ASSIGNMENT TITLE	:	ANALYSIS OF AN RLC NETWORK
MODULE	:	L5 – ELECTRICAL ENGINEERING
MODULE CODE	:	ACDA5001
PROGRAMME	:	BENG ELECTRICAL & ELECTRONIC ENGINEERING
STUDENT NAME	:	FREDDIE WILSON
STUDENT NO	:	2413880

1 ABSTRACT

1.1. In this assignment, the task was to identify and calculate a capacitor size to bring the power factor of an RLC circuit to >0.95. This was performed by placing an 18.5 μF capacitor in parallel with the inductive load to achieve a theoretical unity power factor of 1. Using the circuit, phasor diagrams were created to analyse the voltage and current at different frequencies above and below resonance. Finally, this report investigates the three-phase electrical power distribution system, what power factor correction is, and its importance throughout electrical systems.

TABLE OF CONTENTS

1	ABSTRACT	2
2	INTRODUCTION	5
3	METHOD	6
4	RESULTS	12
5	DISCUSSION	18
6	CONCLUSION	20
7	REFERENCES	21

TABLE OF FIGURES

Figure 1: Parallel RLC Circuit	6
Figure 2: Graph of Circuit Current and Load Voltage Over Time	7
Figure 3: Graph of Power Factor Against Capacitance	10
Figure 4: Graph of Phase Angle Against Capacitance	11
Figure 5: Phasor Diagram at Resonance	14
Figure 6: 50 Hz Phasor Diagram	15
Figure 7: 12.5 Hz Phasor Diagram	16
Figure 8: 62.5 Hz Phasor Diagram	17

LIST OF ACRONYMS AND UNITS

Acronyms	Definition
A	Amperes
AC	Alternating Current
DC	Direct Current
Hz	Hertz
VA	Volt Ampere
W	Watt
VAR	Volt Ampere Reactance
V	Volts
RMS	Root Mean Square
f	Frequency
RLC	Resistor, Inductor, Capacitor
PFC	Power Factor Correction
PF	Power Factor
F	Farads
H	Henry
Z	Impedance
Ω	Ohms
Φ	Phase Angle

2 INTRODUCTION

2.1 Alternating current is generated by rotating a coil within a magnetic field [1, p. 19]. This AC is sinusoidal and periodically alternates between positive and negative magnitudes, with the rate of this change equal to the frequency (f). As a result, alternating currents produce responses in resistors, inductors, and capacitors that do not occur with direct current (DC) [1, p. 18]. Other characteristics of AC circuits are the phase angle (Φ) between current and voltage, which is necessary to determine the power factor, and optimise efficiency and impedance (Z), which is a combination of resistance (Ω) and reactance (X) and determines the circuit's total opposition to AC [1, p. 20-21].

2.2 An RLC circuit consists of resistors (R), inductors (L), and capacitors (C). In an RLC circuit, the capacitor stores energy in an electric field and then discharges, causing current to flow into the inductor. The inductor's magnetic field will then also be discharged, returning current to the capacitor [2]. All components in a parallel RLC circuit share the same voltage; however, their respective currents differ in magnitude and phase. Additionally, the circuit's resistor dissipates real power, whereas the capacitor and inductor exchange reactive power. The values of these components determine the circuit's overall impedance, phase angle, resonance frequency, and overall power factor.

2.3 Power factor is a measure of how effectively electrical power is used within the circuit. It is the ratio of real power, measured in Watts (W), to apparent power, measured in Volt-amperes (VA). Power factor is measured between 0 and 1, with 1 representing unity, meaning that the real power consumed equals the apparent power and that voltage and current are in phase [3]. The primary technique used for power factor correction is adding capacitors in parallel. This works by injecting a leading current to counteract the lagging current caused by inductive loads such as motors and transformers [4].

2.4 Resonance occurs in a parallel RLC circuit when the inductive reactance (X_L) and capacitive reactance (X_C) are equal, which causes maximum impedance [5], resulting in a purely resistive circuit. Therefore, the total current which is supplied by the source is I meaning the total circuit current will be in phase with the supply voltage [6]. Additionally, at resonant frequency, the current through the inductor (I_L) and the capacitor (I_C) are greater than the supply current [7].

2.5 From the information provided in [8], maximum power transfer from the source to the load occurs when the total impedance of the load matches the complex conjugate of the impedance created at the source. When these conditions are met, the circuit source delivers the maximum possible energy to the load. Additionally, the maximum power transfer theorem is essential in ensuring that all electronic devices and circuits are efficient. As well as ensuring signal integrity in communications and audio loudspeaker applications, and, finally, in electrical machines such as transformers, where low efficiency can result in significant financial penalties and energy waste.

3 METHOD

3.1 Figure 1 below shows a model of a parallel RLC circuit before the power factor correction (PFC) capacitor was calculated and added to the system, derived from the student number 2413880.

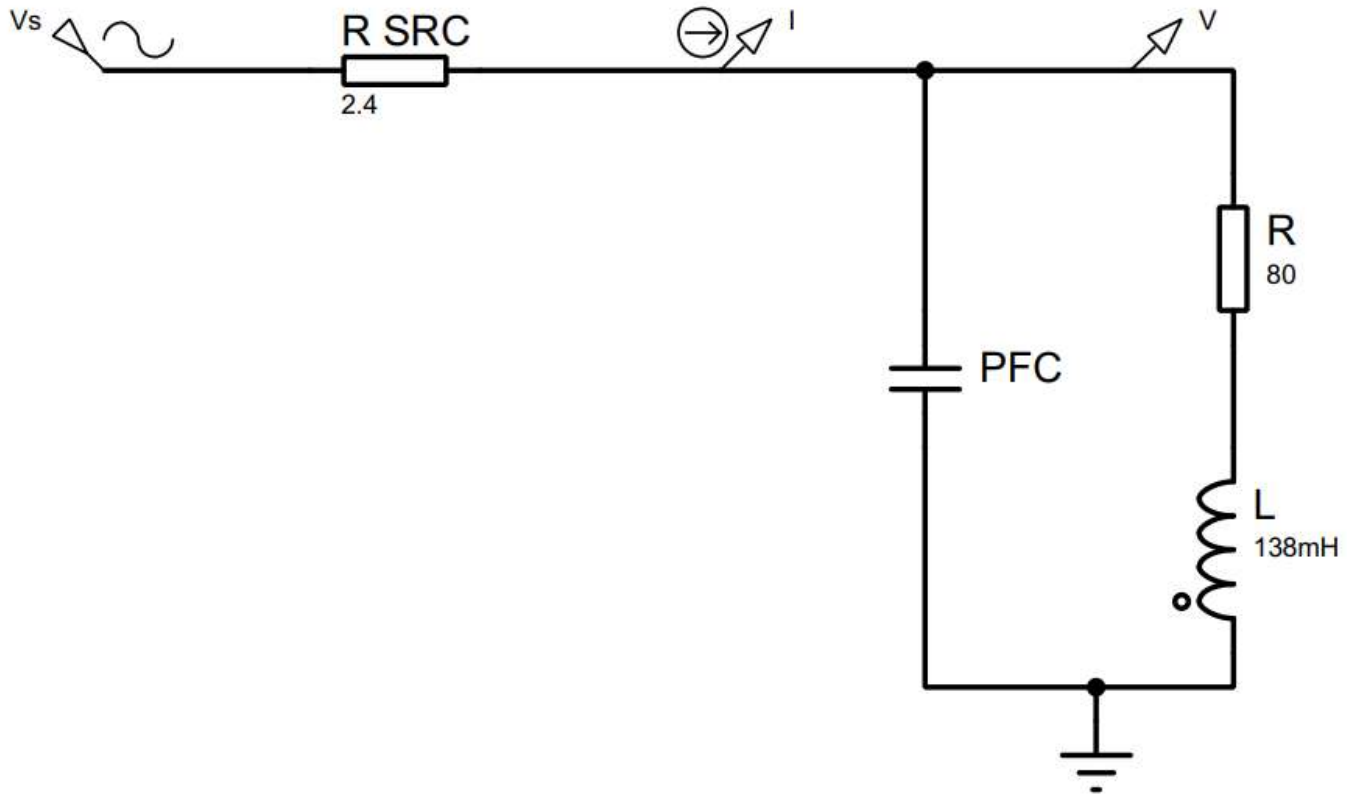

Figure 1: Parallel RLC Circuit

3.2 Figure 2 shows a graph of the RLC circuit current against the voltage before PFC was applied. To calculate the power factor of the circuit, the following phase angle formula was used:

$$Phase\ Angle\ \Phi = \frac{t}{T} \times 360$$

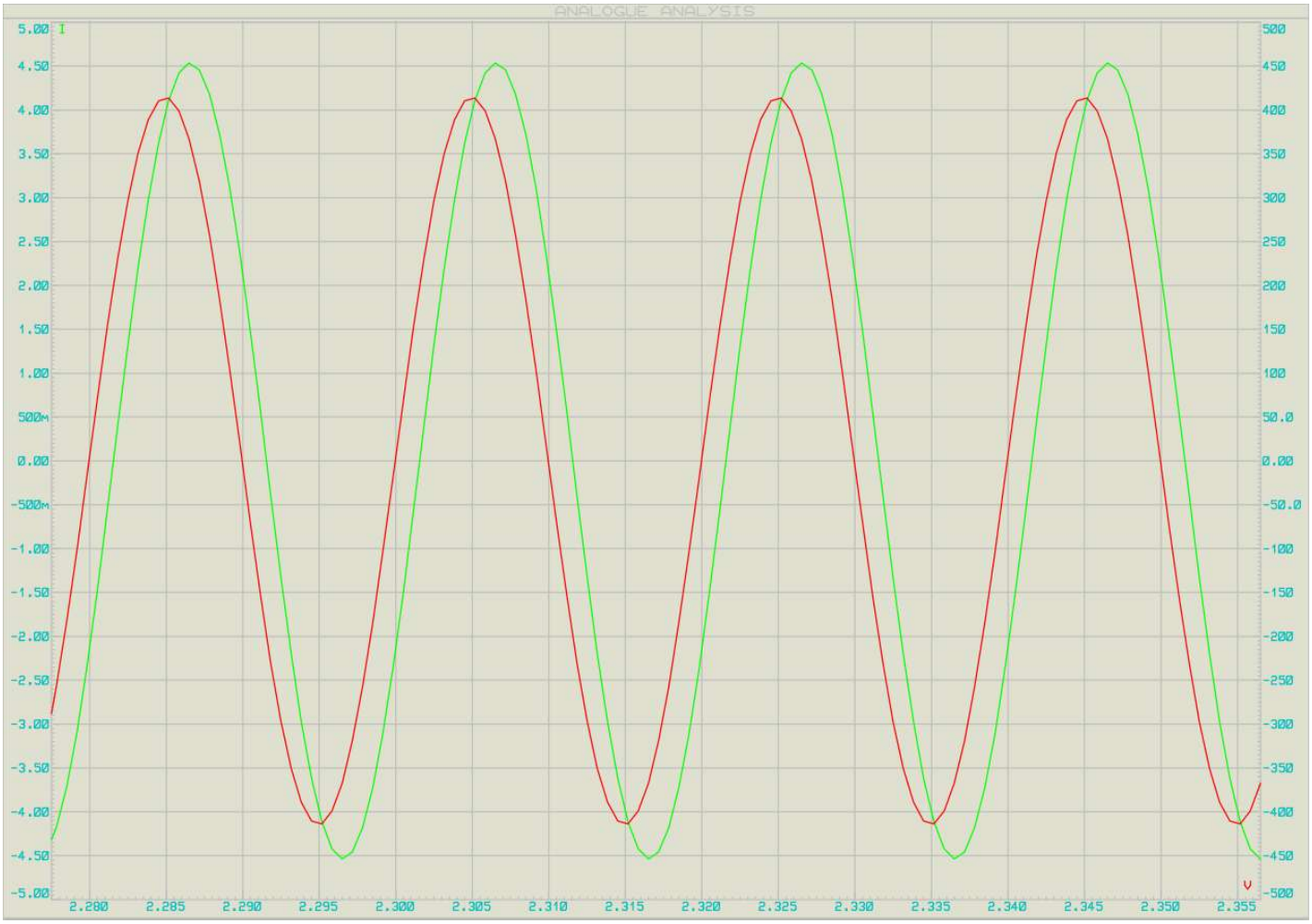

Figure 2: A Graph of Circuit Current and Load Voltage Over Time

From Figure 2, the time difference between the voltage and current was 1.8 ms. Using the phase angle formula, this yielded a phase angle of 32.4°.

$$\frac{1.8 \times 10\text{^} - 3}{0.02} \times 360 = 32.4°$$

3.3 To calculate the active, reactive and apparent power of the circuit, the following formulas were used:

$$Active\ Power\ (P) = VI\ \times Cos\ (\Phi)$$

$$Reactive\ Power\ (Q) = VI\ \times Sin\ (\Phi)$$

$$Apparent\ Power\ (S) = VI$$

By calculating the maximum and subsequent RMS values for the circuits voltage and current from the graph shown in Figure 2, the following results were obtained:

$$V_{max}\ = V_{pk} = 415V$$

$$V_{RMS} = \frac{415}{\sqrt{2}} = 293.45V$$

$$I_{max}\ = I_{pk} = 4.53A$$

$$I_{RMS}\ = \frac{4.53}{\sqrt{2}} = 3.2A$$

Therefore, by substituting the RMS values for voltage and current and the calculated phase angle of 32.4°. The results for active, reactive and apparent power are as follows:

$$Active\ Power\ (P) = 293.45\ \times 3.2\ \times Cos\ (32.4°) = 792.86\ W$$

$$Reactive\ Power\ (Q) = 293.45 \times 3.2\ \times Sin\ (32.4°) =\ 503.16\ VAR$$

$$Apparent\ Power\ (S) = 293.45\ \times 3.2 = 939.04\ VA$$

3.4 Then the power factor of the circuit was calculated using the formula:

$$Power\ Factor = Cos\ (\Phi)$$

Using the phase angle of 32.4°, this produced a power factor of 0.84.

$$\text{Cos}\ (32.4°) = 0.84$$

3.5 To calculate an appropriate size for the power factor correction capacitor for the circuit to achieve a perfect theoretical power factor of 1, the following formula was used:

$$C = \frac{1}{2\pi f X_C}$$

Firstly, the current flowing through the capacitor needed to be calculated by using the current through the circuit's inductive and resistive components and multiplying by the sine of the phase angle:

$$I_C = 3.2 \sin(32.4°) = 1.71\ A$$

From this, the capacitive reactance was calculated by using the formula:

$$X_C = \frac{V_{RMS}}{I_C}$$

Substituting in the RMS voltage and current through the capacitor yielded the result of:

$$X_C = \frac{293.45}{1.71} = 171.61\ \Omega$$

Subsequently, giving a capacitance of:

$$C = \frac{1}{2\pi \times 50 \times 171.61} = 18.5\ \mu F$$

3.6 Figure 3 below shows a graph of the power factor of the circuit against the value of the power factor correction capacitor. From the graph, it is evident that the power factor increases as the capacitor value approaches the calculated value of 18.5 μF for a unity power factor, then decreases as the system becomes capacitor dominant.

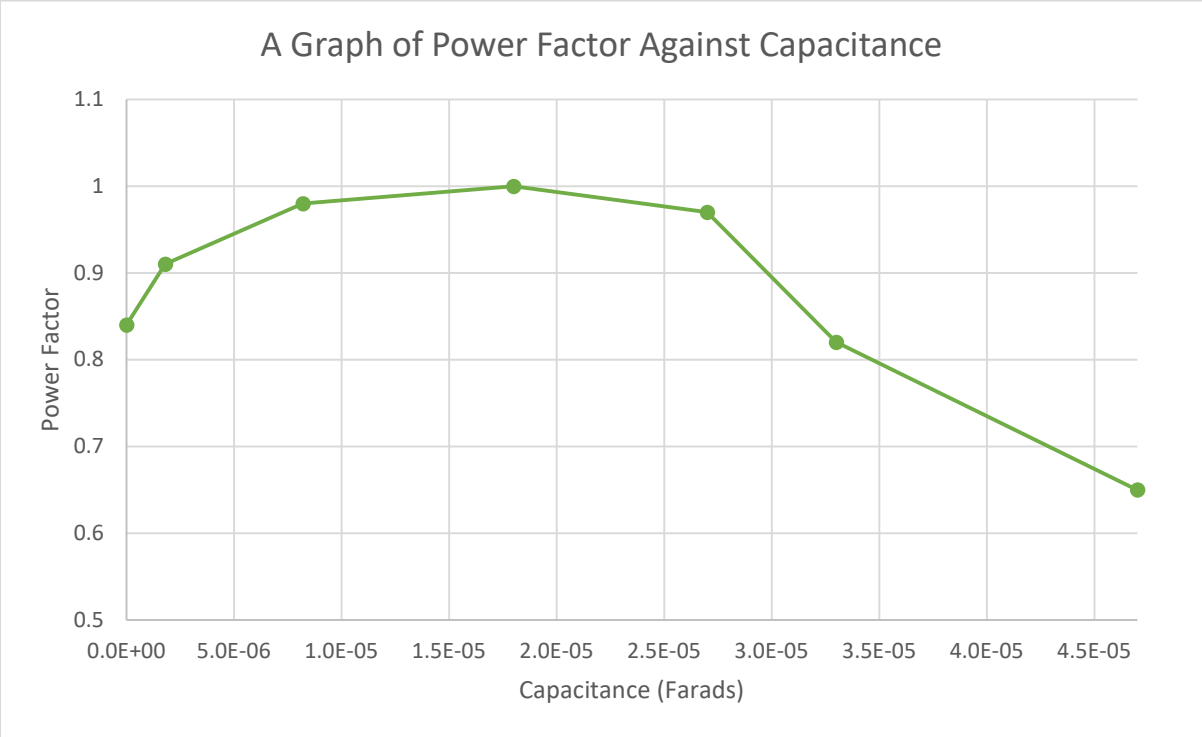

Figure 3: A Graph of Power Factor Against Capacitance

3.7 Figure 4 below shows a graph of the phase angle of the circuit against the value of the power factor correction capacitor. From the graph, the phase angle decreases as the capacitor value approaches the calculated value of 18.5 μF for a unity power factor, then increases as the system becomes capacitor dominant. The graphs of power factor versus capacitance in Figure 3 and phase angle versus capacitance in Figure 4 show contrasting trends, as expected.

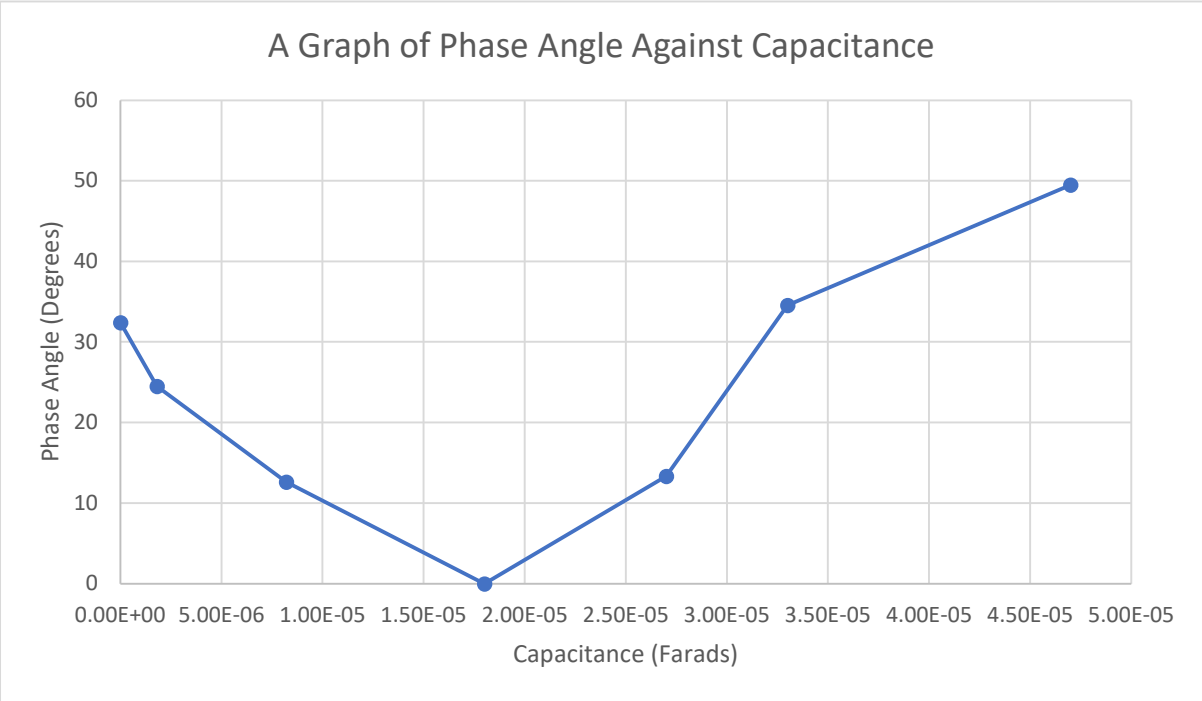

Figure 4: A Graph of Phase Angle Against Capacitance

3.8 According to the research shown in [9], as the power factor approaches 1, the capacitor's process of storing energy in its electric field and releasing it to the circuit in each half cycle remains unchanged. However, the net reactive power in the circuit cancels out because the rate of energy exchange between the capacitor and the inductor is equal. This means energy continually oscillates between the capacitor's electric field and the inductor's magnetic field, leading to a purely resistive circuit.

4 RESULTS

4.1 To calculate the frequency at which resonance in the circuit occurred, the following formula was used:

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Substituting in the circuit's inductance, resistance and capacitance yielded a frequency of 37.5 Hz as shown below:

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{(138mH \times 18.5\mu F)} - \frac{80^2}{(138mH)^2}} = 37.5\ Hz$$

Simulating frequencies of 37.5 Hz, 50 Hz, 12.5 Hz, and 62.5 Hz, the following formulas and method were used to calculate the capacitive current (I_C), the current through the inductive and resistive components (I_{LR}), the total current (I), the RMS voltage (V_{RMS}), phase angle between voltage and total current (Φ), and the phase angle between voltage and the current through the inductive and resistive components (Φ_{LR}).

$$X_C = \frac{1}{2\pi f C}$$

$$X_L = 2\pi f C$$

$$Z_{LR} = \sqrt{R^2 + {X_L}^2}$$

$$I_{LR} = \frac{V_{LR}}{Z_{LR}}$$

$$V_{RMS} = \frac{V_{pk}}{\sqrt{2}} \text{ (Values from Proteus simulation)}$$

$$I_C = \frac{V_C}{X_C}$$

$$\Phi_{LR} = tan^{-1}\left(\frac{X_L}{R}\right)$$

$$\Phi = tan^{-1}\left(\frac{I_{LR}sin\Phi_{LR} \sim I_C}{I_{LR}cos\Phi_{LR}}\right)$$

$$I = \sqrt{(I_{LR}cos\Phi_{LR})^2 + (I_{LR}sin\Phi_{LR} \sim I_C)^2}$$

Using the previous list of formulas and values taken from the Proteus simulation, a table of values was created, shown below.

Table I: Simulation Results at Different Frequencies

Frequency	**12.5 Hz**	**37.5 Hz**	**50 Hz**	**62.5 Hz**
Capacitive Reactance (X_C)	688.24 Ω	229.41 Ω	172.06 Ω	137.65 Ω
Inductive Reactance (X_L)	10.84 Ω	32.5 Ω	43.35 Ω	54.19 Ω
Impedance (Z_{LR})	80.73 Ω	86.35 Ω	90.99 Ω	96.63 Ω
Voltage (V_{RMS})	293.45 V	291.33 V	293.45 V	293.45 V
Capacitor Current (I_C)	0.43 A	1.27 A	1.71 A	2.13 A
Current Through RL (I_{LR})	3.63 A	3.37 A	3.23 A	3.04 A
Phase Angle in RL (Φ_{LR})	7.72°	22.11°	28.45°	34.11°
Circuit Phase Angle (Φ)	0.92°	0°	3.45° (Leading)	9.59° (Leading)
Total Current (I)	3.60 A	3.07 A	2.84 A	2.55 A
Power Factor (PF)	0.999	1	0.998	0.986

At resonance, the circuit's efficiency improved with no reactive losses, yielding the following results for active, reactive, and apparent power.

$$Active\ Power\ (P) = 291.33\ \times 3.07\ \times Cos\ (0°) = 894.38\ W$$

$$Reactive\ Power\ (Q) = 291.33 \times 3.07\ \times Sin\ (0°) =\ 0\ VAR$$

$$Apparent\ Power\ (S) = 291.33\ \times 3.07 = 894.38\ VA$$

Therefore, substituting the values at each frequency, the first phasor diagram for the RLC circuit at resonance is shown in Figure 5 below. At this frequency, I_C was calculated at 1.27 A, I_{LR} was 3.37 A, I was 3.07 A, V_{RMS} was 291.33V, Φ_{LR} was 22.11°, and finally Φ was 0°. Resulting in a unity power factor.

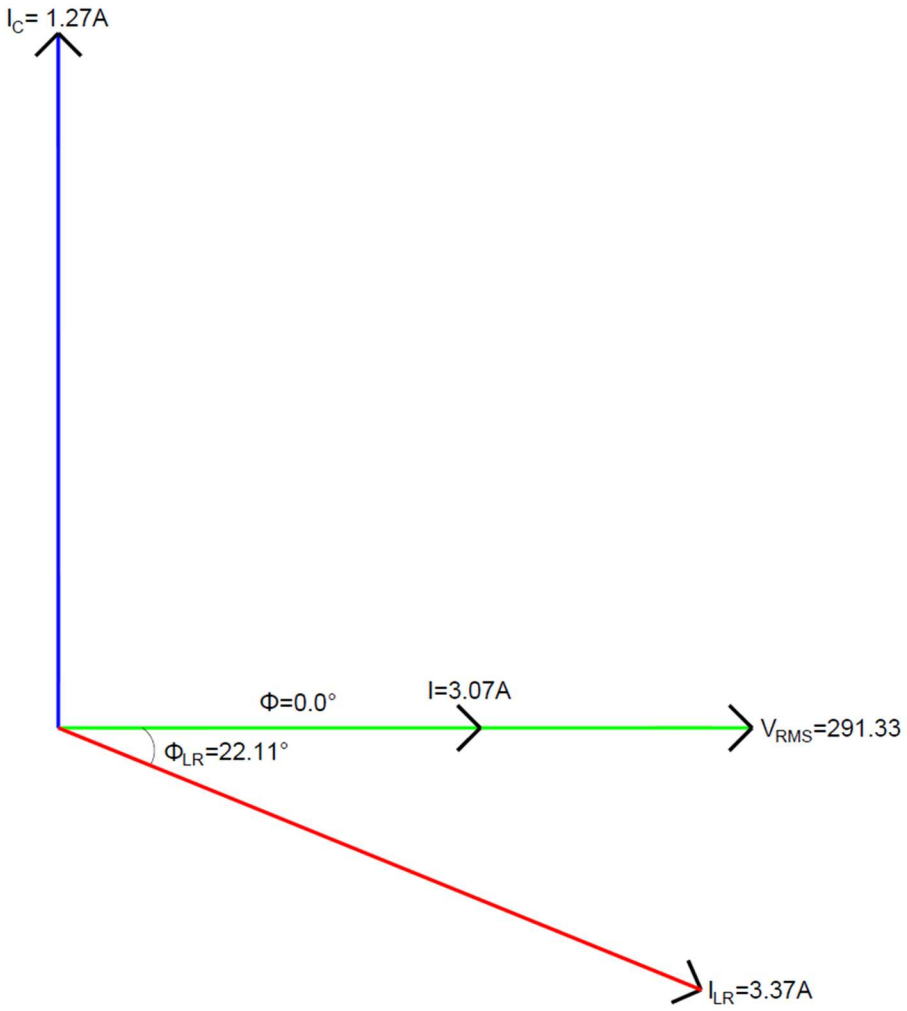

Figure 5: Phasor Diagram at Resonance

4.2 The next phasor diagram was created using a frequency of 50 Hz, which is typical of a UK mains source. At this frequency, I_C was calculated at 1.71 A, I_{LR} was 3.23 A, I was 2.84 A, V_{RMS} was 293.45V, Φ_{LR} was 28.45°, and finally Φ was 3.45°. Resulting in a power factor of 0.998.

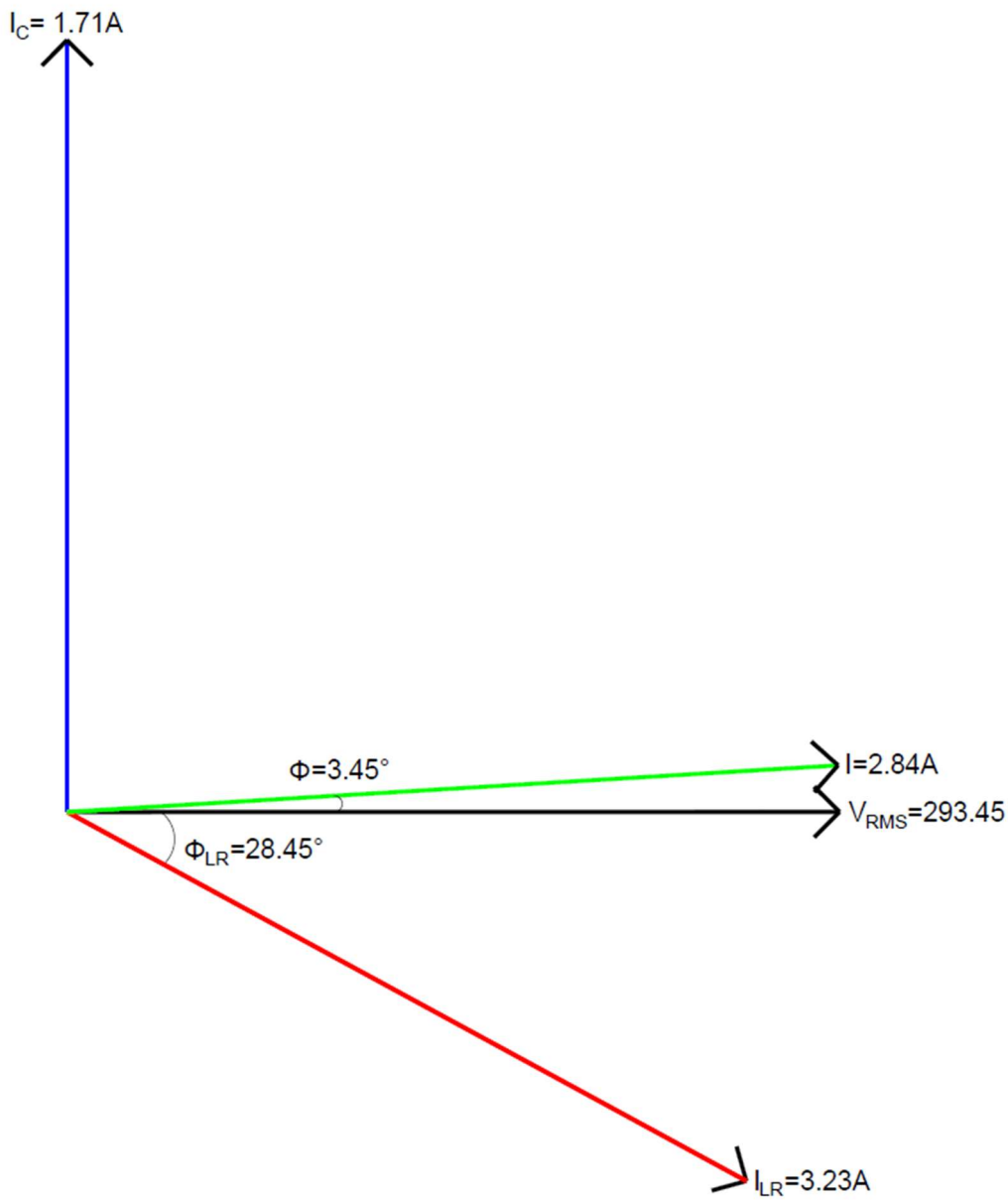

Figure 6: 50 Hz Phasor Diagram

4.3 The following phasor diagram was created using a frequency of 12.5 Hz, which is 25 Hz less than the resonant frequency. At this frequency, I_C was calculated at 0.43 A, I_{LR} was 3.63 A, I was 3.60 A, V_{RMS} was 293.45V, Φ_{LR} was 7.72°, and finally Φ was 0.92°. Resulting in a power factor of 0.999.

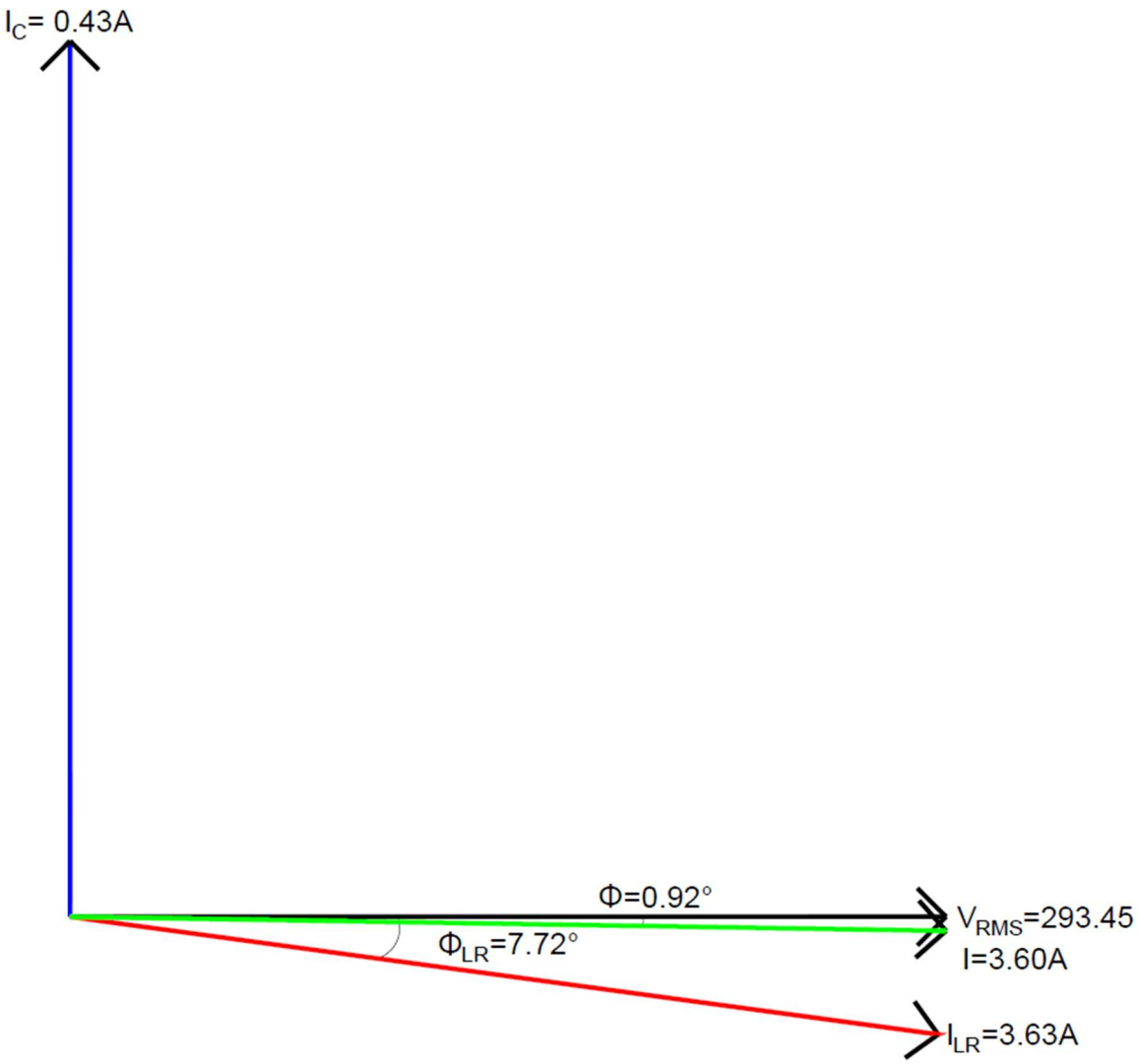

Figure 7: 12.5 Hz Phasor Diagram

4.4 The following phasor diagram was created using a frequency of 62.5 Hz, which is 25 Hz greater than the resonant frequency. At this frequency, I_C was calculated at 2.13 A, I_{LR} was 3.04 A, I was 2.55 A, V_{RMS} was 293.45V, Φ_{LR} was 34.11°, and finally Φ was 9.59°. Resulting in a power factor of 0.986.

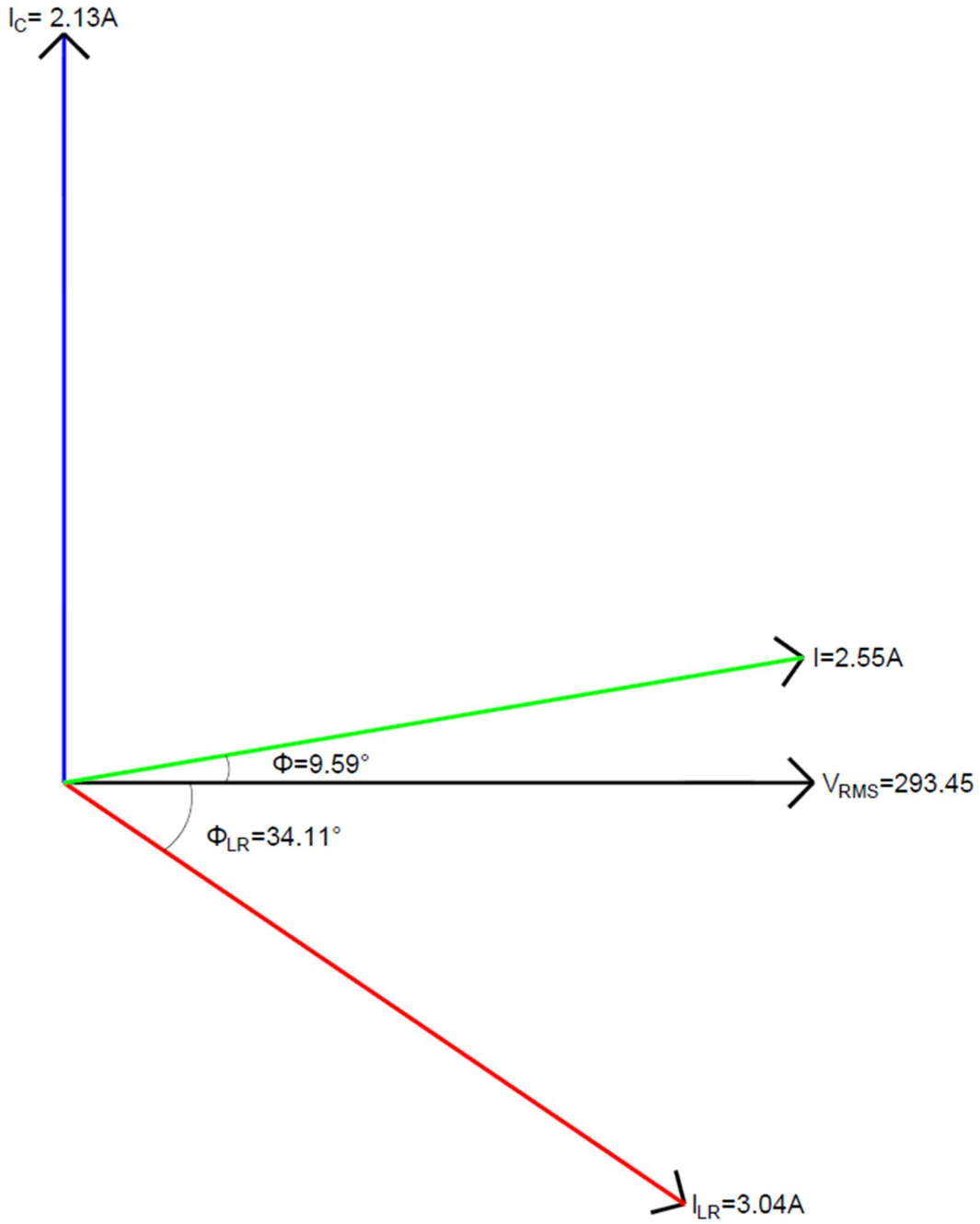

Figure 8: 62.5 Hz Phasor Diagram

5 DISCUSSION

5.1 Firstly, the current was greatest at 12.5 Hz, which was expected as X_L was 10.8 Ω at this frequency. Additionally, X_C was 685 Ω, indicating that current mainly flowed through the inductive branch of the circuit, resulting in a minimal leading phase angle of 0.92° and a power factor of 0.999 because the reactive effects were minimal. This shows the relationship between the frequency and reactance in the circuit. As frequency increases, inductive reactance increases proportionally, whereas capacitive reactance decreases proportionally. At low frequencies, the circuit behaves almost purely resistive. The total current was at its lowest at the highest frequency simulated at 62.5 Hz. At higher frequencies, the capacitor becomes dominant, which introduces a leading phase angle. This highlights the importance of selecting a frequency close to the circuit's resonance, as operating the system too far from this range results in suboptimal performance and efficiency.

Secondly, at the resonant frequency (f_r) the phase angle (Φ) was 0°, resulting in a unity power factor. This resulted in a purely resistive circuit where Z = R, and the current and voltage are perfectly in phase. The total current at resonance was 3.07 A, which is less than at 12.5 Hz. This is because in a parallel RLC circuit configuration, X_L increases with frequency, in turn reducing the branch current.

Moreover, when the circuit was simulated with a frequency of 50 Hz, the total current was reduced to 2.84 A, since X_L is increased with the higher frequency. At this frequency, the capacitor has cancelled all inductive effects in the circuit, resulting in a capacitive phase angle of 3.45° and a PF of 0.998. Furthermore, similar results were obtained at 62.5 Hz, where the capacitive effects of the circuit exceed the inductive effects, resulting in a leading phase angle of 9.59° and a PF of 0.986. Additionally, the total current was further reduced to 2.55 A due to an increase in X_L. Overall, the observed results match theoretical predictions and expectations for a parallel RLC circuit.

5.2 The circuit achieved resonance when the capacitive reactance and inductive reactance were equal and opposite, cancelling each other out [5], which was calculated to occur at 37.5 Hz. This condition is critical for eliminating reactive power losses and ensuring system efficiency. In parallel RLC circuits, the resonant frequency does not always result in maximum current flow, as inductive reactance increases with frequency, which is a key difference between parallel and series RLC circuits.

5.3 Before a power factor correction capacitor was added, the circuit achieved a power factor of 0.84, which is less than the minimum of 0.95. This meant the system needed to be provided with 939 Volt-Amperes of apparent power to deliver 793 Watts of real power, indicating that 503 VARs are lost due to current oscillations in the circuit [10]. Upon calculating a suitable capacitor value to bring the circuit's power factor to the required >0.95, a value of 0.998 at 50 Hz was achieved, which is approximately a unity power factor with negligible reactive losses. In a practical system, this means better compliance with utility company requirements and overall lower running costs.

5.4 Maximum power transfer is when the load impedance matches the impedance of the source [8]. The results show that the highest current occurred at 12.5 Hz because the R-L branch of the circuit had its lowest impedance at that frequency. This demonstrates that a unity power factor does not necessarily result in maximum current in parallel RLC circuit applications and depends on the impedance provided by the circuit branches.

5.5 Improving the power factor of a system and operating it with a supply frequency close to its respective resonance greatly improves the overall efficiency of the circuit. In this RLC circuit, the power factor is approximately unity at 50 Hz, meaning most of the power supplied is converted into useful work rather than wasted in transmission. This is a fundamental principle in AC circuits: a poor power factor results in an increased apparent power requirement, leading to lower efficiency, higher costs, and even penalties from the electric company or government.

5.6 Electricity in the UK and the developed world is generated and transmitted using a three-phase system. The reasons for three-phase over single-phase systems are the economic benefits of delivering more power with fewer conductors, as well as a much more constant power flow to the load [11]. In industrial settings, this reduces mechanical stress and vibration in equipment.

Moreover, as shown in [12], although two-phase systems can provide a rotating magnetic field and deliver constant torque and power to the load, three-phase systems carry more power and eliminate the need for an additional neutral wire, as required in two-phase systems. Subsequently, if loads are unbalanced, a four-wire configuration is used; however, the system remains more effective and practical than a two-phase system.

Additionally, systems with more than three phases are not used, as more phases require extra material costs due to the need for additional conductors, larger transformers, and more complex switchgear and distribution systems, such as larger transmission line towers [12].

6 CONCLUSION

6.1 In this RLC circuit simulation, it is evident to see the effects that the source frequency and capacitor size had on the circuit from the values of the total and branch currents to the phase angle and power factor. Firstly, at lower measured frequencies, such as 12.5 Hz, the impedance from the inductive elements is minimal, allowing a large current to flow and a near-unity power factor. At higher frequencies, the results from the simulations and subsequent calculations showed an increase in inductive reactance (X_L) and a decrease in capacitive reactance (X_C), altering the current and phase angle values. At the calculated resonant frequency of 37.5 Hz, these reactances were equal and opposite, cancelling each other. When this occurred, the power factor was 1, and no reactive power was exchanged within the circuit. However, resonance does not result in maximum current in this configuration, as it depends on the branch impedances.

6.2 In AC circuits, a high PF is critical to ensure minimal energy is wasted as reactive power, and the circuit can deliver power to the load effectively. Before any PFC was applied, the system had a power factor of 0.84, requiring a large amount of apparent power to deliver the real power to the load. As the PF approached 1, this difference was reduced, improving efficiency. In practice, a high PF indicates better compliance with utility grid requirements and industry standards, thereby reducing overall running costs.

7 REFERENCES

[1] K. R. Ekanem, "View of A Comprehensive Study of the Principles and Trends in AC Circuits: Essential Component in Electro-mechanical Systems and Industries," *Reapress.com*, 2025. https://www.imes.reapress.com/journal/article/view/22/17 (accessed Dec. 19, 2025).

[2] M. Elton, "Analyzation of the Resistor-Inductor-Capacitor Circuit," *Undergraduate Journal of Mathematical Modeling: One + Two*, vol. 7, no. 2, Mar. 2017, doi: https://doi.org/10.5038/2326-3652.7.2.4876.

[3] K. C. Obute , O. Oduyemi, M. C. Okonkwo, and C. O. Ikaraoha, "Imperatives of Power Factor on the Performance of Electric Power System," *IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE)* , vol. 15, no. 4, pp. 01-20, Aug. 2020, Available: www.iosrjournals.org

[4] J. Ware, "POWER FACTOR CORRECTION 22," 2006. Available: https://electrical.theiet.org/media/1687/power-factor-correction-pfc.pdf

[5] T. Zhan, "Recommendation Of RLC Circuit and Its Uses in Electronic Devices," *Highlights in Science Engineering and Technology*, vol. 81, pp. 281–285, Jan. 2024, doi: https://doi.org/10.54097/0yyssj09.

[6] W. Storr, "Parallel Resonance and Parallel RLC Resonant Circuit," *Basic Electronics Tutorials*, Jun. 25, 2013. https://www.electronics-tutorials.ws/accircuits/parallel-resonance.html (accessed Dec. 23, 2025).

[7] A. Ahmad, "Understanding Resonance In Parallel RLC Circuits," *Eepower.com*, Jul. 06, 2023. https://eepower.com/technical-articles/understanding-resonance-in-parallel-rlc-circuits/# (accessed Dec. 23, 2025).

[8] Electricalampere, "Maximum Power Transfer Theorem: Formula & Statement," *Electrical Ampere*, Aug. 22, 2025. https://electricalampere.com/maximum-power-transfer-theorem/ (accessed Dec. 24, 2025).

[9] K. Hadiningrum, R. F. Muldiani, and D. Pratama, "The Effect of Capacitance on the Power Factor Value of Parallel RLC Circuits," *Current Journal: International Journal Applied Technology Research*, vol. 1, no. 2, pp. 120–127, Oct. 2020, doi: https://doi.org/10.35313/ijatr.v1i2.27.

[10] S. Singh, "Reactive power management in utility grids with renewable energy," *Elsevier eBooks*, pp. 115–133, Oct. 2024, doi: https://doi.org/10.1016/b978-0-443-19021-6.00006-5.

[11] "Introduction to Electric Power Systems | Electrical Engineering and Computer Science | MIT OpenCourseWare," *MIT OpenCourseWare*, 2025. https://ocw.mit.edu/courses/6-061-introduction-to-electric-power-systems-spring-2011/ (accessed Dec. 30, 2025).

[12] Electrical Technology, "Why 3-Phase Power? Why Not 6, 12 or More for Transmission?," *ELECTRICAL TECHNOLOGY*, Nov. 18, 2021. https://www.electricaltechnology.org/2021/11/why-3-phase-power-why-not-6-12-or-more.html (accessed Dec. 30, 2025).