

Fault tree analysis

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Introduction

Fault tree analysis (FTA) is a tool used to systematically assess the probability of an undesirable event occurring, this process starts with a top-level event and branches leading down showing the combination of events that need to occur or not occur to fulfil the main event.

Primarily, FTA is used to evaluate potential failure modes before an action is taken or a product is developed. By identifying these failure modes early, organizations can reduce the cost of failures or minimize the potential physical harm and reputational harm they could cause, making it a commonly used tool for risk assessments [1]

A fault tree consists of Boolean logic gates, these demonstrate how different operations interact to create possible failure modes.

Boolean logic gates:

AND gate: Multiple conditions must be true (positive or negative) for the for the event to occur.

(Example there must be both students AND a teacher for class to begin.)

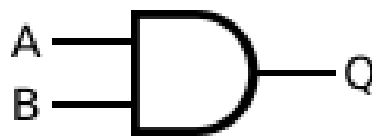


Figure 1 And gate [2]

OR gate: Either condition must be true, or both conditions can be true.

(Example wood OR coal can be used to fuel a fire.)



Figure 2 OR gate [2]

NOT Gate: Outputs the opposite of the input (Inverter)

(Example Two people sat on a seesaw, if one is touching the ground the other person will do the opposite, so they must be in the air)

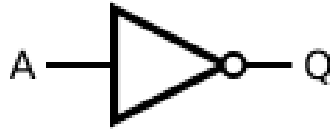


Figure 3 NOT gate [2]

XOR Gate: Returns true if either condition is true, but not both simultaneously.

(Example For there to be a winner in arm-wrestling either player must win for there to be a winner, but both can't win at the same time)



Figure 4 XOR gate [2]

Fault tree analysis calculation pre-Human factors

The probability of non-availability is calculated first, before the analysis can occur. This is done using the equation $(1 - \text{probability of availability})$. Converting the values in this way ensures that all probabilities are expressed using the same criteria (success), allowing the FTA calculations to be performed consistently.

Chance of leaky Kettle = $1 - 0.001 = 0.999$

Chance of water supply failure = $1 - 0.01 = 0.99$

Mains supply failure chance = $1 - 0.01 = 0.99$

Event	Original Table Value	Success or Failure	Corrected Value to Use in Tree
Tea availability	0.98	Success	0.98
Coffee availability	0.98	Success	0.98
Chance of leaky Kettle	0.001	Failure	0.999
Chance of water supply failure	0.01	Failure	0.99
Kettle reliability	0.99	Success	0.99
Plug in operation reliability	0.89	Success	0.89
Switch on reliability	0.89	Success	0.89
Mains supply failure chance	0.01	Failure	0.99
Cup availability	1	Success	1
Cup reliability	0.99	Success	0.99
Milk availability	0.95	Success	0.95
Milk freshness	0.95	Success	0.95
Sugar availability	0.88	Success	0.88
Sweetener Availability	0.99	Success	0.99

Table 1 Value conversion table

Hot cup of tea with milk and sugar

The Completed drink consists of a Hot cup of tea with milk and sugar for later comparison.

Key: P = probability

$$P(\text{Hot Cup of Tea with Milk}) = P(\text{Hot Water}) \times P(\text{Cup}) \times P(\text{Ingredients in Cup})$$

$$P(\text{Hot Cup of Tea with Milk}) = 0.7678 \times 0.99 \times 0.9011 = 0.6849$$

$$P(\text{Hot Cup of Tea with Milk}) = 0.6849$$

Calculations for Hot water

$$P(\text{Hot Water}) = P(\text{Water in Kettel}) \times P(\text{Power to Element})$$

$$P(\text{Hot Water}) = 0.98901 \times 0.7763 = 0.7678$$

Calculations for water in kettle

$$P = (\text{Water in Kettel}) = P(\text{Water Supply}) \times P(\text{No Leaks})$$

$$P = (\text{Water in Kettel}) = 0.99 \times 0.999 = 0.98901$$

$$P(\text{Water in Kettel}) = 0.98901$$

Calculations for Power to Element

$$P(\text{Power to Element})$$

$$= P(\text{Plugged in}) \times P(\text{Switched on}) \times P(\text{Mains Supply})$$

$$\times P(\text{KettleOK})P(\text{Power to Element}) = 0.89 \times 0.89 \times 0.99 \times 0.99$$

$$= 0.77633721$$

$$P(\text{Power to Element}) = 0.7763$$

Calculations for Ingredients in cup

$$P(\text{Ingredients in cup}) = P(\text{Correct Drink}) \times P(\text{Sugar}) \times P(\text{Milk})$$

$$P(\text{Ingredients in cup}) = 0.9996 \times 0.9988 \times 0.9025 = 0.9011$$

$$P(\text{Ingredients in cup}) = 0.9011$$

Calculations for Correct drink

Correct drink selected is tea for later comparison

$$P(\text{Correct Drink}) = 0.98 + 0.98 - (0.98 \times 0.98)$$

$$P(\text{Correct Drink}) = 1.98 - 0.9604 = 0.9996$$

$$P(\text{Correct Drink}) = 0.9996$$

Calculations for Sugar

Correct drink selected includes sugar for later comparison

$$P(\text{Sugar}) = 0.88 + 0.99 - (0.88 \times 0.99)$$

$$P(\text{Sugar}) = 1.87 - 0.8712 = 0.9988$$

$$P(\text{Sugar}) = 0.9988$$

Calculations for Milk

$$P(\text{Milk}) = 0.95 \times 0.95 = 0.9025$$

$$P(\text{Milk}) = 0.9025$$

Calculations for Cup

$$P(Cup) = P(In\ Good\ Order) \times P(Available)$$

$$P(Cup) = 0.99 \times 1 = 0.99$$

$$P(Cup) = 0.99$$

Fault tree analysis calculation post-Human factors

Integration of Human factors into the fault tree

When calculating Bill's likelihood of successfully preparing a drink under fatigue, it is known that his ability to plug in and switch on the kettle decreases to 75% of its normal level, while his ability to select a cup decreases to 90% of normal.

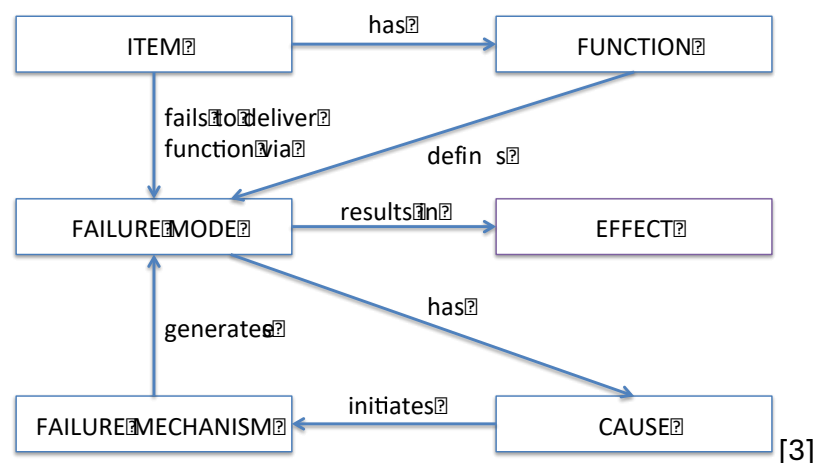
The first step is to update the effected probabilities to reflect the fatigued conditions. For the plug-in operation and the switch-on action their original values are simply reduced by 75% of normal.

Conversely, selecting a cup requires a different approach. As the initial assumption was to independently reduce both cup availability and cup reliability to 90% of their normal values. However, this can be proven to be incorrect by breaking down a systems model of failure.

A Systems Model of Failure Analysis

The item and its function dictate the environment where failure is possible, this being the failure mode. The failure mode is driven by the cause, In this case Bill waking up in the middle of the night, which initiates the failure mechanism, fatigue. However, the exact failure mode is still unknown, but the effect is known, Bill's ability to select a cup has decreased to 90% of its normal level. This analysis leads to the conclusion that a new, unknown failure mode must be responsible for the effect, as neither cup availability nor cup reliability should be impacted by the failure mechanism of fatigue.

Knowing this, the possible failure mode can be hypothesised by using the known failure mechanism, fatigue, to generate possible failure modes. This could be selecting bowl instead of a cup, as this is a possible yet an unlikely failure mode that could result in the reduced cup selection probability.



Item	Cup
Function	To hold drink
Failure mode	Unknown
Failure mechanism	Fatigue
Cause	Bill being woken in the middle of the night
Effect	Reduction in ability to select Cup

Table 2 A Systems Model of Failure Analysis Cup example

[1] [3]

Ultimately, this demonstrates that the probabilities for both cup availability and cup reliability remain unchanged. However, introducing a new failure mode (or modes) would be inappropriate, as any associated probabilities would be unsubstantiated due to a lack of supporting information. Instead, the method applied introduces a reduction at the final AND gate output, representing the adjusted probability of successfully selecting a cup [4]

Incorrect assumptions	Original Value	New Adjusted Value (with Human Error)
Cup availability (should not be calculated)	1	0.90
Cup reliability (should not be calculated)	0.99	0.891
Cup	0.99	0.8019

Table 3 Incorrect assumption

Correct adjusted Probabilities	Original Value	New Adjusted Value (with Human Error)
Plug in operation reliability	0.89	0.6675
Switch on reliability	0.89	0.6675
Cup	0.99	0.891

Table 4 Adjusted probabilities

Fatigued hot cup of tea with milk and sugar

$$\begin{aligned}
 P(\text{Fatigued Hot Cup of Tea with Milk}) \\
 &= P(\text{Fatigued Hot Water}) \times P(\text{Fatigued Cup}) \times P(\text{Ingredients in Cup})
 \end{aligned}$$

$$\begin{aligned}
 P(\text{Fatigued Hot Cup of Tea with Milk}) &= 0.4319 \times 0.8910 \times 0.9011 \\
 &= 0.34676391519
 \end{aligned}$$

$$P(\text{Fatigued Hot Cup of Tea with Milk}) = 0.3468$$

Calculations for Ingredients in cup

Calculations have not changed for ingredients in cup.

$$P(\text{Ingredients in cup}) = P(\text{Correct Drink}) \times P(\text{Sugar}) \times P(\text{Milk})$$

$$P(\text{Ingredients in cup}) = 0.9996 \times 0.9988 \times 0.9025 = 0.9011$$

$$P(\text{Ingredients in cup}) = 0.9011$$

Calculations for Correct drink

Tea is selected per proforma

$$P(\text{Correct Drink}) = 0.98 + 0.98 - (0.98 \times 0.98)$$

$$P(\text{Correct Drink}) = 1.98 - 0.9604 = 0.9996$$

$$P(\text{Correct Drink}) = 0.9996$$

Calculations for Sugar

Sugar is selected per proforma

$$P(\text{Sugar}) = 0.88 + 0.99 - (0.88 \times 0.99)$$

$$P(\text{Sugar}) = 1.87 - 0.8712 = 0.9988$$

$$P(\text{Sugar}) = 0.9988$$

Calculations for Milk

$$P(\text{Milk}) = 0.95 \times 0.95 = 0.9025$$

$$P(\text{Milk}) = 0.9025$$

Calculations for Fatigued Cup

As the failure mode is unknown the reduction is applied to the overall cup selection probability.

$$P(Cup) = P(In\ Good\ Order) \times P(Available)$$

$$P(Cup) = 0.99 \times 1 = 0.99$$

$$P(Cup) = 0.99$$

$$P(Cup) \times 0.90 = P(Fatigued\ Cup)$$

$$0.99 \times 0.90 = 0.8910$$

$$P(Fatigued\ Cup) = 0.8910$$

Calculations for Fatigued Hot water

$$P(\text{Fatigued Hot Water}) = P(\text{Water in Kettel}) \times P(\text{Fatigued Power to Element})$$

$$P(\text{Fatigued Hot Water}) = 0.98901 \times 0.4367 = 0.4319$$

Calculations for Fatigued Power to Element

$$P(\text{Fatigued Power to Element}) =$$

$$P(\text{Fatigued Plugged in}) \times P(\text{Fatigued Switched on}) \times P(\text{Mains Supply}) \\ \times P(\text{KettleOK})$$

$$P(\text{Fatigued Power to Element}) = 0.6675 \times 0.6675 \times 0.99 \times 0.99 = 0.43668968062$$

$$P(\text{Fatigued Power to Element}) = 0.4367$$

Calculations for water in kettle

$$P = (\text{Water in Kettel}) = P(\text{Water Supply}) \times P(\text{No Leaks})$$

$$P = (\text{Water in Kettel}) = 0.99 \times 0.999 = 0.98901$$

$$P(\text{Water in Kettel}) = 0.98901$$

Application and effect of human factors analysis

After conducting a baseline Fault Tree Analysis (FTA) for the process of making the correct drink, the failure mechanism fatigue was introduced to evaluate its impact on operational success.

The calculations showed that the baseline success rate was 68.49% but with the introduction of the failure mechanism the success rate decreased to 34.68%. This equates to a percentage decrease of 49.36% underpinning the significant impact that changes in subsystems success rate can affect the Top-event/overall system.

$$\frac{68.49 - 34.68}{68.49} \times 100 = 49.36\%$$

Within this exercise the negative effect increase was caused by the human factor failure mechanism of fatigue caused by waking up in the middle of the night, however this can be implemented to identify weaknesses from any part a system/process. Applying this within a wider systems engineering framework allows for the evaluation of systems before its implementation. This creates a lowered risk environment for the identification of the possible failure modes, failure mechanisms and their causes to see their possible effect early in its lifecycle. This subsequently results in being able to calculate what failure modes have effects that are tolerable and those that need to be solved before system implementation, this calculation normally consists of a balance of cost, reliability and safety.[5]

The recall of Takata airbags illustrates this point, as the Japanese manufacturer Takata was forced into a global recall due to a faulty inflator seal that allowed moisture to destabilize the ammonium nitrate propellant used, resulting in millions of airbag units no longer fit for purpose. This caused Takata to file for bankruptcy in 2017, however if a proactive fault identification method was used, like an FTA, early in the design phase, environmental egress from inadequate seals could have been identified as a critical failure mode. This would have allowed for early-stage solutions, such as selecting a more stable propellant or engineering a more robust seal. [6], [7]

Item	Airbag
Function	Deploy during a crash to reduce likelihood of severe injury to the occupants
Failure mode	Unstable ammonium nitrate
Failure mechanism	Environmental egress
Cause	faulty inflator seal
Effect	Increase of death and injury due to improper deployment

Table 5 A Systems Model of Failure Analysis Air bag example

	Original probabilities %	Fatigue probabilities %	Percentage change %
Tea Milk Sugar	68.49	34.68	49.36
Hot water	76.78	43.19	43.75
Power to element	77.63	43.67	43.74
Cup	99	89.1	10
Plugged in	89	66.75	25
Switched on	89	66.75	25

Table 6 Percentage change of probability

Conclusion

The results of the FTA clearly demonstrate that the introduction of the failure mechanism fatigue significantly compromised operational reliability of the system, demonstrating the tool's effectiveness in identifying faults/weaknesses within systems. However, the importance of this method is made more apparent by the Takata airbag recall. A proactive fault identification process, such as a FTA, could have identified environmental egress as a critical failure mode when paired with ammonium nitrate propellant long before production. By identifying these catastrophic safety risks early, Takata could have avoided their bankruptcy but more importantly could have prevented the deaths that were caused by this fault not initially being identified. In summary, these examples highlight that FTA is not only a tool for simple tasks, but a versatile and essential safeguard for all systems.

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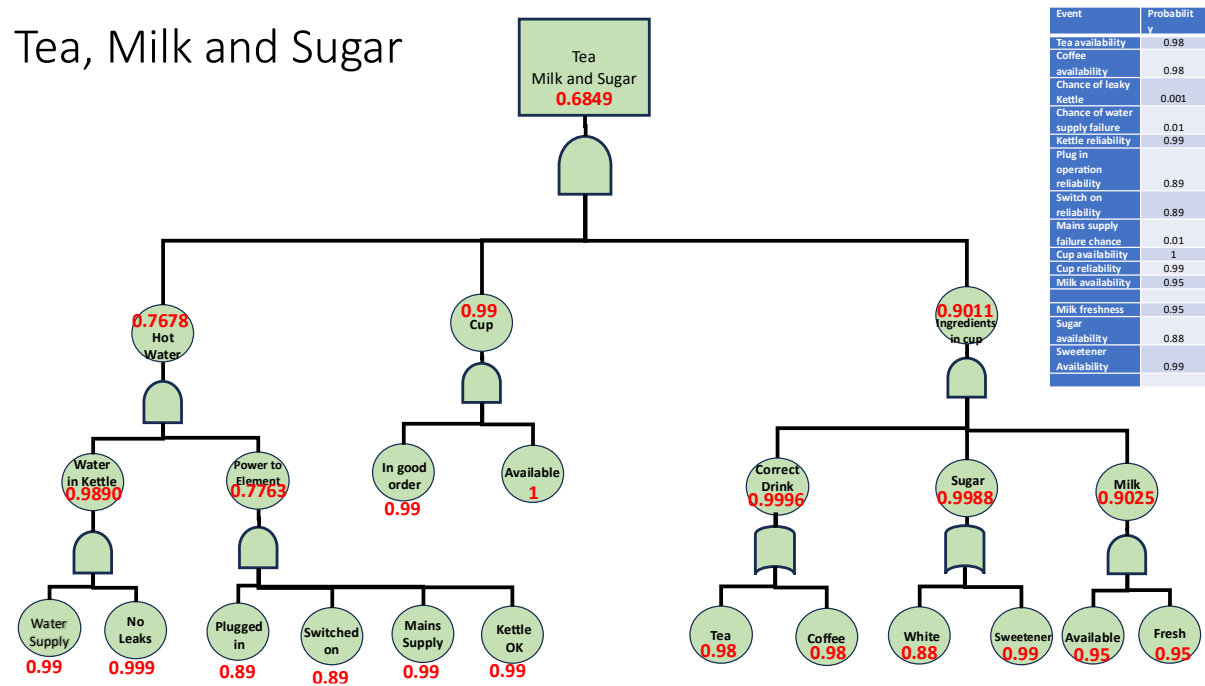
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Annex

Annex A fault tree

Tea, Milk and Sugar



Annex B Fatigued fault tree

Fatigued Tea, Milk and Sugar

