrete. It is seen from Figs. 7a and 8 that the predictions of the UMIST scabbing formulae are generally in good

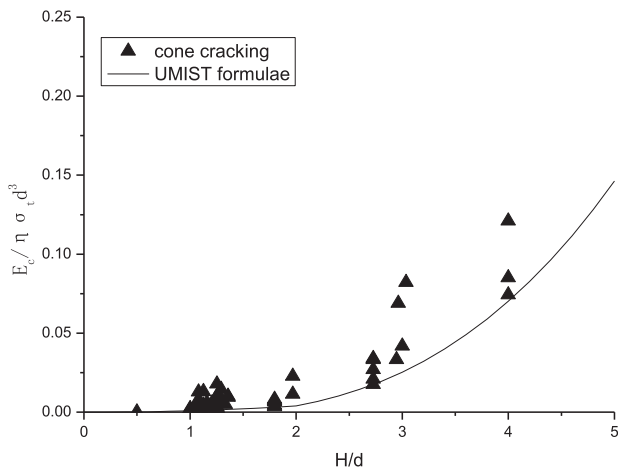


Fig. 5. Comparison of the UMIST formulae for cone cracking (Eq. (3)) with the experimental data for flat-nosed projectiles [21,22].

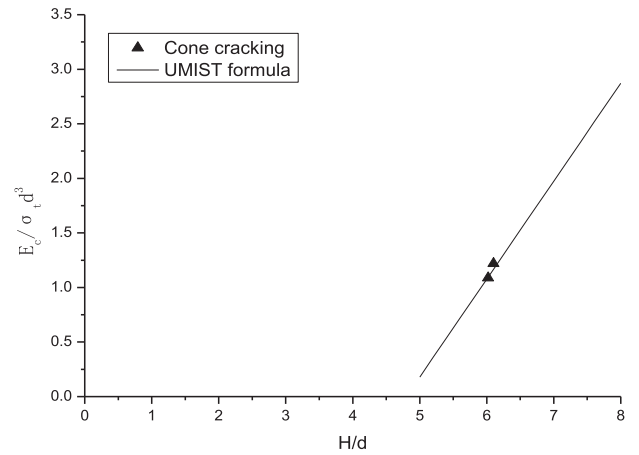


Fig. 6. Comparison of the UMIST formula for cone cracking (Eq. (7)) with the experimental data for flat-nosed projectiles [21].

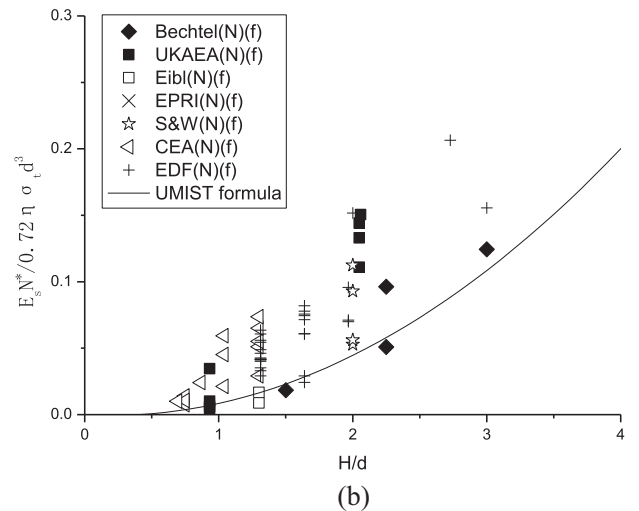
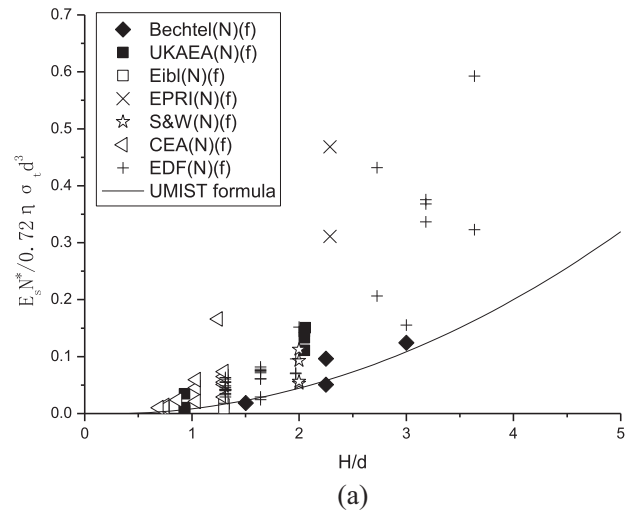


Fig. 7. (a) Comparison of the UMIST scabbing formula (Eq. (4)) with the experimental data for flat-nosed projectiles [6]. (b) Enlarged detail of the partial Fig. 7a.

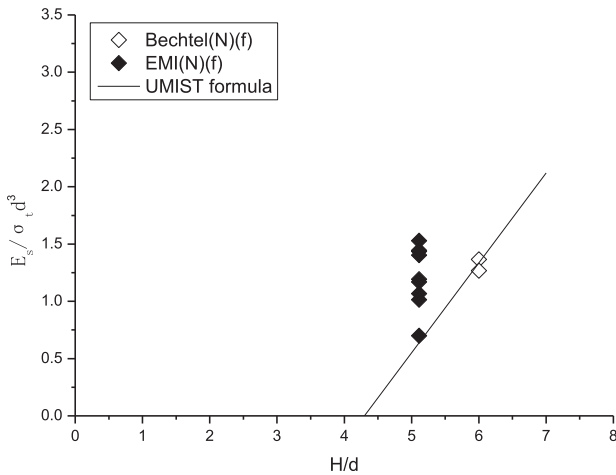


Fig. 8. Comparison of the UMIST scabbing formula (Eq. (8)) with the test data for flat-nosed projectiles [6].

agreement with the test results for flat-nosed projectiles. However, it should be noted that the UMIST scabbing formula Eq. (4) overpredicts the test data a little bit for $H/d < 2.5$ as can be seen from the enlarged detail shown in Fig. 7b. It implies that for $H/d < 2.5$ the UMIST formula (Eq. (4)) may not be that safe to be used for design of structures to resist scabbing due to impact by flat-ended projectiles. The thick reinforced concrete targets damage failure mode with scabbing are rare. It can be seen from Fig. 8 that the predictions of the UMIST formula are agreement with experiment data. However, the thick reinforced concrete targets damage failure mode with scabbing are rare to demonstrate that its protective ability for scabbing is higher than thin reinforced concrete targets.

It is also seen from Fig. 9 that the UMIST scabbing formula (Eq. (4)) significantly underestimate the experimental data for non-flat-nosed missiles. In other words, the UMIST scabbing formula is too conservative for design of structures to resist scabbing resulting from impact by non-flat-faced (i.e. conical, spherical-nosed) missiles.

2.2.4. Perforation

Figs. 10 and 11 show comparisons between the UMIST perforation formulae (Eq. (5)) and the test data for flat-faced missiles [6]

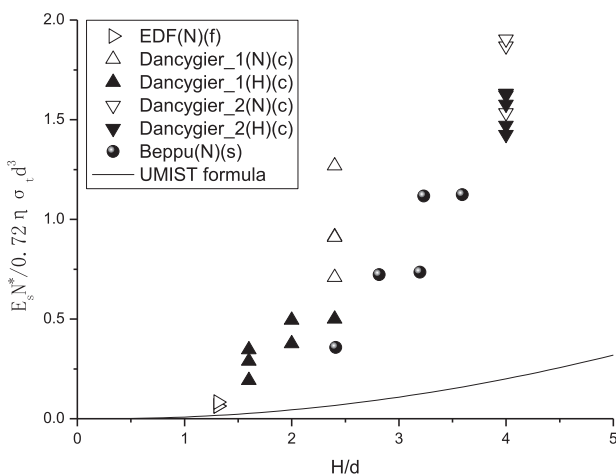


Fig. 9. Comparison of the UMIST scabbing formula (Eq. (4)) with the experimental data for non-flat-nosed projectiles [6,23,24].

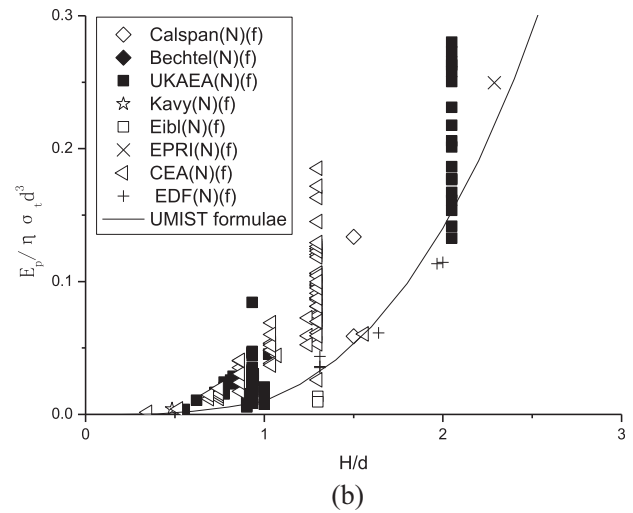
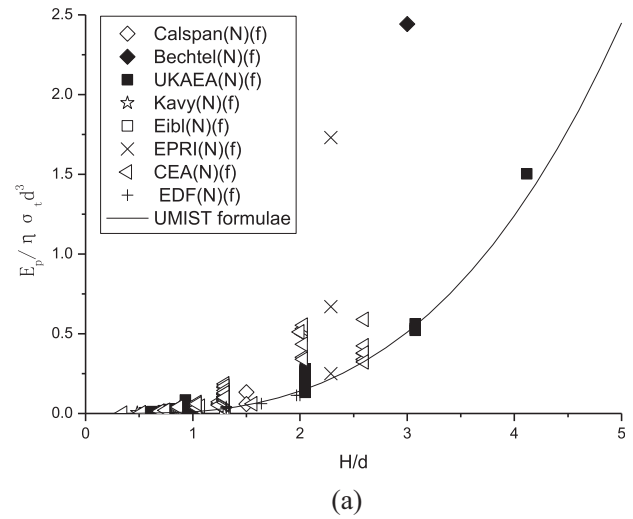


Fig. 10. (a) Comparison of the UMIST perforation formulae (Eq. (5)) with the experimental data for flat-nosed projectiles [6]. (b) Enlarged detail of the partial Fig. 10a.

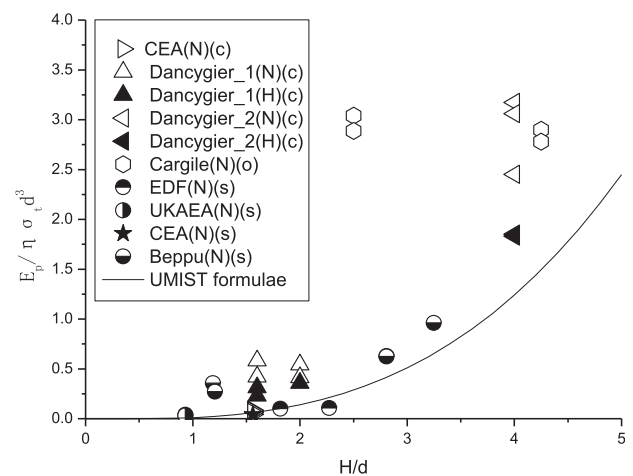


Fig. 11. Comparison of the UMIST perforation formulae (Eq. (5)) with the experimental data [6,23,25,26] for non-flat-nosed projectiles.

and non-faced-missiles [6,23,25,26] respectively for $H/d < 5$. Comparison is also made between the UMIST perforation formula (Eq. (9)) and the test results for flat-faced as well as non-flat-ended projectiles for $H/d > 5$ [6,18,24,25,27–30] in Fig. 12. In the legends of Figs. 10–12, (f) (c) (s) and (o) indicate flat, conical, spherical and ogival nosed projectiles respectively whilst (N) (H) denote normal and high strength concrete. The predictions of the UMIST perforation formulae (Eq. (5)) are generally in reasonable agreement with the test results for flat-nosed projectiles as well as non-flat-ended missiles as can be seen from Figs. 10–12 except for very large H/d where the UMIST perforation formula (Eq. (9)) significantly overestimate the experimental result as can be seen from Fig. 11. In other words, the UMIST perforation formulae fail to produce consistent results as compared to the experimental data especially for very thick concrete targets.

Furthermore, closer examination reveals that the UMIST perforation formulae (Eq. (5)) overpredict the test data for thin concrete targets with $H/d < 2.5$ a little bit for flat-faced projectiles as well as non-flat-ended missiles, as can be seen from the enlarged detail shown in Figs. 10b and 11, respectively. Closer examination also reveals that for thick concrete targets with $H/d \geq 5$ the UMIST perforation formula (Eq. (9)) overestimates the test results for $H/d < 7.5$ as well. It implies that the UMIST formulae (Eq. (4) and Eq. (9)) may not be safe to be used for design of structures to resist perforation due to impact by flat-ended projectiles as well as non-flat-ended missiles for thin concrete targets with $H/d < 2.5$, intermediate thick concrete targets with $5 \leq H/d < 7.5$ and very thick concrete targets with $H/d > 15$ approximately. It has to be mentioned here that care should be exercised where assessments are required for $H/d \leq 0.5$, as also noted in Refs. [2,4].

3. Formulation of a unified approach

In the following sections a unified approach for concrete impact is formulated by closely following the study in Ref. [3] and by introducing a mean resistive pressure which caters for the effects of various parameters such as nose shape, impact velocity and unconfined compressive strength as proposed in Refs. [31,33]. Various empirical equations are obtained which can be used to predict the depth of penetration, through-thickness cone cracking, scabbing and perforation of reinforced concrete targets subjected to projectile impact at normal incidence.

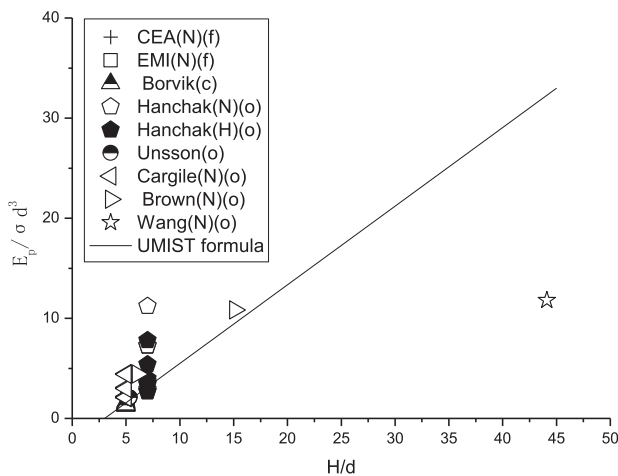


Fig. 12. Comparison of the UMIST perforation formula (Eq. (9)) with the test results [6,18,24,25,27–30].

3.1. Penetration

The depth of penetration of concrete targets struck transversely by projectiles with different nose shapes has been examined in Ref. [33]. For completeness the equation for depth of penetration is quoted as follows

$$\frac{x}{d} = \frac{4}{\pi} \frac{E_k}{\sigma d^3} \quad (10)$$

where x is penetration depth, $E_k = (1/2)MV_i^2$ is projectile impact energy with M and V_i being projectile mass and impact velocity, respectively, d projectile diameter and σ is mean resistive pressure which can be expressed by the following equation

$$\sigma = \left(\alpha + \beta \sqrt{\frac{\rho_t V_i}{Y}} \right) Y \quad (11)$$

where α can be determined theoretically by spherical or cylindrical cavity expansion analysis [32] and β evaluated from experiments; ρ_t is the density of the concrete target material; Y is a measure of shear strength for concrete which can be expressed as [33]

$$Y = \begin{cases} 1.4f'_c + 45, & f'_c \leq 75 \text{ MPa} \\ 150, & 75 \text{ MPa} < f'_c < 150 \text{ MPa} \\ f'_c, & f'_c \geq 150 \text{ MPa} \end{cases} \quad (12)$$

where f'_c is the unconfined compressive strength of concrete.

The form of the UMIST penetration formula for a flat-ended projectile is the same as that of the newly suggested penetration formula except for the mean resistive pressure which has a different expression. Close examination of Eqs. (2) and (11) reveal that for relatively low impact velocities the values of σ_t (Eq. (2)) and σ (Eq. (11)) are almost the same and that for higher impact velocities the difference between these two parameters becomes quite large. In fact, the difference between the values of σ_t (Eq. (2)) and σ (Eq. (11)) increases with increasing impact velocity.

3.2. Cone cracking, cabbng and perforation

According to the experimental observations, reinforced concrete slabs struck normally by projectiles can be categorized into two regimes, namely, thin target and thick target. The response of a thin concrete slab is global bending together with local failure whereas the response of a thick concrete slab is localized [2].

For a thin reinforced concrete target subjected to impact by a projectile the critical energies E_c , E_s and E_p for cone cracking, scabbing and perforation can generally be written as [3,21]

$$\frac{E_c}{\phi \eta \sigma d^3} = C_1 \left(\frac{H}{d} \right) + C_2 \left(\frac{H}{d} \right)^2 + C_3 \left(\frac{H}{d} \right)^3 + C_4 \left(\frac{H}{d} \right)^4 \quad (13)$$

$$\frac{E_s}{\phi \eta \sigma d^3} = C_5 \left(\frac{H}{d} \right) + C_6 \left(\frac{H}{d} \right)^2 + C_7 \left(\frac{H}{d} \right)^3 + C_8 \left(\frac{H}{d} \right)^4 \quad (14)$$

$$\frac{E_p}{\phi \eta \sigma d^3} = C_9 \left(\frac{H}{d} \right) + C_{10} \left(\frac{H}{d} \right)^2 + C_{11} \left(\frac{H}{d} \right)^3 + C_{12} \left(\frac{H}{d} \right)^4 \quad (15)$$

where η is coefficient of reinforcement, ϕ is projectile nose shape effect and C_1 – C_{12} are empirical constants.

For a thick reinforced concrete target struck by a projectile the critical energies E_c , E_s and E_p for cone cracking, scabbing and perforation can generally be written in the following forms [3,21]

Table 1

Values of various parameters for concrete targets.[33].

	α	β	ϕ
Conical-nosed ($\theta < 90^\circ$)	$\frac{1}{2} \left[1 + \ln \frac{2E}{(5-4\nu)Y} \right]$	$2 \sin \frac{\theta}{2}$	$1 + \frac{2}{\tan(\frac{\theta}{2})}$
Conical-nosed ($90^\circ \leq \theta < 180^\circ$)	$\frac{1}{2} \left[1 + \ln \frac{2E}{(5-4\nu)Y} \right]$	$\sqrt{2}$	$1 + \frac{2}{\tan(\frac{\theta}{2})}$
Flat-nosed	$\frac{1}{2} \left[1 + \ln \frac{2E}{(5-4\nu)Y} \right]$	$\sqrt{2}$	1
Ogival-nosed	$\frac{2}{3} \left[1 + \ln \frac{E}{3(1-\nu)Y} \right]$	$3/4\psi$	$1 + \psi$
Hemispherical-nosed	$\frac{2}{3} \left[1 + \ln \frac{E}{3(1-\nu)Y} \right]$	$3/2$	$3/2$

$$\frac{E_c}{\sigma d^3} = \frac{\pi}{4} \left(\frac{H}{d} - \chi_c \right) \quad (16)$$

$$\frac{E_s}{\sigma d^3} = \frac{\pi}{4} \left(\frac{H}{d} - \chi_s \right) \quad (17)$$

$$\frac{E_p}{\sigma d^3} = \frac{\pi}{4} \left(\frac{H}{d} - \chi_p \right) \quad (18)$$

where χ_c , χ_s and χ_p are constants determined by experiments.

3.3. Comparison with available experimental data and discussion

The values of various parameters in the equations given in Sections 3.1 and 3.2 can be determined experimentally. For concrete targets, the values of α , β and ϕ are listed in Table 1. α and β are constants in Eq. (11), ϕ is projectile nose shape effect, θ is the cone angle of the missile, ψ is the caliber-radius-head of the ogival nose projectile.

For conical-nosed projectiles η can be determined by following expression.

$$\eta = \begin{cases} 0.5 + \frac{3}{8} \sin^4 \left(\frac{\theta}{2} \right) \left(\frac{d}{C_r} \right) r_t & \frac{d}{C_r} < \sqrt{\frac{d}{d_r}} \\ 0.5 + \frac{3}{8} \sin^4 \left(\frac{\theta}{2} \right) \left(\sqrt{\frac{d}{d_r}} \right) r_t & \frac{d}{C_r} \geq \sqrt{\frac{d}{d_r}} \end{cases} \quad (19)$$

where θ is the cone angle of the missile, d_r is the rebar diameter, C_r is the rebar spacing and r_t is the total bending reinforcement ($r_t = 4r$ with r being % EWEF, defined as $r = \pi d_r^2 / 4HC_r$).

For ogival-nosed projectiles

$$\eta = \begin{cases} 0.5 + \frac{3}{128\psi^2} \left(\frac{d}{C_r} \right) r_t & \frac{d}{C_r} < \sqrt{\frac{d}{d_r}} \\ 0.5 + \frac{3}{128\psi^2} \left(\sqrt{\frac{d}{d_r}} \right) r_t & \frac{d}{C_r} \geq \sqrt{\frac{d}{d_r}} \end{cases} \quad (20)$$

where $\psi = S/d$ is the caliber-radius-head of the ogive nose projectile. S is the radius of the ogive nose.

Based on the available experimental data and using polynomial curve-fitting technique, the various empirical constants in Eqs. (13)–(18) have been obtained. Hence, Eqs. (13)–(18) can be recast into the following forms:

For thin concrete targets

$$\frac{E_c}{\phi \eta \sigma d^3} = 0.0003 \left(\frac{H}{d} \right) + 0.0003 \left(\frac{H}{d} \right)^4 \quad 0 < \frac{H}{d} < 5 \quad (21)$$

$$\frac{E_s}{\phi \eta \sigma d^3} = -0.0066 \left(\frac{H}{d} \right) + 0.0120 \left(\frac{H}{d} \right)^2 \quad 0 < \frac{H}{d} < 5 \quad (22)$$

and

$$\frac{E_p}{\phi \eta \sigma d^3} = 0.0048 \left(\frac{H}{d} \right) + 0.0018 \left(\frac{H}{d} \right)^2 \quad 0 < \frac{H}{d} \leq 1.3 \quad (23a)$$

$$\frac{E_p}{\phi \eta \sigma d^3} = -0.0300 \left(\frac{H}{d} \right) + 0.0220 \left(\frac{H}{d} \right)^3 \quad 1.3 < \frac{H}{d} < 4 \quad (23b)$$

For thick concrete targets

$$\frac{E_c}{\sigma d^3} = \frac{\pi}{4} \left(\frac{H}{d} - 4.7 \right) \quad \frac{H}{d} \geq 5 \quad (24)$$

$$\frac{E_s}{\sigma d^3} = \frac{\pi}{4} \left(\frac{H}{d} - 4.3 \right) \quad \frac{H}{d} \geq 5 \quad (25)$$

and

$$\frac{E_p}{\sigma d^3} = \begin{cases} \frac{\pi}{4} \left(\frac{H}{d} - 3.0 \right) & \frac{H}{d} \geq 4, V_i < 300 \text{ (m/s)} \\ \frac{\pi}{4} \left(\frac{H}{d} - 2.0 \right) & \frac{H}{d} \geq 4, V_i \geq 300 \text{ (m/s)} \end{cases} \quad (26)$$

The response (energy absorbing) mechanisms are different for thin and thick concrete slabs. Hence, the function of E_c is not necessarily continuous for $H/d = 5$. The same argument can be applied to the functions of E_s and E_p .

3.3.1. Penetration

Comparisons are made of the present formula (Eq. (10)) with test data [5–20] for the depth of penetration of concrete targets struck normally by flat-nosed projectiles in Fig. 13a and by non-flat-nosed projectiles in Fig. 14a. In the legends of Fig. 13, (f) (c) and (o) indicate flat, conical and ogival nosed projectiles respectively whilst (N) (H) denote normal and high strength concrete. It is seen from Fig. 13 that the present penetration formula is in good agreement with the experimental results available in the literature.

3.3.2. Cone cracking

Comparisons are made between the test data for cone cracking of reinforced concrete targets with the present formulae for $H/d < 5$ (Eq. (21)) in Fig. 14 [21,22] and for $H/d \geq 5$ (Eq. (24)) in Fig. 15 [21]. It can be seen from Figs. 14 and 15 that the present formulae for cone cracking are in good agreement with the available experimental data. It should be mentioned here that the experimental data presented in Figs. 14 and 15 were acquired for flat-nosed missiles only and, at the present time, there is paucity of the test results for non-flat-ended projectiles.

3.3.3. Scabbing

Comparisons are made between the experimental results for scabbing of reinforced concrete targets with the present formulae for $H/d < 5$ (Eq. (22)) in Fig. 16a for flat-nosed projectiles [6] and in Fig. 17 for non-flat-nosed missiles [23,24]. Comparison is also made between the experimental data for scabbing of reinforced concrete targets with the present formulae and for $H/d \geq 5$ (Eq. (25)) in Fig. 18 [6]. In the legends of Figs. 16–18, (f) (c) and (s) indicate flat, conical

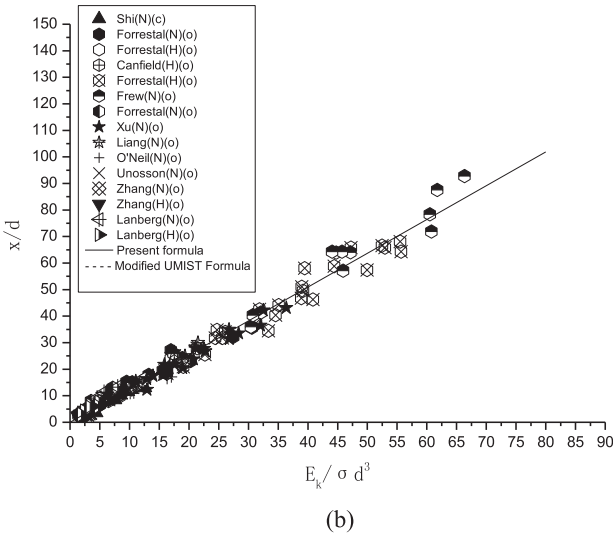
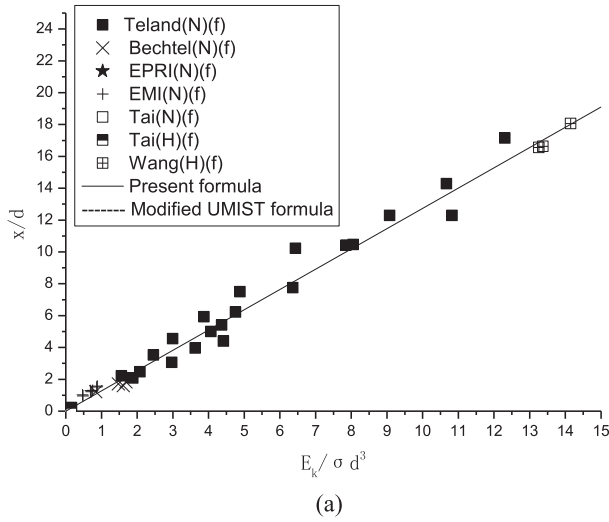


Fig. 13. Comparisons of the present penetration formula (Eq. (10)) with the test data for the depth of penetration: (a) flat-nosed projectiles; (b) non-flat-nosed missiles [5–20].

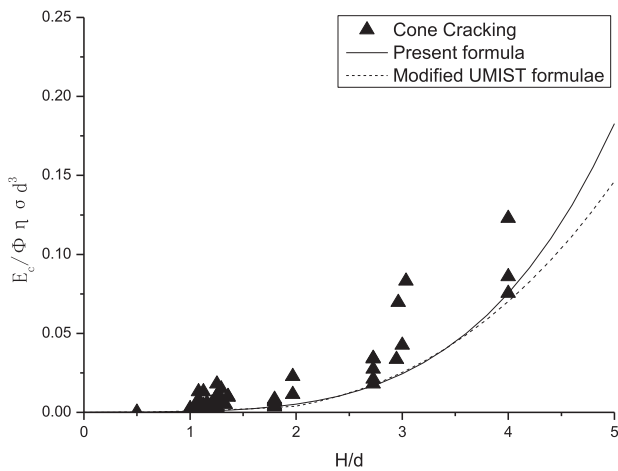


Fig. 14. Comparison of the present formula (Eq. (21)) with the test data [21,22] for cone cracking of reinforced concrete targets with $H/d < 5$.

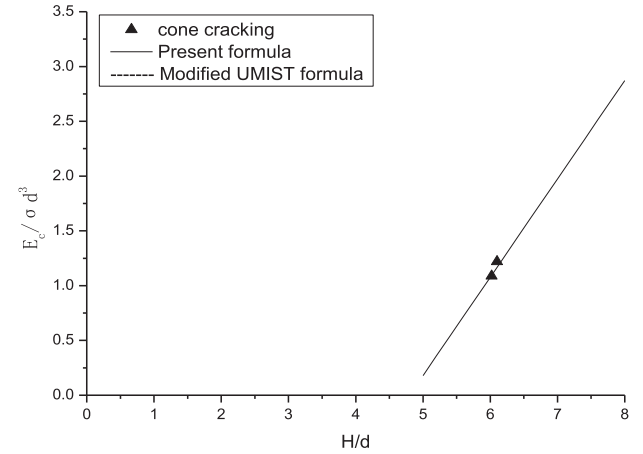


Fig. 15. Comparison of the present formula (Eq. (24)) with the test data [21] for cone cracking of reinforced concrete targets with $H/d \geq 5$.

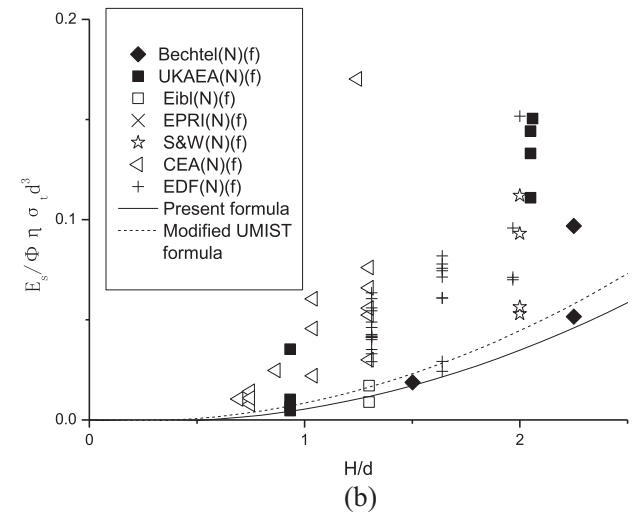
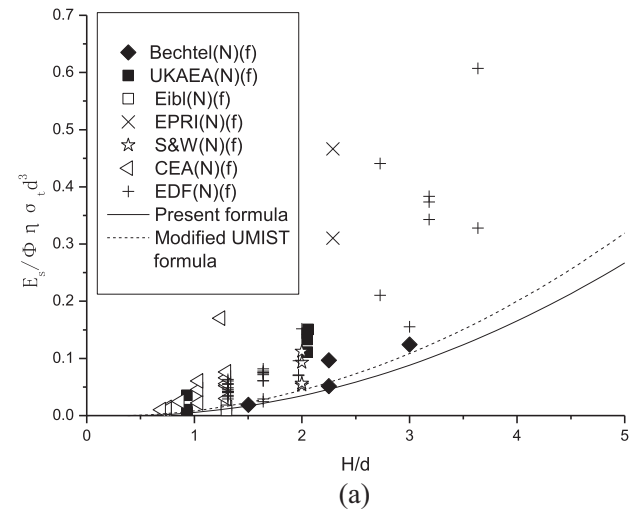


Fig. 16. (a) Comparison of the present formula (Eq. (22)) with the test data [6] for scabbing of reinforced concrete targets with $H/d < 5$ struck by flat-nosed projectile. (b) Enlarged detail of the partial Fig. 16a.

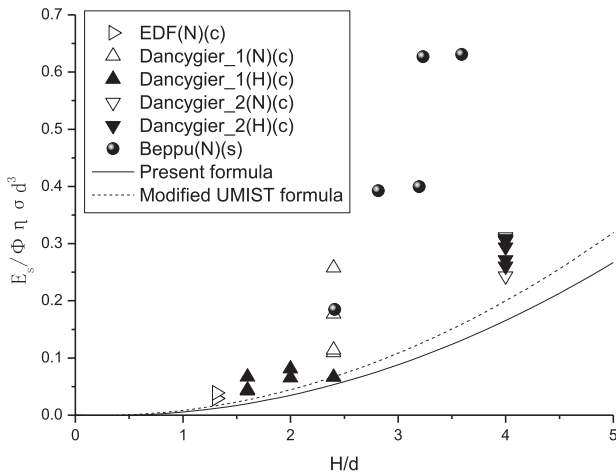


Fig. 17. Comparison of the present formula (Eq. (25)) with the test data [6,23,24] for scabbing of reinforced concrete targets with $H/d < 5$ struck by non-flat-nosed projectile.

and spherical nosed projectiles respectively whilst (N) (H) denote normal and high strength concrete. It is clear from Figs. 16a,b,17,18 that the present formulae for scabbing are in good agreement with the available test data.

3.3.4. Perforation

Comparisons are made between the test data for perforation of reinforced concrete targets with the present formulae for $H/d < 4$ (Eq. (23)) in Fig. 19a for flat-nosed projectiles [6] and in Fig. 20a for non-flat-nosed missiles [6,23,25,26]. Comparisons are also made between the experimental results [6,18,24,25,27–30] for perforation of reinforced concrete targets with the present formulae for $H/d \geq 4$ (Eq. (26)) in Fig. 21a for impact velocities less than 300 m/s and in Fig. 21b for impact velocities greater than 300 m/s. In the legends of Figs. 19–21, (f) (c), (s) and (o) indicate flat, conical, spherical and ogival nosed projectiles respectively whilst (N) (H) denote normal and high strength concrete. It is clear from Figs. 19a, b, 20a, 20b, 21 that the present perforation formulae are in good agreement with the test data available in the literature.

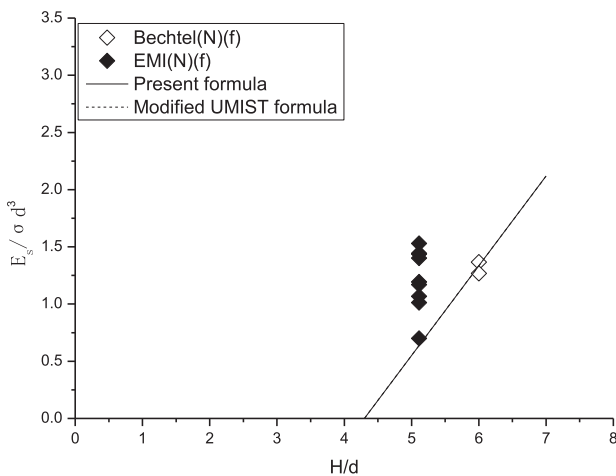


Fig. 18. Comparison of the predictions of the present model with the test data [6] for scabbing of reinforced concrete targets ($H/d \geq 5$) struck by flat-nosed projectile.

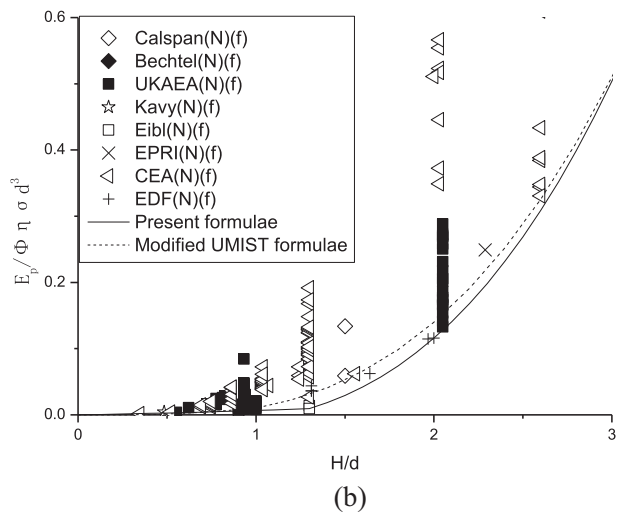
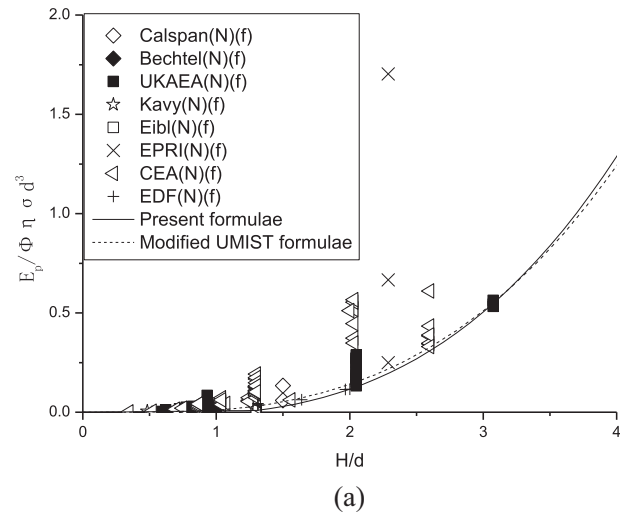


Fig. 19. (a) Comparison of the predictions of the present model with the test data [6] for perforation of reinforced concrete targets ($H/d < 4$) struck by flat-nosed projectile. (b) Enlarged detail of the partial Fig. 19a.

3.3.5. Further discussion

Unfortunately, the UMIST equations and the newly proposed formulae can not be presented in one graph due to the use of different expressions for the mean resistive pressure (see Eqs (2) and (11) for more details). In order to make direct comparison between the UMIST equations and the newly proposed formulas, Tables 2–4 are given which contain the test data of the critical conditions respectively for through-thickness cone cracking (cone cracking limits), scabbing (scabbing limits) and perforation (ballistic limits) available in the literature and the predicted values from the UMIST formulae and the newly suggested equations are also listed in the tables. V_{cracking} , V_{scabbing} and V_{50} are the experimentally obtained cone cracking, scabbing and ballistic limits respectively whilst V_c , V_s and V_b are the predicted cone cracking, scabbing and ballistic limits respectively. It can be seen from Tables 2–4 that the newly suggested empirical equations produce more consistent results than the UMIST formulae as compared to the experimental observations. It also can be seen from Figs. 16, 19–21 that the present equations are better than the modified UMIST formulae.

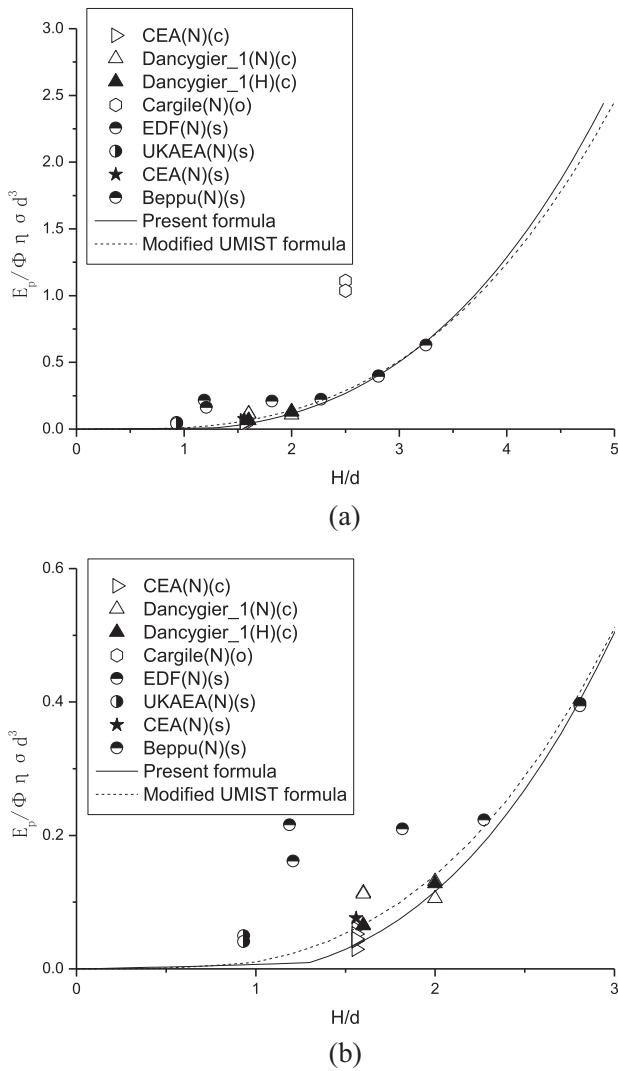


Fig. 20. (a) Comparison of the predictions of the present model with the test data [6,23,25,26] for perforation of reinforced concrete targets ($H/d < 4$) struck by non-flat-nosed projectile. (b) Enlarged detail of the partial Fig. 20a.

It should be mentioned here that all the test results presented in Figs. 5–21 were for concrete slabs which failed either in cone cracking or in scabbing or in perforation. For example, the test data presented in Figs. 5 and 6, Figs. 14 and 15 were for concrete slabs which failed in through-thickness cone cracking which include just cone cracking (cracking limit condition), moderate cone cracking and severe cone cracking. The same argument can be applied to Figs. 7–9, Figs. 16–18 for scabbing and Figs. 10–12, Figs. 19–21 for perforation. In other words, the lines predicted from various empirical equations are demarcation lines or failure boundaries above which failure occurs and below which no failure happens.

It has to be mentioned here that care should be exercised where assessments are required for $H/d \leq 0.5$, as also noted in Refs. [2,4].

3.4. The modified UMIST formulae

The modified UMIST formulae (see Appendix for more details) are obtained directly by substituting σ for σ_t of the Eq. (1) with $N^* = 0.72$ for depth of penetration; $\phi\sigma$ for σ_t of the Eqs. (3)–(5) with $N^* = 0.72$ for $H/d < 5$ and σ for σ_t of the Eqs. (7)–(9) with the

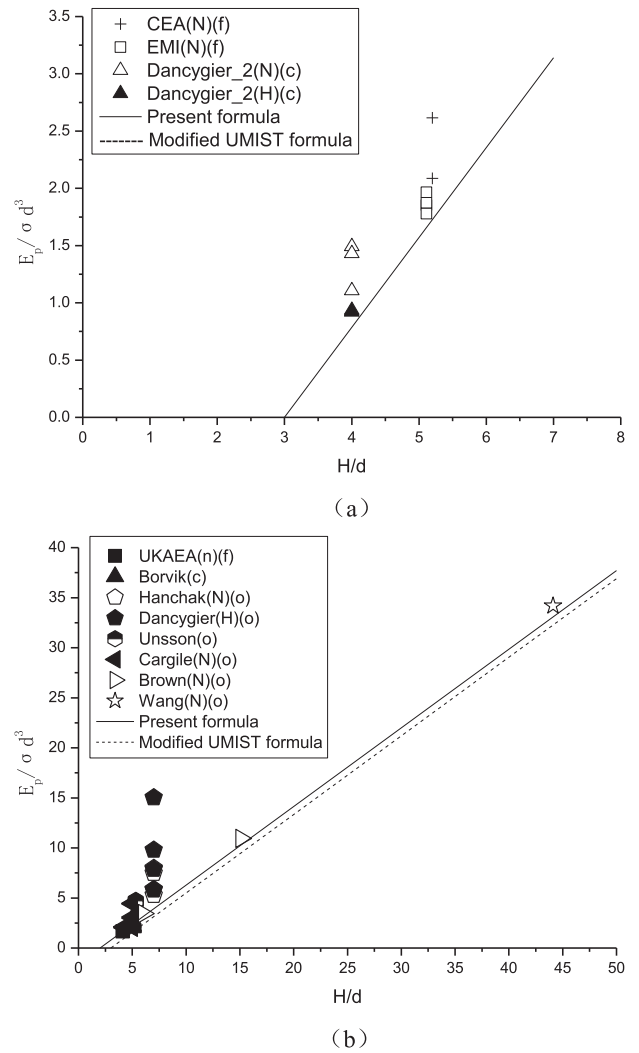


Fig. 21. Comparison of the present formula (Eq. (26)) with the test data [6,18,24,25,27–29] for perforation of reinforced concrete targets with $H/d \geq 4$ (a) $V_i < 300$ (m/s); (b) $V_i \geq 300$ (m/s).

coefficients and the functional forms being kept unchanged. It should be noted here that the modified UMIST formulae for penetration (Eq. (A1)) and cone cracking (Eq. (A5)), scabbing (Eq. (A6)), perforation (Eq. (A7)) for thick concrete targets are the same as those of the present empirical formulae given in Section 3 except for perforation where the newly suggested empirical formula for $V_i \geq 300$ m/s is slightly different from that of the modified UMIST formula. The results predicted from the modified UMIST formulae are also shown in Figs. 13–21. As can be seen from Figs. 13–21 the modified UMIST formulae are generally in reasonable agreement with the experimental data available in the literature.

4. Conclusions

A new set of empirical equations have been derived to predict the depth of penetration, through-thickness cone cracking, scabbing and perforation of reinforced concrete targets struck by projectiles at normal incidence within a unified framework. The present work represents an extension and further development of the UMIST formulae. The main conclusions are as follows:

Table 2

Comparison of the predictions of the present model and UMIST formulae with experimentally obtained cracking limits.

Reference	Test no	Nose shape	Mass (kg)	d (mm)	V_{cracking} (m/s)	H (mm)	f'_c (MPa)	d_r (mm)	C_r (mm)	UMIST V_c (m/s)	Present V_c (m/s)	UMIST error (V_c/V_{cracking})	Present error (V_c/V_{cracking})
Reid and Wen [21]	8	Flat	2622	600	3.7	300	33.5	16	150	3.28	3.75	0.89	1.01
	36	Flat	2622	75	2.07	300	31.3	16	150	2.01	2.106	0.97	1.02
	72	Flat	404	167	4.27	300	32.5	16	150	4.14	4.47	0.97	1.05
	66	Flat	396	110	5.07	300	36.3	16	150	5.06	5.00	1.0	0.99
	51	Flat	1066	50	8.2	300	29.2	16	150	7.96	7.89	0.97	0.96
	72	Flat	404	167	4.27	300	32.5	16	150	4.14	4.47	0.97	1.05
	77	Flat	399	50	14.35	300	32	16	150	13.42	13.26	0.94	0.93
Sinclare [22]	E5	Flat	48	150	4.94	161	27.5	16	175	5.26	4.51	1.06	0.91

Table 3

Comparison of the predictions of the present model and UMIST formulae with experimentally obtained scabbing limits.

Ref	Test No	Nose-shape	Mass (kg)	d (mm)	V_{scabbing} (m/s)	H (mm)	f'_c (MPa)	d_r (mm)	C_r (mm)	UMIST V_s (m/s)	Present V_s (m/s)	UMIST error (V_s/V_{scabbing})	Present error (V_s/V_{scabbing})
Bechtel [6]	4F	Flat	96.94	203	37.21	304.8	39.813	0	0	41.99	34.92	1.13	0.94
	18F	Flat	96.49	203.2	63.14	457.2	35.19	0	0	67.78	57.68	1.07	0.92
UKAEA [6]	di 30	Flat	310	89	5.3	83	39.97	2.5	10	6.61	5.11	1.25	0.96
EIBL [6]	vii	Flat	1019	200	8.2	260	43	12	80	11.30	9.39	1.38	1.15
EDF [6]	iii-19	Flat	160	305	77.5	500	35.5	16	125	84.98	69.91	1.1	0.90
Bechtel [6]	4 hard	Flat	3.65	25.4	67.1	152.4	40.94	9.5	152.4	67.14	64.39	1.0	0.96
	5 hard	Flat	1.71	25.4	98.21	152.4	40.94	9.5	152.4	102.60	96.95	1.04	0.99
EMI [6]	380	Flat	2.25	45	155	230	35	6	40	147.14	137.80	0.95	0.89
Dancygier and Yankelevsky [23]	6 PC-H	Conical	0.16	25	144	60	104	0	0	46.47	128.07	0.32	0.89

1. The UMIST formulae can be used with reasonable confidence to predict the depth of penetration, cone cracking, scabbing and perforation of reinforced normal strength concrete targets subjected to impact by large mass low velocity missiles. On the other hand, for scabbing of reinforced concrete targets struck by non-flat-ended missiles the UMIST formulae are too conservative whilst they significantly overpredict the perforation of very thick concrete targets as compared to the experimental data. This is not surprising since the UMIST formulae were originally developed for flat-faced missiles and nuclear power industry where the threat by large mass low velocity projectile impact is a major concern.
2. The predictions of the new set of empirical equations have been shown to be in good agreement with the experimental data available in the literature for reinforced concrete targets subjected to impact by projectiles with a wide range of impact conditions. It has been demonstrated that these new equations are not only applicable for flat-nosed projectiles, but also for non-flat-nosed projectiles; that not only applicable for low velocity impact, but also for high velocity impact and that not only applicable for normal strength concrete, but also for high strength concrete.
3. The new empirical formulae have been verified against a wider range of parameters, i.e., $0.059 \text{ kg} < M < 2622 \text{ kg}$,

Table 4

Comparison of the predictions of the present model and UMIST formulae with experimentally obtained ballistic limits.

Reference	Test No	Nose shape	Mass (kg)	d (mm)	V_{50} (m/s)	H (mm)	f'_c (MPa)	d_r (mm)	C_r (mm)	UMIST V_b (m/s)	Present V_b (m/s)	UMIST error (V_b/V_{50})	Present error (V_b/V_{50})
UKAEA [6]	di 13	flat	159.4	150	6.67	83	42.65	2.5	32	5.31	6.99	0.80	1.05
	di 42	flat	310	89	3.58	80.3	38.27	2.5	15	4.87	4.18	1.36	1.17
	di 60	flat	54.48	83	3.73	83	38.8	0	0	5.64	4.53	1.51	1.21
EIBL [6]	vi 1	flat	1019	200	8.2	260	37	12	80	15.00	8.03	1.83	0.98
CEA [6]	M 53	flat	310	100	15.16	260	44	0	0	18.89	18.01	1.25	1.19
EDF [6]	III-31	flat	240	200	75.8	400	36	16	125	96.39	82.81	1.27	1.09
CEA [6]	M507A	conical	133	100	18.7	156	45	6	50	18.63	21.06	1.0	1.13
	M515	conical	252	100	16.68	156	48	6	50	10.89	17.17	0.65	1.03
Dancygier and Yankelevsky [23]	5SX-R	conical	0.16	25	158	50	35	0.5	7	65.10	147.88	0.41	0.94
	5SX-H	conical	0.16	25	206	50	110	0.5	7	89.77	195.28	0.44	0.95
Dancygier et al. [24]	7	conical	1.5	50	289	200	113	8	200/100	359.98	264.66	1.25	0.92
Borvick et al. [27]	C75-4	conical	0.20	20	318.4	100	80.1	0	0	355.29	326.75	1.12	1.03
	C75	conical	0.20	20	302.6	100	80.1	0	0	355.29	326.75	1.17	1.08
	C150-6	conical	0.20	20	317.5	100	129.2	0	0	355.29	331.62	1.12	1.04
	C200-3	conical	0.20	20	366.2	100	180.5	0	0	355.42	335.19	0.97	0.92
Beppu et al. [26]	3	spherical	0.05	25	307.43	70.2	25	0	0	235.67	310.02	0.77	1.01
Cargile et al. [25]	6	ogive	2.27	50.8	307.91	215.9	39.9	0	0	251.19	293.4	0.82	0.95
Brown [29]	7	ogive	2.32	50.8	288.56	215.9	39.9	0	0	247.39	289.83	0.86	1.0
	1	ogive	0.77	37	475	224	39.3	0	0	503.18	453.61	1.06	0.96
	2	ogive	0.77	37	939	559	39.3	0	0	1499.8	900.07	1.6	0.96
Wang et al. [30]		ogive	52	144	1278.8	6000	35	0	0	3211.3	1277.0	2.51	1.0

- 7.6 mm < d < 600 mm, $0 < V_i < 1300$ m/s, $13.5 \text{ MPa} < f_c < 210 \text{ MPa}$.
4. The modified UMIST formulae have also been found to be in reasonable agreement with the experimental data available in the literature.

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Appendix

The UMIST formulae originally developed for flat-faced projectiles [2,3] can be directly extended to obtain the modified UMIST formulae by substituting σ for σ_t of the Eq. (1) with $N^* = 0.72$ for depth of penetration; $\phi\sigma$ for σ_t of the Eqs. (3)–(5) and σ for σ_t of the Eqs. (7)–(9) with $N^* = 0.72$ whilst the coefficients and the functional forms are kept unchanged.

Depth of penetration

$$\frac{x}{d} = \left(\frac{2}{\pi}\right) \frac{MV_i^2}{\sigma d^3} \quad (\text{A1})$$

Cone cracking, scabbing and perforation for $H/d < 5$

$$\frac{E_c}{\eta\phi\sigma d^3} = -0.00031\left(\frac{H}{d}\right) + 0.00113\left(\frac{H}{d}\right)^2 \quad \left(0 < \frac{H}{d} \leq 2\right) \quad (\text{A2a})$$

$$\frac{E_c}{\eta\phi\sigma d^3} = -0.00325\left(\frac{H}{d}\right) + 0.00130\left(\frac{H}{d}\right)^3 \quad 2 < \frac{H}{d} < 5 \quad (\text{A2b})$$

$$\frac{E_s}{\eta\phi\sigma d^3} = -0.005441\left(\frac{H}{d}\right) + 0.01386\left(\frac{H}{d}\right)^2 \quad 0 < \frac{H}{d} < 5 \quad (\text{A3})$$

$$\frac{E_p}{\eta\phi\sigma d^3} = -0.00506\left(\frac{H}{d}\right) + 0.01506\left(\frac{H}{d}\right)^2 \quad 0 < \frac{H}{d} \leq 1 \quad (\text{A4a})$$

$$\frac{E_p}{\eta\phi\sigma d^3} = -0.01\left(\frac{H}{d}\right) + 0.02\left(\frac{H}{d}\right)^3 \quad 1 < \frac{H}{d} < 5 \quad (\text{A4b})$$

Cone cracking, scabbing and perforation for $H/d \geq 5$

$$\frac{E_c}{\sigma d^3} = \frac{\pi}{4} \left[\frac{H}{d} - 4.7 \right] \quad (\text{A5})$$

$$\frac{E_s}{\sigma d^3} = \frac{\pi}{4} \left[\frac{H}{d} - 4.3 \right] \quad (\text{A6})$$

$$\frac{E_p}{\sigma d^3} = \frac{\pi}{4} \left[\frac{H}{d} - 3.0 \right] \quad (\text{A7})$$

η is estimated by Eqs. (19) and (20).

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Glossary

- x : penetration depth into semi-infinite target
 d : projectile diameter
 N^* : nose shape factor
 H : target thickness
 M : projectile mass
 V_i : projectile impact velocity
 f_c : unconfined compressive strength of concrete
 E_k : Initial kinetic energy of missile

E_c : critical impact energy for through-thickness cone cracking
 E_s : critical impact energy for scabbing
 E_p : critical impact energy for perforation
 χ_i : Constants defined in Eqs. (16)–(18), $i = c, s, p$
 σ_i : mean dynamic resistive pressure, defined by Eq. (2)
 σ : mean dynamic resistive pressure, defined by Eq. (11)
 η : effect of rebar quantity and spacing, defined in Eq. (6)
 d_r : rebar diameter
 C_r : rebar spacing
 r_t : total bending reinforcement
 r : each way each face (% EWEF)
 α : parameter defined in Eq. (11)
 β : parameter defined in Eq. (11)

ρ_c : concrete density
 Y : a measure of shear strength for concrete
 ϕ : projectile nose shape factor
 θ : cone angle of a conical-nosed missile
 ψ : caliber-radius-head of an ogival-nosed projectile
 S : radius of an ogive nose
 $V_{cracking}$: limit velocity for cone cracking obtained by experiment
 $V_{scabbing}$: limit velocity for scabbing obtained by experiment
 V_{50} : limit velocity for perforation obtained by experiment
 V_c : limit velocity for cone cracking obtained by model
 V_s : limit velocity for cone scabbing obtained by model
 V_b : limit velocity for perforation obtained by model