

THE FOLLOWING 3 QUESTIONS
AND ANSWERS ARE FROM THE
2019 ADVANCED ENGINE DESIGN
RENT EXAM.

THE LOGIC WILL HELP YOU
REVISE.

Marked.

QUESTION ONE

You are required to carry out first and second order balancing on a single cylinder engine. The engine is a 450cc off road bike and requires to have minimum vibration. Piston mass is 450 gms, conrod mass is 420gms. Stroke is 60 mm and con-rod length between centres is 130mm. The crankshaft has a total mass of 3kg.

- a) Explain, using diagrams, the principle of using constant amplitude forward and reverse rotating vectors to represent the reciprocating and rotating first order forces. Calculate the total mass moment in kg.mm of the resultant first order forward vector.
(2 marks)
- b) Calculate the required offset in mm of the centre of gravity of the crankshaft from its centre-line of rotation to achieve 100% balance of the first order forward vector.
(2 marks)
- c) Calculate the mass moment in kg.mm of the resultant reverse rotating first order vector. If the counter rotating balance shaft has a mass of 2kg calculate its required centre gravity offset from its centre-line or rotation to achieve the required mass moment.
(2 marks)
- d) Explain, using a sketch, how you would drive the balance shaft. If the crankshaft has rotated 270 degrees from TDC show the position of the balance shaft using a suitable sketch.
(2 marks)
- e) Explain, using diagrams, the principle of using constant amplitude forward and reverse rotating vectors to represent the reciprocating second order forces. Calculate the total mass moments in Kg.mm of the forward and reverse second order vectors.
(2 marks)
- f) If balance shafts are used to cancel the second order forces and each balance shaft has a mass of 2kg, calculate the required offset of their respective centres of gravity.
(1 mark)
- g) If the crankshaft has rotated 270 degrees from TDC, show the positions of the second order balance shafts using a suitable sketch and identifying the directions of rotation of each shaft.
(3 marks)
- h) Using another sketch, show how both the second order balance shafts could be driven from the crankshaft.
(2 marks)
- i) If the engine is running at 7000 rpm crankshaft speed and the torque on the first order and two second order balance shafts can be assumed to be 3 Nm at each shaft pulley, calculate the total power loss driving the three balance shafts.
(4 marks)

QUESTION TWO

You are required to carry out first and second order balancing on a 120 degree bank angle 800cc vee twin 4 stroke engine. There are two crankshaft options. First, a crankshaft with both con-rods on a common crankpin, and second, a crankshaft with offset crankpins. Piston mass is 330gms and con rod mass 330gms. Stroke is 50 mm and con rod length between centres is 140mm.

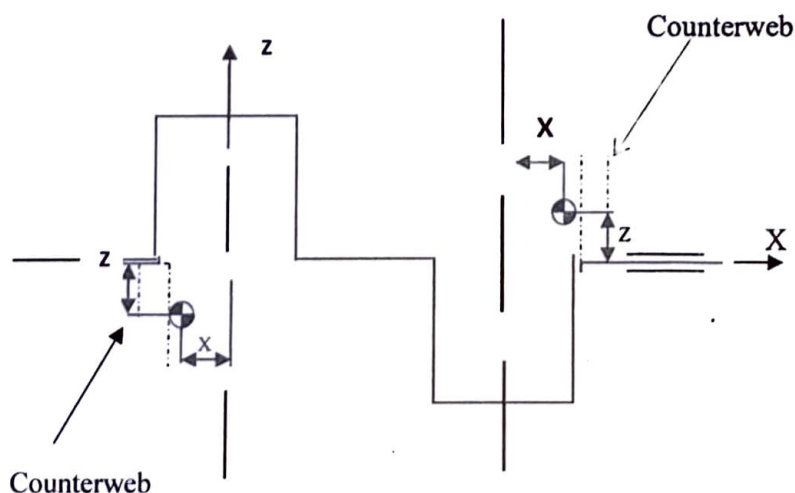
- a) For the common crankpin crankshaft, calculate the mass moment of the resultant first order forward rotating vector in kg.mm. If the crankpin is along bank 1, show the direction of the resultant first order rotating vector using a suitable sketch. (2 marks)
- b) Calculate the mass moment in Kg.mm and direction of the resultant first order reverse rotating vector if the crankpin is along bank 2. Use a suitable sketch to show direction. (2 marks)
- c) Calculate the mass moment in Kg.mm and direction of the resultant second order forward rotating vector if the crankpin is along Bank 1. Use a suitable sketch. (3 marks)
- d) Calculate the mass moment in Kg.mm and direction of the resultant second order reverse rotating vector if the crankpin is along Bank 1. Use a suitable sketch. (3 marks)
- e) The crankshaft is now to incorporate offset crankpins and putting pin 1 along Bank 1, calculate the offset angle required to cancel out the reverse rotating first order vectors. (2 marks)
- f) With this offset angle and pin 2 along Bank 2, calculate the mass moment in Kg.m and direction of the resultant second order forward rotating vector. Use a sketch to illustrate your answer. (2 marks)
- g) With pin 2 along Bank 2, calculate the mass moment in Kg.m and direction of the resultant second order reverse rotating vector. Use a sketch to illustrate your answer. (4 marks)
- h) Are the resultant second order forward and reverse rotating vectors the same magnitude? If the crankshaft now turns clockwise ninety degrees, show the mass moment in Kg.m and direction of the resultant second order forward and reverse rotating vectors using a suitable sketch. (2 marks)

QUESTION THREE

You are required to analyse the crankshaft and flywheel on an in-line 2 cylinder 2 stroke engine with crankpins at 180° to each other.

- The engine has 3 main bearings.
 - The individual piston and gudgeon pin mass is 300 gms.
 - The individual con-rod mass is 360 gms.
 - The stroke is 50 mm.
 - The bore-centre spacing (pin to pin) is 100 mm.
- (a) Calculate the mass of the required bob-weights which would be needed for balancing the crankshaft on a balancing machine if 100% of the forward rotating vectors are to be balanced. (3marks)
- (b) The crankshaft is to be fully balanced using 2 equal counter webs on the outer sides of the 2 respective crankpins.

The diagram shows the x and z coordinate axes for the crankshaft.



If the centre of mass offset z for an individual crankpin including is 3 mm, calculate the x offset from the middle of the crankpin to achieve the required balance if the total crankshaft mass is 6 kg.

(4 marks)

- c) It is now required to add a counter-rotating balance shaft to cancel the resultant first order reverse rotating moment of the mass moments along the crankshaft. Calculate the magnitude of the reverse rotating moment in kg.mm.mm. (2 marks)
- d) If the balance shaft is basically a circular shaft with two eccentric masses at each end at a distance 150mm apart, calculate the mass moment in kg.mm of each eccentric mass to achieve 100% balance of the reverse rotating moment. (3 marks)

- 1.
- e) If the crankpins are now phased by 90° so that when crankpin 1 is at TDC, crankpin 2 is 90° past TDC, calculate the magnitude of the resultant first order moment of the mass moments in kg.mm.mm along the crankshaft. (3 marks)
- f) Looking from the front of the crankshaft with pin 1 at TDC, show the direction of the resultant first order forward rotating moment of the mass moments along the crankshaft. Explain how you calculated the direction. (3 marks)
- g) Looking from the front of the crankshaft with pin 1 at TDC, show the direction of the resultant first order reverse rotating moment of the mass moments along the crankshaft. Explain your reasoning. (2 marks)

1.

EXAM

RESIT

JULY 2019

QUESTION 1.

$$\begin{aligned}
 M_{rec} &= 450 + \frac{1}{3} \times 420 \\
 &= 590 \text{ gms} \\
 &= \underline{\underline{0,59 \text{ kg.}}}
 \end{aligned}$$

$$\begin{aligned}
 M_{rot} &= \frac{2}{3} \times 420 \\
 &= 280 \text{ gms} \\
 &= \underline{\underline{0,28 \text{ kg.}}}
 \end{aligned}$$

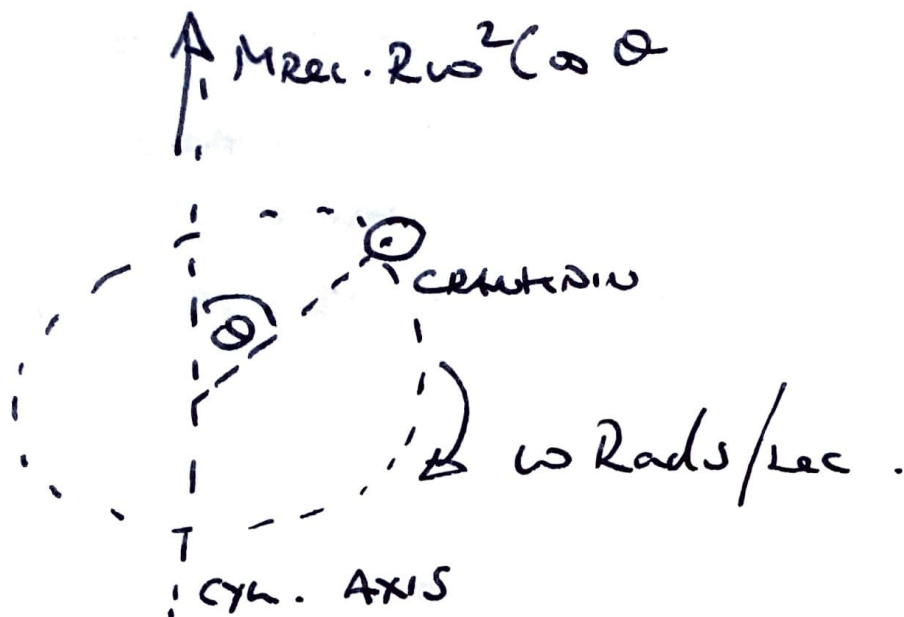
$$R = \frac{60}{2} = \underline{\underline{30 \text{ mm}}}$$

$$L = \underline{\underline{130 \text{ mm.}}}$$

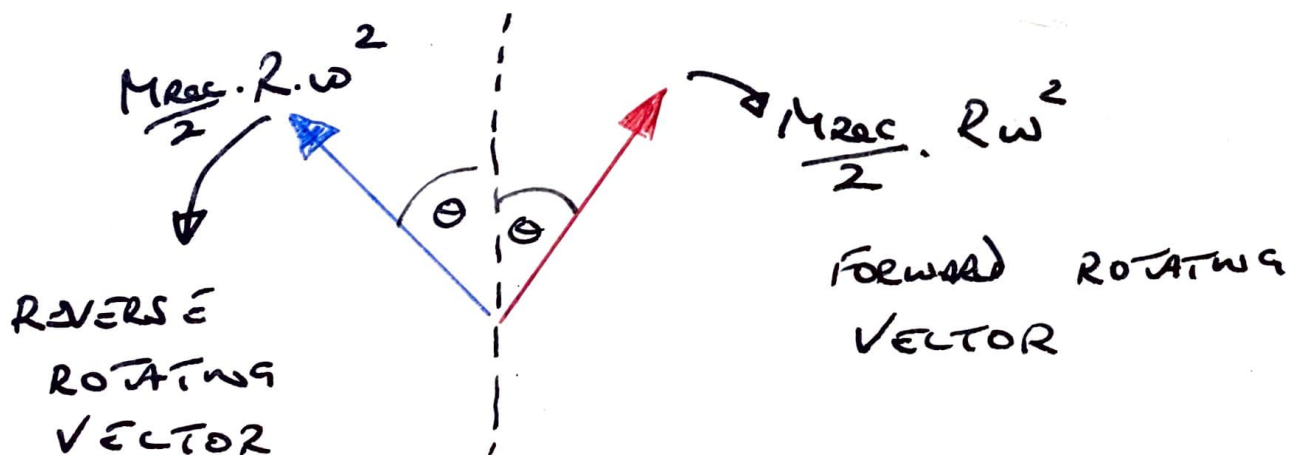
(a) THE RECIPROCATING MASS RESULTS IN FIRST ORDER FORCES ALONG THE CYLINDER AXIS GIVEN BY:

$$F = M_{rec} \cdot R \omega^2 \cos \theta$$

2.



THIS VERTICAL FIRST ORDER FORCE IS MATHEMATICALLY IDENTICAL TO TWO CONSTANT AMPLITUDE COUNTER ROTATING FORCES AS SHOWN BELOW:

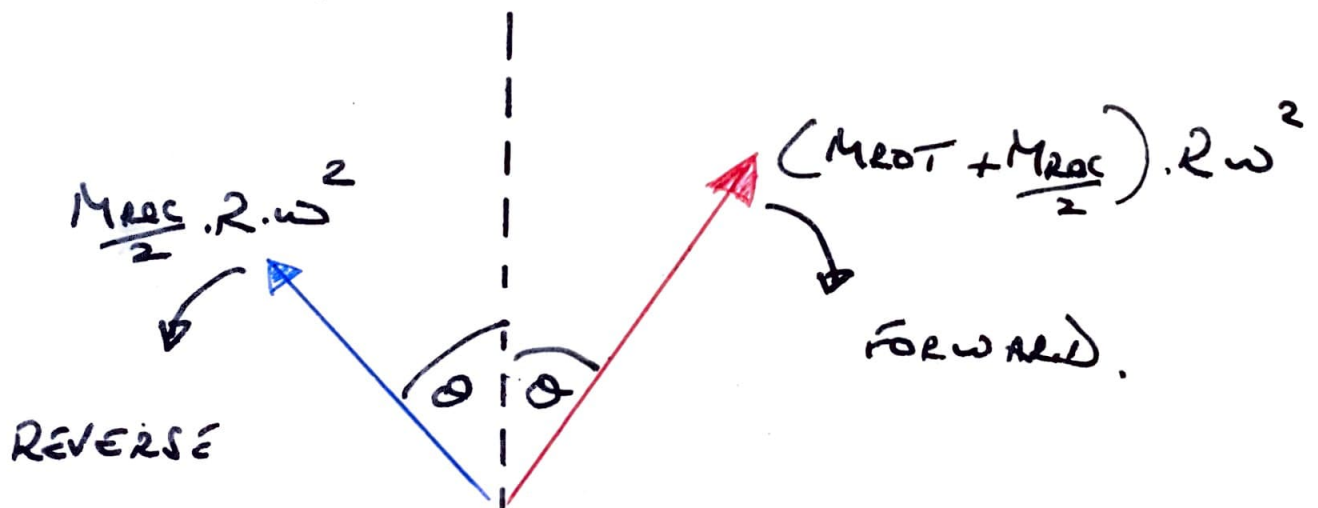


3.

THE ROTATING MASS OF THE COUPLER CREATES A CENTRIFUGAL FORCE ACTING ALONG DIRECTION OF CRANKPIN AS SHOWN BELOW:



THE TOTAL FIRST ORDER FORCES CAN BE REPRESENTED AS BELOW:

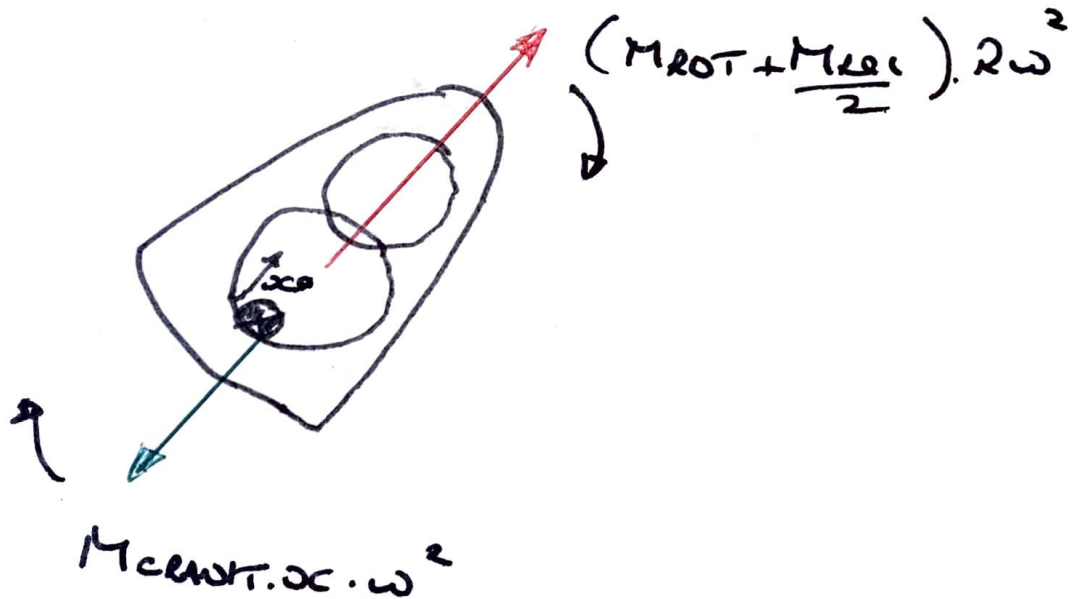


4.

First order followed

$$\begin{aligned}
 \text{MASS MOMENT} &= \left(M_{\text{ROT}} + \frac{M_{\text{REC}}}{2} \right) \cdot R \\
 &= \left(0,28 + \frac{0,59}{2} \right) \cdot 30 \\
 &= (0,28 + 0,295) \cdot 30 \\
 &= \underline{\underline{17,25 \text{ kg} \cdot \text{mm}}}
 \end{aligned}$$

(b)



To fully balance mass

$$M_{\text{CENTR.}} \cdot x \cdot \omega^2 = \left(M_{\text{ROT}} + \frac{M_{\text{REC}}}{2} \right) R \omega^2$$

$$\text{IE } M_{\text{CENTR.}} \cdot x = \left(M_{\text{ROT}} + \frac{M_{\text{REC}}}{2} \right) \cdot R$$

$$3 \cdot x = 17,25$$

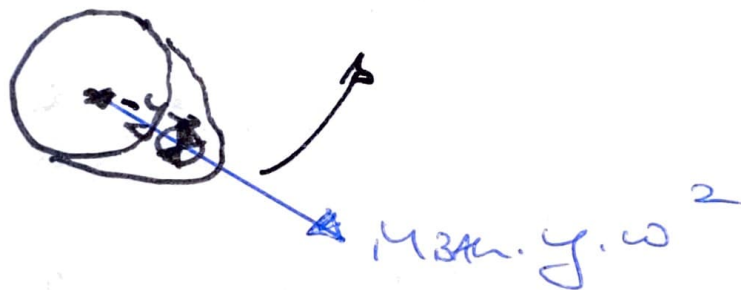
$$x = \underline{\underline{5,75 \text{ mm}}}$$

5.

(c) MASS MOMENT OF
REVERSE VECTOR $= \frac{M_{\text{ecc}}}{2} \cdot R$

$$= \frac{0,59}{2} \cdot 30$$

$$= \underline{\underline{8,85 \text{ kg} \cdot \text{mm}}}$$



$$M_{\text{BM}} \cdot y = 8,85$$

$$2 \cdot y = 8,85$$

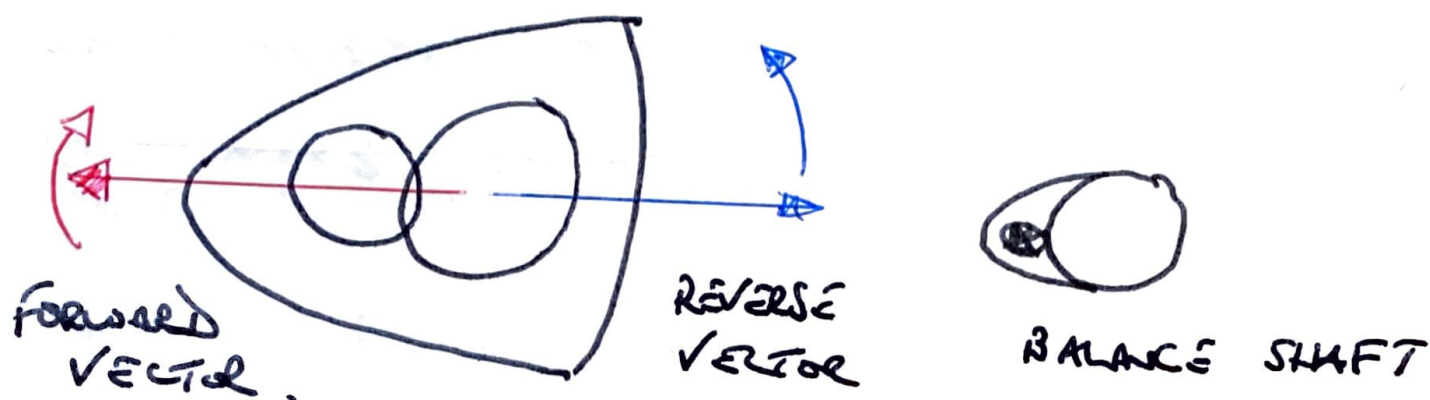
$$y = \underline{\underline{4,43 \text{ mm}}}$$

(d) USE GEARS OF EQUAL DIAMETER



TO CREATE WORK
BUT OPPOSITE
ROTATIONAL SPEED.

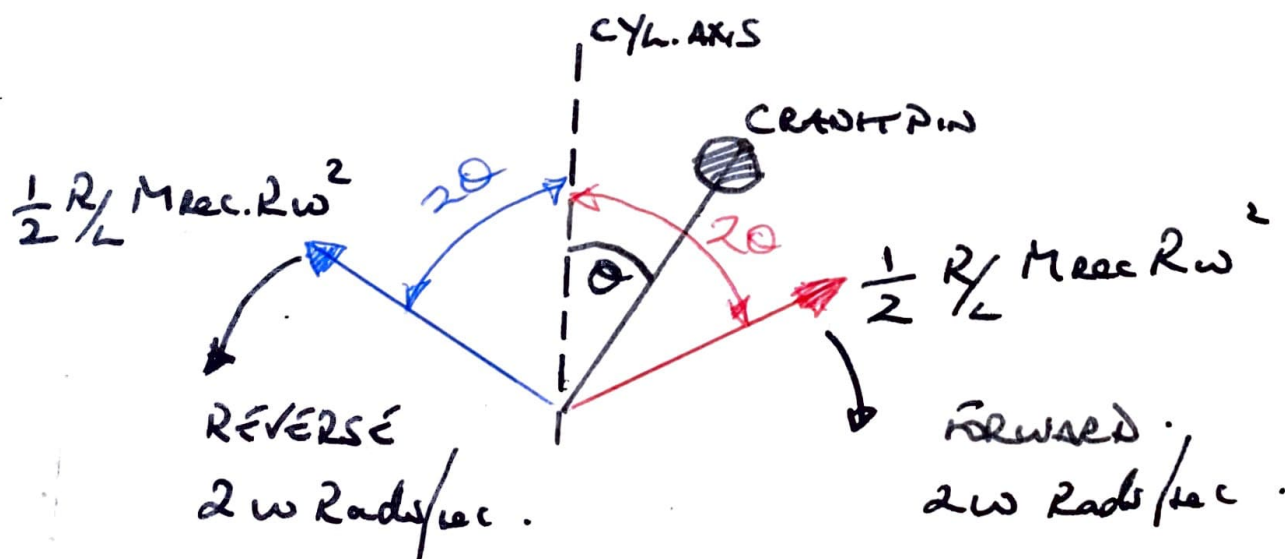
6.



(c) Vertical second order force

$$= \frac{R}{L} M_{rec} R \omega^2 \cos 2\theta$$

This can be represented as two constant amplitude counter rotating vectors as shown below:



7.

MASS MOMENT OF SECOND ORDER FOLLOWED

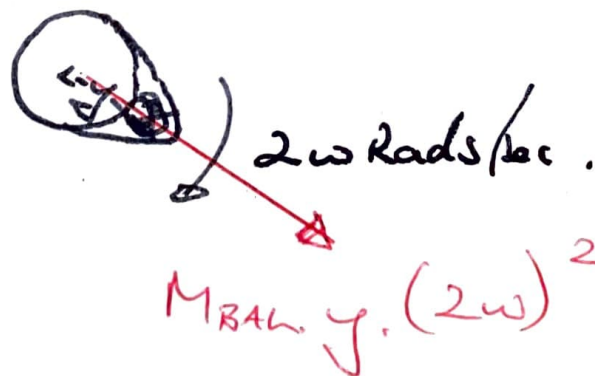
= MASS MOMENT OF SECOND ORDER
REVERSED

$$= \frac{1}{2} \frac{R}{L} M_{2ec} \cdot R$$

$$= \frac{1}{2} \cdot \frac{30}{130} \times 0,59 \times 30$$

$$= \underline{\underline{2,0 \text{ kg.m}}}$$

(f) TO BALANCE SECOND ORDER VIBRATIONS
WE BALANCE SHIFTS ROTATING AT
TWICE ENGINE SPEED.



$$\begin{aligned} \text{BALANCING FORCE} &= M_{BAL} \cdot y \cdot (2\omega)^2 \\ &= 4 M_{BAL} \cdot y \cdot \omega^2 \end{aligned}$$

#8

$$\text{MASS MOMENT} = 4 \text{ MBAL. } y$$

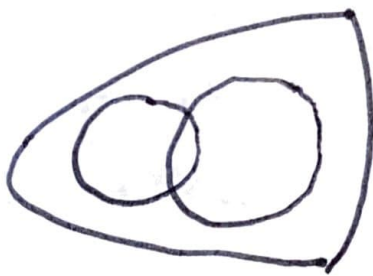
For 100% BALANCE MTRC

$$4 \cdot \text{MBAL. } y = 2,0$$

$$4 \cdot x 2 \cdot y = 2$$

$$y = \frac{2}{8} = \underline{\underline{0,25 \text{ mm.}}}$$

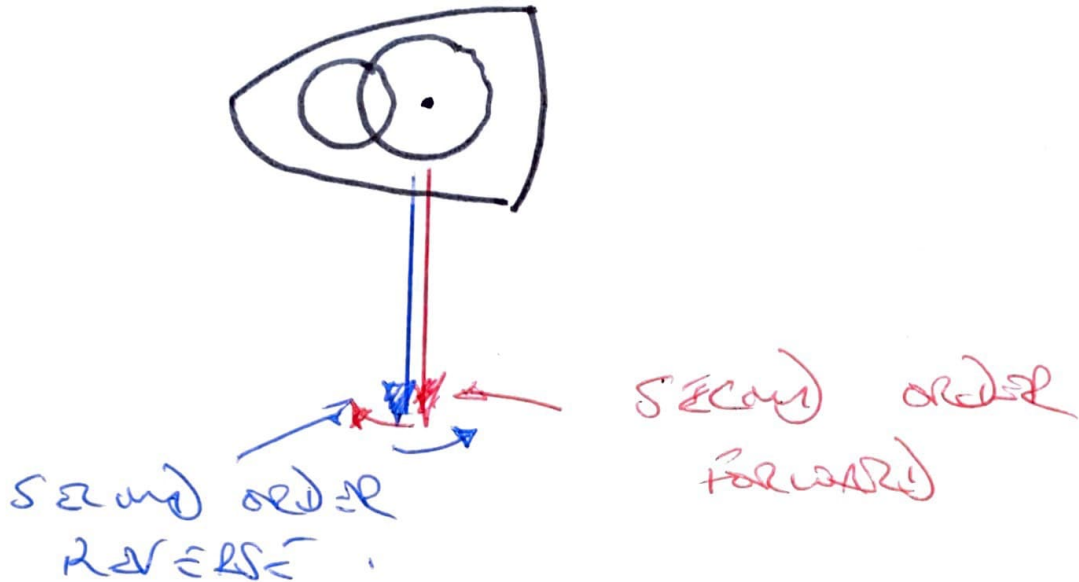
(9)



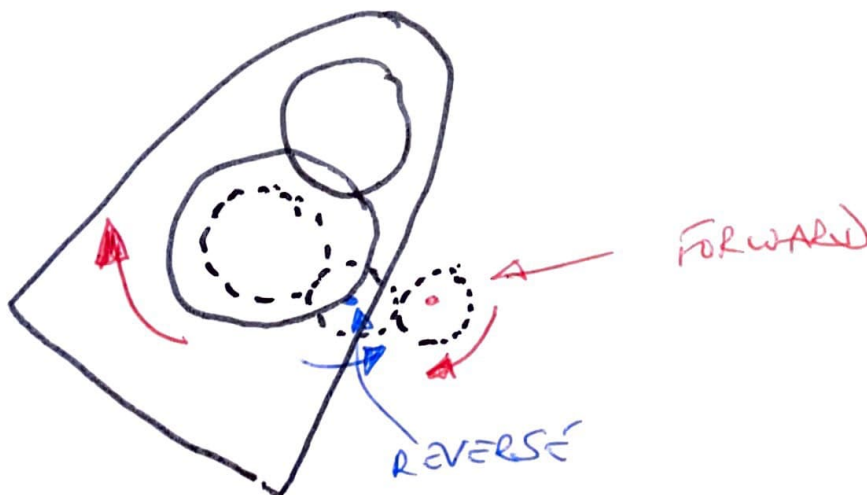
CRANKSHAFT ROTATES 270° FROM TDC
 SECOND ORDER BEWARED ROTATES 540° FROM TDC
 " " REVERSE " 540° FROM TDC.

89.

DIAGRAM BELOW SHOWS DIRECTION OF SECOND ORDER VALUES.



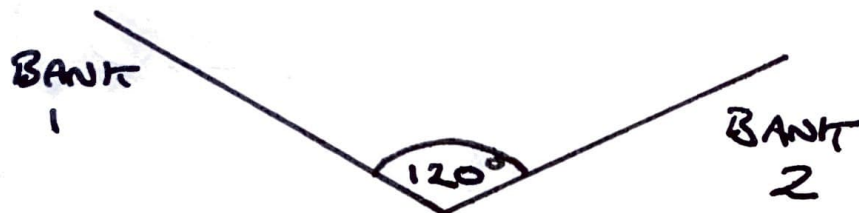
(h) GEAR DRIVE. USE SECONDARY GEARS OF HALF DIAMETER TO DOUBLE SPEED OF ROTATION.



Q.10.

(2) DON'T WORRY ABOUT THIS QUESTION.

11. QUESTION 2.



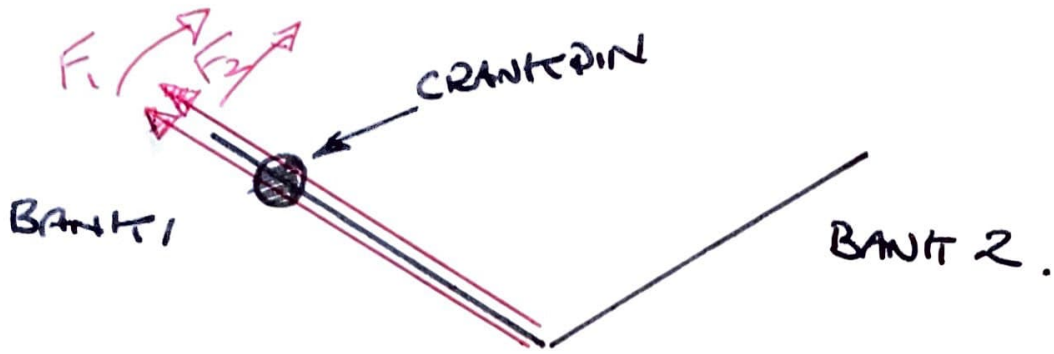
$$\begin{aligned} M_{rec} &= 330 + \frac{1}{3} \times 330 \\ &= 440 \text{ gms} \\ &= \underline{\underline{0,44 \text{ kg.}}} \end{aligned}$$

$$\begin{aligned} M_{rot} &= \frac{2}{3} \times 330 \\ &= 220 \text{ gms} \\ &= \underline{\underline{0,22 \text{ kg.}}} \end{aligned}$$

$$R = \frac{50}{2} = \underline{\underline{25 \text{ mm}}}$$

$$L = \underline{\underline{140 \text{ mm.}}}$$

12.



$$F_1 = F_2 = (M_{ROT} + \frac{M_{REC}}{2}) \cdot R \cdot \omega^2$$

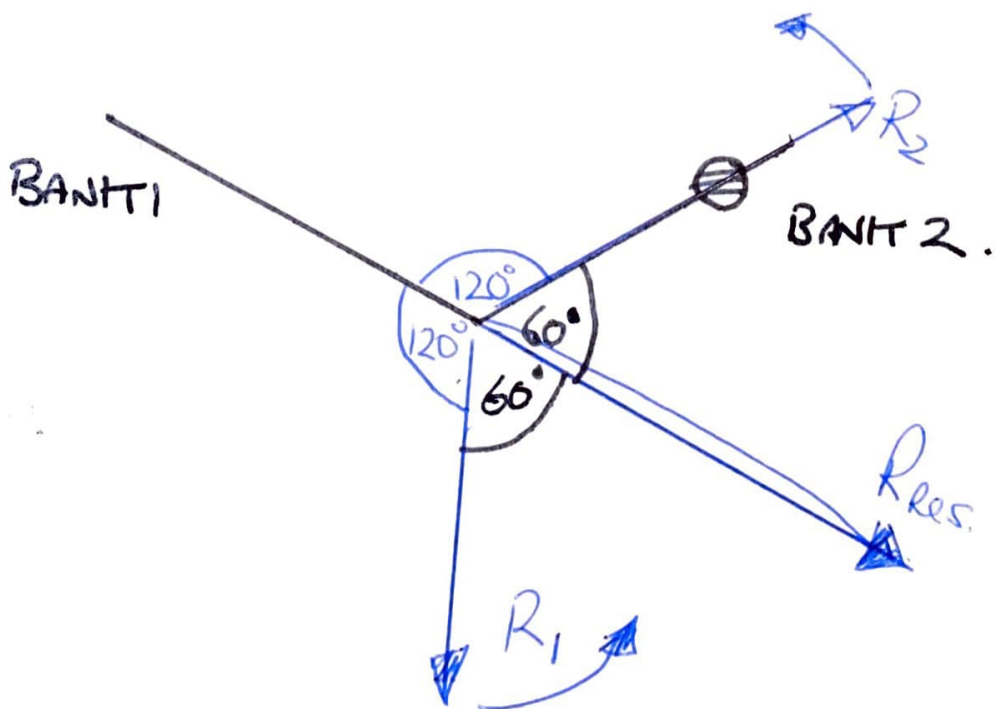
MASS MOMENT OF RESULTANT

$$\text{FIRST ORDER VECTOR} = 2 \left(M_{ROT} + \frac{M_{REC}}{2} \right) \cdot R$$

$$= 2 (0,22 + 0,22) \times 25$$

$$= \underline{\underline{22 \text{ kg.m.}}}$$

(b)



13.

$$R_1 = R_2 = \frac{M_{rec}}{2} \cdot R \omega^2$$

RESULTANT RADIAL VECTOR

$$= \frac{M_{rec}}{2} \cdot R \omega^2 \cos 60^\circ$$

$$+ \frac{M_{rec}}{2} R \omega^2 \cos 60^\circ$$

RESULTANT RADIAL MASS MOMENT

$$= \frac{M_{rec}}{2} \cdot R \cos 60^\circ + \frac{M_{rec}}{2} \cdot R \cos 60^\circ$$

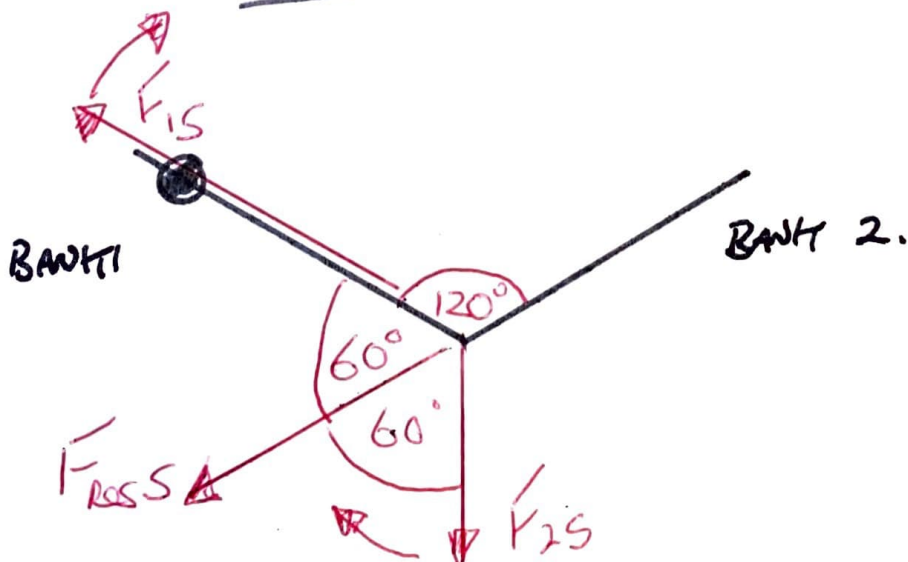
$$= \frac{M_{rec}}{2} \cdot R \times 0,5 + \frac{M_{rec}}{2} \cdot R \times 0,5$$

$$= \frac{M_{rec}}{2} \cdot R$$

$$= 0,22 \times 25$$

$$= 11 \text{ kg} \cdot \text{mm.}$$

(c)



$$14. F_{1S} = F_{2S} = \frac{1}{2} \cdot \frac{R}{L} M_{rec} \cdot R \cdot \omega^2.$$

$$F_{205S} = F_{1S} \cos 60^\circ + F_{2S} \cos 60^\circ$$

MASS MOMENT OF RESULTANT STROUD

order forward vector

$$= \frac{1}{2} \frac{R}{L} M_{rec} \cdot R \cdot \cos 60 + \frac{1}{2} \frac{R}{L} M_{rec} \cdot R \cdot \cos 60$$

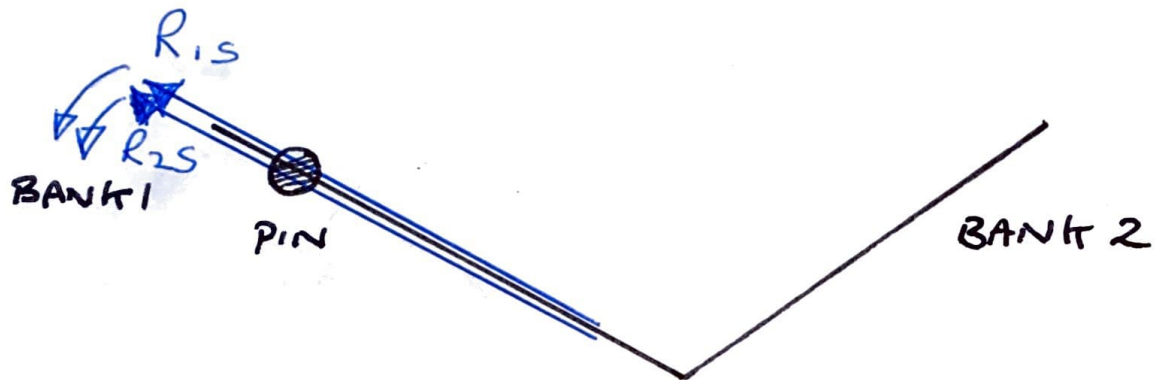
$$= \frac{1}{2} \cdot \frac{R}{L} M_{rec} \cdot R$$

$$= \frac{1}{2} \cdot \frac{25}{140} \cdot 0,44 \cdot 25$$

$$= \underline{\underline{0,98 \text{ kg} \cdot \text{mm}}}$$

14 a.

(d)



$$R_{1s} = R_{2s} = \frac{1}{2} \frac{R}{L} M_{rec.} R \omega^2$$

MASS MOMENT OF RESULTANT ~~SCREW~~
OR ~~25~~ REVERSE VECTOR

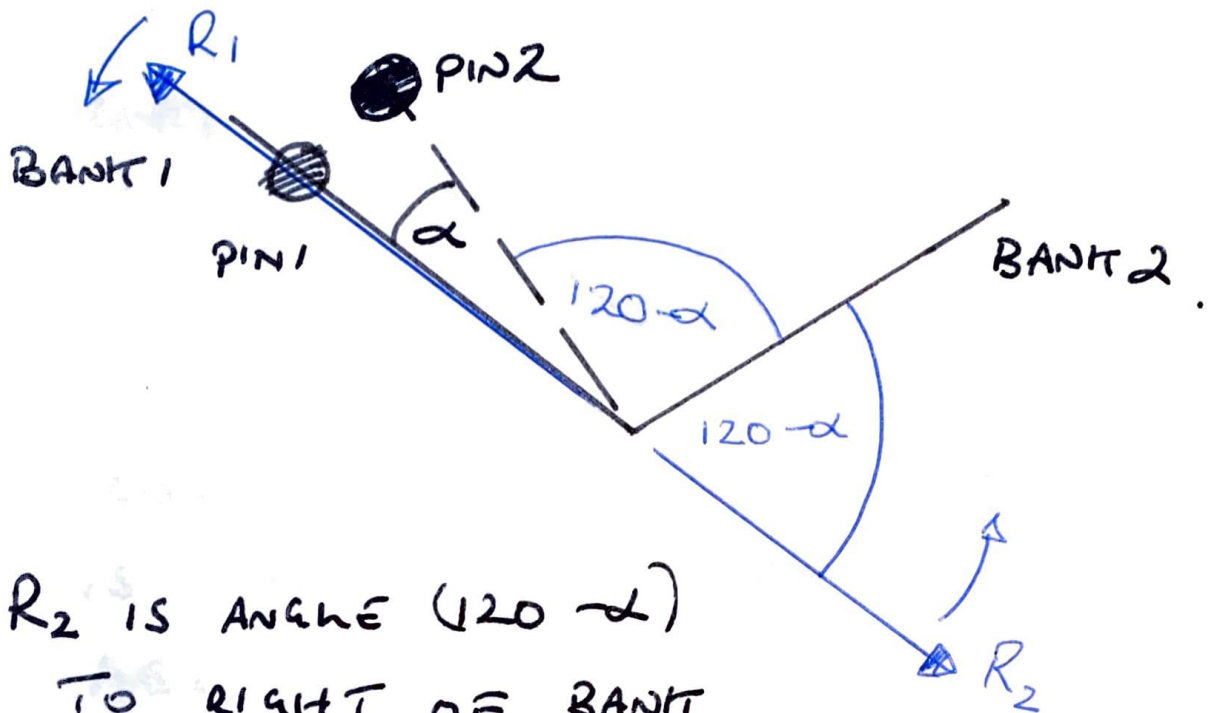
$$= 2 \times \frac{1}{2} \frac{R}{L} M_{rec.} R$$

$$= \frac{25}{140} \times 0,44 \times 25$$

$$= \underline{\underline{1,96 \text{ kg} \cdot \text{mm}}}$$

15.

(2) SPLIT THE CRANKPINS AS SHOWN.



R_2 IS ANGLE $(120 - \alpha)$
TO RIGHT OF BANK
2 AND NEEDS TO BE
OPPOSITE BANK 1.

$$\therefore \alpha + (120 - \alpha) + (120 - \alpha) = 180.$$

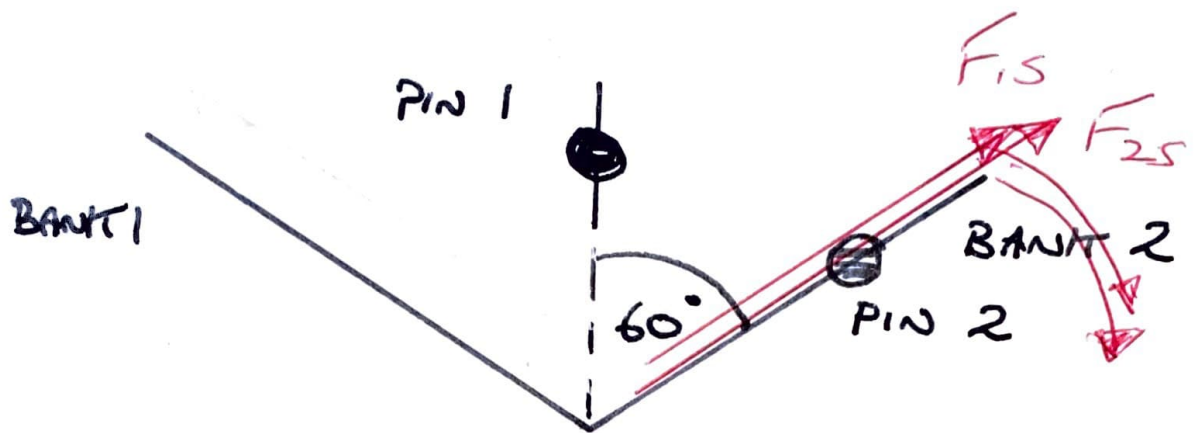
$$240 - \alpha = 180$$

$$\alpha = 240 - 180$$

$$= 60^\circ$$

16.

(f)



RESULTANT SECOND ORDER FORWARD
IS ALONG BANIT 2 AND HAS
MBS MOMENT

$$= 2 \cdot \frac{1}{2} R/L \cdot M_{rec} \cdot R$$

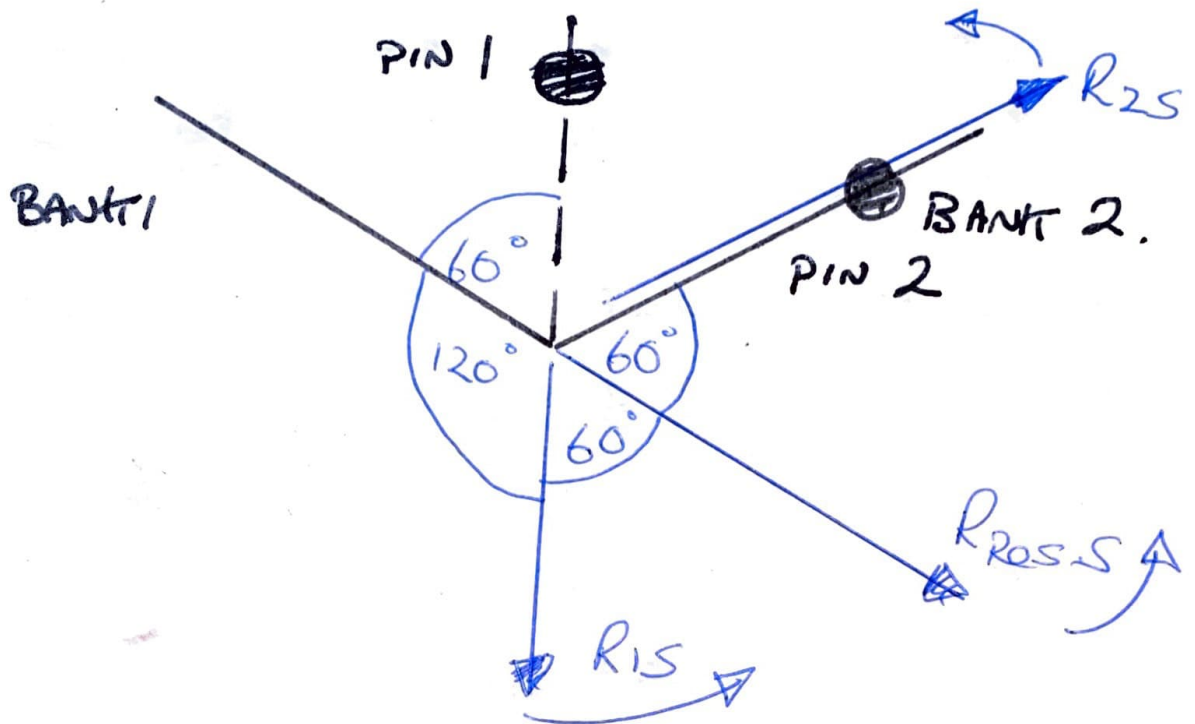
$$= \frac{25}{140} \cdot 0,44 \cdot 25$$

$$= \underline{\underline{1,96 \text{ kg.m.}}}$$

$$= \underline{\underline{1,96 \times 10^{-3} \text{ kg.m.}}}$$

17.

(9)



R_{res} IS OPPOSITE BANT 1.

MASS MOMENT

$$= \frac{1}{2} R / M_{rec} \cdot R \cdot \cos 60^\circ + \frac{1}{2} R / M_{rec} \cdot R \cdot \cos 60^\circ$$

$$= \frac{1}{2} \cdot R / M_{rec} \cdot R$$

$$= \frac{1}{2} \times \frac{25}{140} + 0,44 \times 25$$

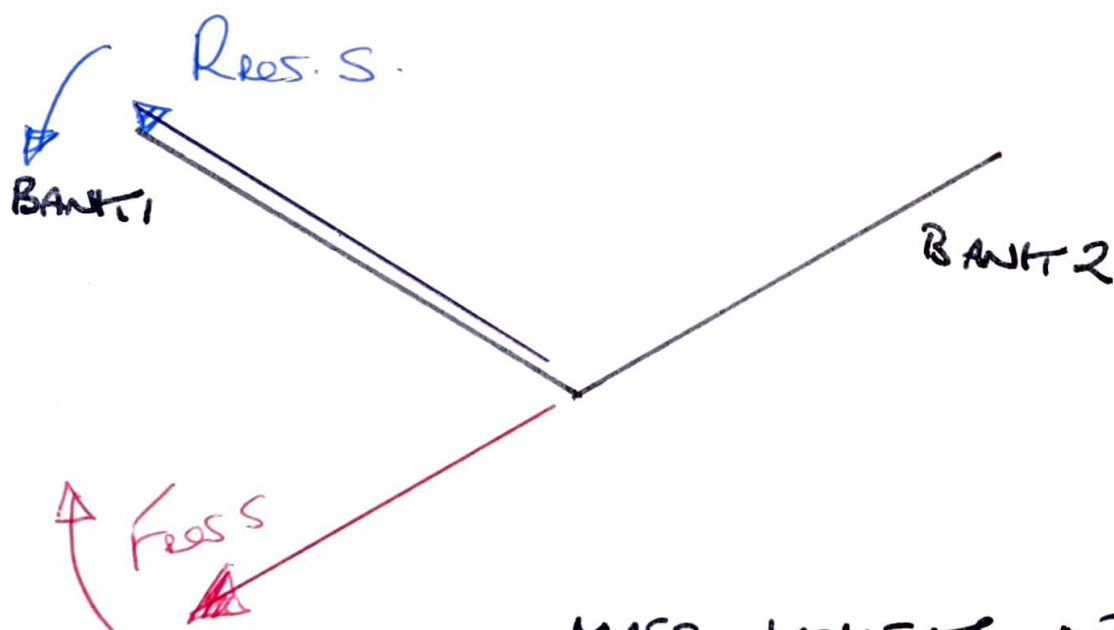
$$= \underline{0,98 \text{ kg.m}}$$

$$= \underline{\underline{0,98 \times 10^{-3} \text{ kg.m}}}$$

(h) THE RESULTANT FORWARD AND REVERSE SECOND ORDER VECTORS ARE NOT THE SAME MAGNITUDE.

IF CRANESHAFT TURNS CLOCKWISE 90° THE FORWARD SECOND ORDER VECTOR TURNS 180° CLOCKWISE AND REVERSE SECOND ORDER TURNS 180° ANTI CLOCKWISE.

DIAGRAM LOOKS AS BELOW:



MASS MOMENTS OF FORWARD AND REVERSE RESULTANTS STAY AS BEFORE IN (f) AND (g).

19.

QUESTION 3

$$M_{\text{rec}} = 300 + \frac{1}{3} \times 360$$

$$= 420 \text{ gms}$$

$$= \underline{\underline{0,42 \text{ kg.}}}$$

$$M_{\text{ROT}} = \frac{2}{3} \times 360$$

$$= 240 \text{ gms}$$

$$= \underline{\underline{0,24 \text{ kg.}}}$$

$$R = \frac{50}{2} = 25 \text{ mm}$$

BOB SPACING = 100 mm.

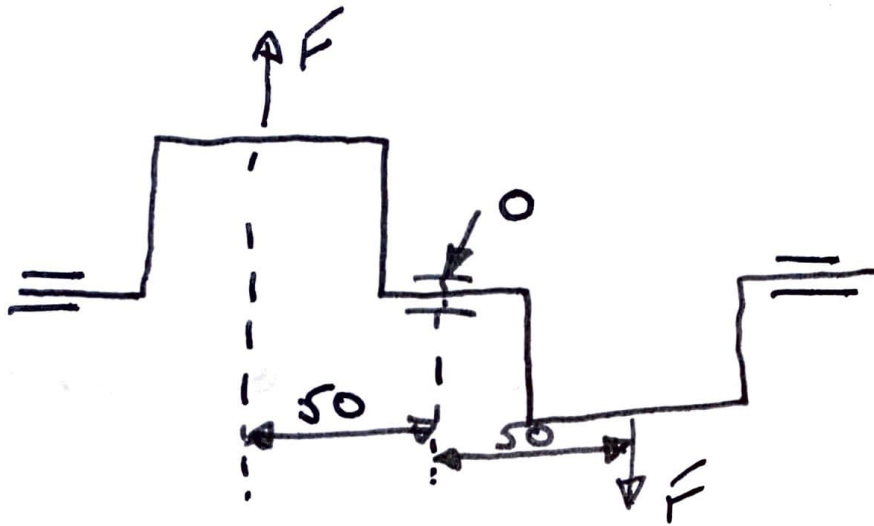
$$(c) \quad \text{BOB WEIGHT MASS} = \frac{M_{\text{ROT}} + M_{\text{rec}}}{2}$$

$$= \frac{0,24 + 0,42}{2}$$

$$= \underline{\underline{0,45 \text{ kg.}}}$$

20.

(b)



$$F = \left(M_{\text{ROT}} + \frac{M_{\text{REC}}}{2} \right) R \omega^2$$

TAKING MOMENTS ABOUT CENTRE O
IN PLANE OF CRANKPINS OF THE
FORCES F

$$M_{11} = 50F + 50F = 100F$$

$$\text{MASS OF EACH CRANKSPAN} = \frac{6}{2} = 3 \text{ kg.}$$

IF CENTRE OF MASS OFFSET AT
EACH SPAN = 3 mm

COUNTERBALANCE FORCE AT EACH SPAN

$$= M_{\text{CRANKSPAN}} \cdot Z \cdot \omega^2$$

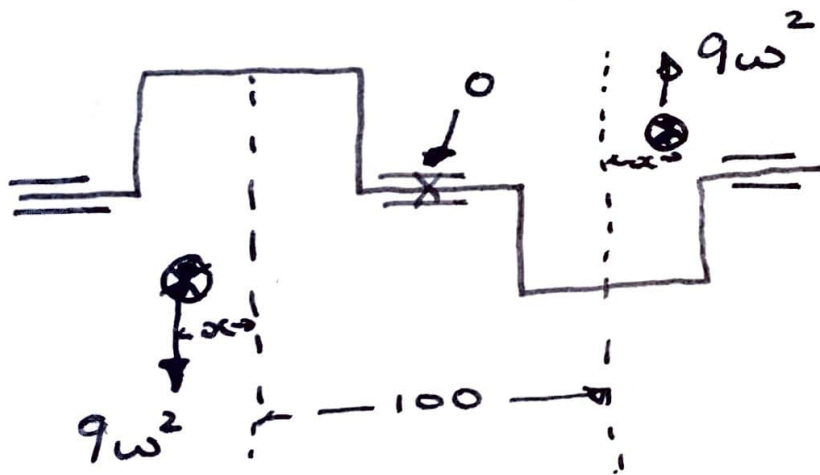
21.

$$= 3.3 \cdot \omega^2$$

$$= 9\omega^2$$

$$\underline{\underline{=}}$$

DIAGRAM BELOW SHOWS COUNTERBALANCE FORCES



MOMENT OF COUNTERBALANCE FORCES
ABOUT CENTRE O IN PLANE OF
CENTRIFUGES

$$= (50 + x) \cdot 9\omega^2 + (50 + x) \cdot 9\omega^2$$

$$= (100 + 2x) 9\omega^2$$

$$= 900\omega^2 + 18x\omega^2$$

22.

FOR FULL BALANCE

$$100F = 900\omega^2 + 18 \cdot x \cdot \omega^2$$

$$F = (M_{205} + \frac{M_{225}}{2}) \cdot 2\omega^2$$

$$= 0,45 \times 25 \cdot \omega^2$$

$$= 11,25 \cdot \omega^2$$

$$\therefore 100 \times 11,25 \cancel{\omega^2} = 900 \cancel{\omega^2} + 18 \cdot x \cdot \cancel{\omega^2}$$

$$1125 = 900 + 18x$$

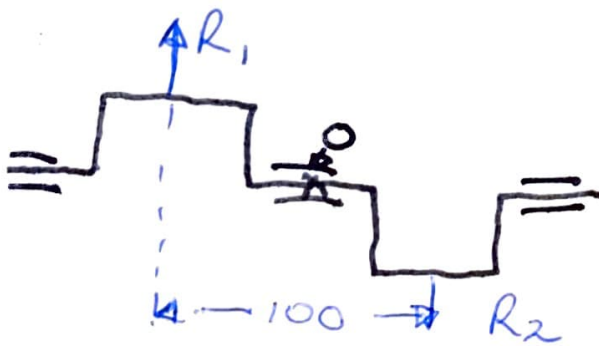
$$225 = 18x$$

$$x = \underline{\underline{12,5 \text{ mm.}}}$$

(c) LOOK AT REVERSE VECTORS

WHEN CRANKPIN 1 IS AT TDC.

DIAGRAM BELOW SHOWS REVERSE VECTORS:



23.

$$R_1 = R_2 = \frac{M_{rec}}{2} \cdot R \omega^2$$

MASS MOMENT OF EACH
REVERSE VECTOR = $\frac{M_{rec}}{2} \cdot R$

$$= \frac{0,42 \times 25}{2}$$

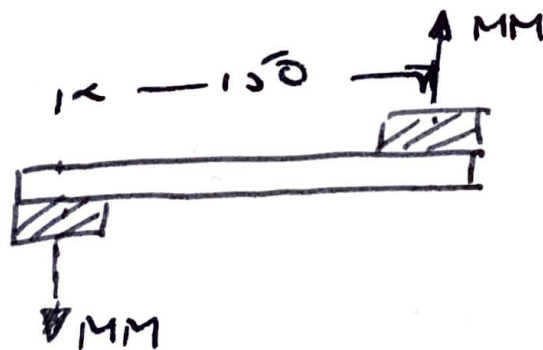
$$= 5,25 \text{ kg.mm.}$$

MOMENT OF MASS MOMENTS OF
REVERSE VECTORS

$$= 100 \times 5,25$$

$$= 525 \text{ kg.mm.mm.}$$

(d)



COUNTERBALANCE SHAFT HAS TWO
EQUAL MASS MOMENTS 150 mm APART

$$\text{MOMENT OF MASS MOMENTS} = 150 \cdot MM$$

24.

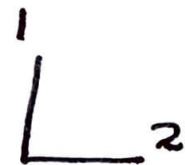
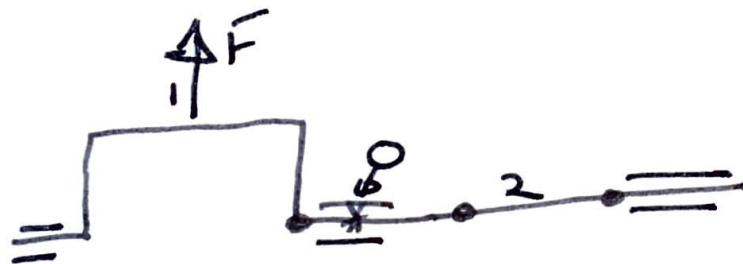
FOR 100% BALANCE OF REVERSE
VERTICALS

$$150 \text{ MM} = 525$$

$$\text{MM} = \underline{\underline{3,5 \text{ kg.mm.}}}$$

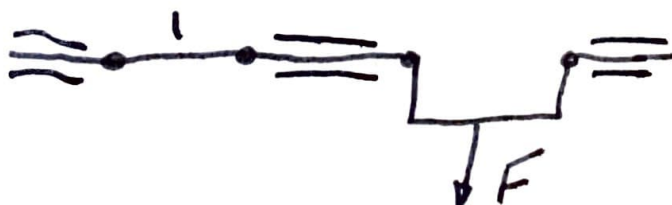
(2) CALCULATE MOMENT OF MASS
MOMENTS ABOUT CENTRE IN
PLANE OF PIN 1.

DIAGRAM SHOWS THE IN PLANE
FORCES



$$M_{11} = 50F$$

FOR M_1 LOOK AT FORCES PERPENDICULAR
TO PLANE OF PIN 1 AS BELOW:



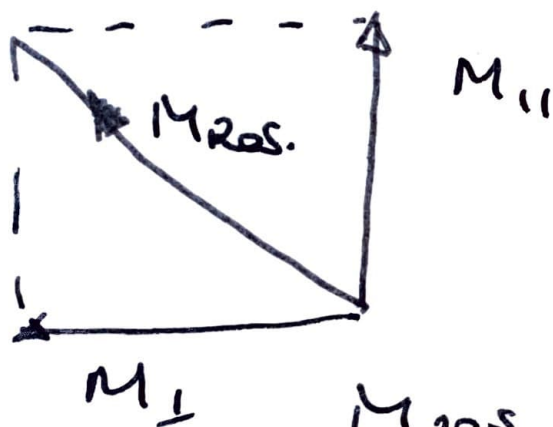
25.

$$M_{\perp} = 50F.$$

USE PYTHAGORAS.

PIN 2 MOVED

REAR OF CRANK TO RIGHT AND
FRONT OF CRANK TO LEFT



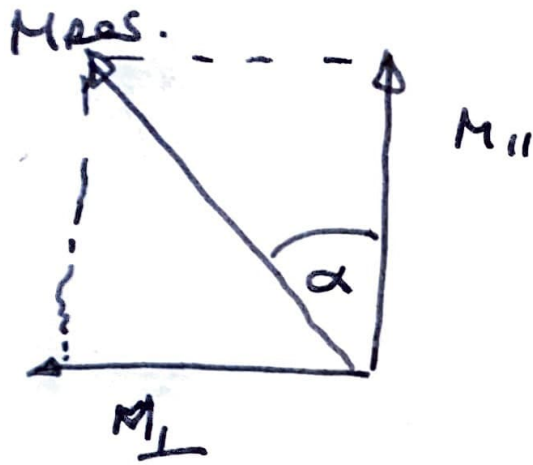
$$\begin{aligned} M_{res} &= \sqrt{M_{\parallel}^2 + M_{\perp}^2} \\ &= \sqrt{(50F)^2 + (50F)^2} \\ &= F \cdot \sqrt{50^2 + 50^2} \\ &= \underline{\underline{70,7F.}} \end{aligned}$$

MOMENT OF MASS MOMENTS

$$\begin{aligned} &= 70,7 \cdot (M_{rot} + \frac{M_{res}}{2}) \cdot R \\ &= 70,7 \times 0,45 \times 25 \\ &= \underline{\underline{795,4 \text{ kg.m.m.m}}} \end{aligned}$$

26 .

(f)



$$\tan \alpha = \frac{M_L}{M_{II}} = \frac{50F}{50F} = 1$$

$$\alpha = \underline{\underline{45^\circ}}$$

(g)

MOMENT OF REVERSE MASS MOMENTS
IS SAME ANGLE THE OTHER SIDE
OF CYLINDER AXIS.

