

EXAMINATION SOLUTIONS : SESSION 2018 /19

Programme:	BEng Motorsport Engineering and Design	Year:	2
Module/Unit :	Vehicle Dynamics		
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Model Answers

Question 1

- a) For an off road race buggy calculate the wheel slip torque if all the vehicle mass is on the rear wheels.
 Vehicle mass = 1500kg
 Limiting friction $\mu = 1.2$
 Rolling radius = 0.32

(2 marks)

Max tractive drive force = $m \cdot g \cdot \mu = 1500\text{kg} \times 9.81 \times 1.2 = 17658 \text{ N}$ for the axle
 For each wheel max drive force – $17658 / 2 = 8829 \text{ N}$

Torque = Force x Radius = $8829 \times 0.32 = 2825 \text{ Nm}$

Cone Clutch

- Contact surfaces are in form of cone
- Normal force on contact surface is $\int p \cdot dA$
- More torque is transmitted

α = semi-cone angle, w = width of cone

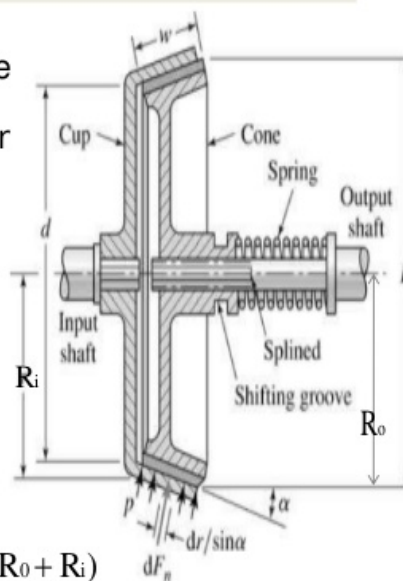
$$\sin \alpha = \frac{R_o - R_i}{w}$$

Uniform pressure theory

$$T = \frac{1}{\sin \alpha} \mu F \frac{2}{3} \frac{(R_o^3 - R_i^3)}{(R_o^2 - R_i^2)}$$

Uniform wear theory

$$T = \frac{1}{\sin \alpha} \mu F \frac{1}{2} (R_o + R_i)$$



A rear wheel drive off road race vehicle has experienced transmission and engine damage due to shock torsional forces entering the drive line when the vehicle lands after a jump. To stop this damage the

engineering director wishes to incorporate a 'mechanical fuse' in the form of a cone clutch incorporated into the driveshaft. The clutch should slip once a drive force of 3000Nm has been exceeded

The clutch has the following dimensions:

Outer diameter 100mm

Inner diameter 95mm

Cone angle $\alpha = 7$ degs

Steel on steel coefficient of friction = 0.2

- b) Calculate the clamp force required to cause clutch slip at 3000Nm.
(6 marks)

$$T = \frac{1}{\sin \alpha} \cdot \frac{2}{3} \mu \cdot F \cdot \frac{(r_0^3 - r_1^3)}{(r_0^2 - r_1^2)}$$

Therefore

$$F = \frac{\sin \alpha}{\mu} \cdot \frac{3}{2} \cdot T \cdot \frac{(r_0^2 - r_1^2)}{(r_0^3 - r_1^3)}$$

$$F = \frac{0.121}{0.2} \cdot \frac{3}{2} \cdot 3000 \cdot \frac{(0.05^2 - 0.0475^2)}{(0.05^3 - 0.0475^3)}$$

$$F = 37257 \text{ N}$$

- c) Discuss any possible errors with the provided clutch assumptions. What practical steps could be taken to ensure the correct clamp load is achieved to cause slip at 3000 Nm drive torque.
(3 marks)

Student should discuss :

Mu value will vary, esp dependant on lubrication.

There is a variation between static and dynamic friction.

Cones may not exactly match- taper angle errors and surface finish issues.

If slip torque is too low – max drive may not be possible and cone clutch will wear rapidly, then clamp force may well drop lower!

If slip torque is too high component damage may result.

Best to measure slip torque and adjust clamp load to suit.

A vehicle is equipped with a steel flywheel of plain section.

Storage capacity = 1Mjoule

Max rpm = 10,000

Max outer diameter = 0.5m

d) Find the mass of the flywheel

(4marks)

$$\text{Kinetic Energy} = KE = \frac{1}{2} \cdot I \cdot \omega^2$$

$$\text{Rpm} = 10,000$$

$$= 10000/60 \times 2\pi = 1047 \text{ rads/sec}$$

$$I = 2KE / \omega^2 = 2 \times 1,000,000 / 1047^2 = 1.8 \text{ kg.m}^2$$

$$\text{Inertia } I = m \cdot k^2$$

$$\text{Radius of gyration } k = 0.707 \times \text{radius (for a plain disc)}$$

$$K = 0.707 \times 0.25 = 0.1768 \text{ m}$$

$$\text{Mass} = I / k^2 = 1.8 / (0.1768^2) = 57 \text{ kg}$$

e) Calculate the thickness of the flywheel..

(3 marks)

$$\text{Density of steel} = 7850 \text{ kg / m}^3$$

$$\text{Radius} = 0.25 \text{ m}$$

$$\text{Area} = 2 \cdot \pi \cdot r^2 = 2 \pi \cdot 0.25^2 = 0.196 \text{ m}^2$$

$$\text{Volume} = \text{thickness} \times \text{area}$$

$$\text{thickness} = \text{volume} / \text{area} = 0.00726 / 0.16 = 0.037 \text{ m} = 37 \text{ mm}$$

f) Discuss flywheel shapes other than a plain disc that could be used to reduce mass and/or package size.

(2 marks)

The student should discuss:

Inertia is proportional to radius of gyration squared, therefore the mass should be concentrated and the outer radius. Example a spoked wheel. Sketches would help.

Kinetic Energy is proportional to velocity squared. Therefore if the speed was increased the radius and or mass could be reduced. Speed is limited by the strength of the material due to centripetal acceleration causing the flywheel to 'burst' if material stresses are exceeded.

Question No	Question	Marks
2a	a) What is the total force required to accelerate the vehicle? 40 kph = 11.11 m/s Accel = 1 m/s² Time to accel = 11.1 secs GVW = 5500kgs + (100kgs x 20 people) = 7500 kg Rolling resistance = 7500 x 0.005 x 9.81 = 367 N Accel resistance = F = M x a = 7500kgs x 1 = 7500 N Aero resistance = 0.5 x cd x area x density of air x velocity squared. = 0.5 x 0.7 x 6 x 1.2 x 5.55 x 5.55 = 77.616 N Total resistive force = 77.616 + 7500 + 367 = 7944 N	4
2b	What is the distance covered during the acceleration phase? Average speed = 5.55 m/s Distance = av speed x time = 5.55 m/s x 11.1 secs = 61.6m	1
2c	How much energy is required to accelerate the vehicle? Energy = force x distance = 7944 x 61.6 = 489400 Joules	1
2d	How much energy is required during the constant speed phase? Rolling resistance = 367 N Aero resistance = 0.5 x 0.7 x 6 x 1.2 x 11.1 x 11.1 = 310.5 N Total resistance 310.5 + 367 = 677 N Distance covered = 600m – (2 x 61.6m) = 476.8m Energy = 677 N x 476.8m = 322925 joules	2
2e	How much energy can be recovered during the deceleration phase? Recoverable energy = 0.5 x mass x velocity squared = 0.5 x 7500kg x 11.1 x 11.1 = 462,000 joules Total resistance = rolling resistance + aero resistance = 367 + 77.62 = 445N Energy due to resistance = 445N x 61.6 m = 27412 joules Available energy = PE – resistance losses = 462,000 x 27412 = 434,600 joules But at 80% efficiency 434600 x 0.8 = 347000 joules	4

2f	a) What is the required flywheel energy storage capacity? Energy required = accel energy required + cruise energy required – recovered energy. Accel energy $\times 1/0.8 = 489400 \times 1.25 = 611000$ Joules Cruise energy $\times 1/0.8 = 322000 \times 1.25 = 404000$ Recovered energy = 347000 joules Flywheel capacity required $611000 + 404000 - 34700 = 667000$ joules. But also require 20% spare $667000 \times 1.2 = 800\,000$ joules. It must also be considered that there must be sufficient stored energy to enable the tram to reach the point at the beginning of deceleration. = accel energy + constant speed energy = $611,000 + 404,000 = 1,015,000 = 1.015$ Mj	3
2g	a) A similar vehicle is equipped with a flywheel of energy capacity 1.2Mj When the vehicle is at the station the flywheel is reaccelerated from 1000 rpm to 5000rpm in preparation for the next section of the journey. What torque is required from the electric drive to accelerate the flywheel in 60 sec $KE = \frac{1}{2} I \omega^2$ $I = 2 \cdot KE / \omega^2$ Initial $\omega = 1000 \times 2\pi / 60 = 104.7$ rads/sec Final $\omega = 5000 \times 2\pi / 60 = 523.6$ rads/sec Difference = $523.6 - 104.7 = 418.9$ rads/sec $I = 2 \times 1200000 / 523.6^2 = 8.754 \text{ kg.m}^2$ T = Inertia $\times$ acceleration Accerleration required = $418.9 / 60 \text{ secs} = 6.982 \text{ rads} / \text{s}^2$ $T = 8.754 \times 6.982 = 61 \text{ Nm}$	5

Question No	Question	Marks
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3a	Find the vertical force acting on the rear axle. $\tan^{-1} 15/100 = \text{angle } 8.53 \text{ degs}$ Force down the slope = mass x gravity x sin (angle) $= 1800 \text{ kg} \times 9.81 \times 1 = 2619 \text{ N}$ Vertical force on the rear wheels = mass x gravity x cos (angle) = $900 \text{ kg} \times 9.81 \times 0.9889 = 8731 \text{ N}$ Weight transfer = force down slope x C of G height / Wheel base $= 2619 \text{ N} \times 0.7\text{m} / 2.8\text{m} = 654.8 \text{ N}$ Total rear axle vertical load = $8731 \text{ N} + 654.8 \text{ N} = 9385.8 \text{ N}$	4
3 b	Therefore wheel load = 4693 N What is the resulting vehicle acceleration ? (open differential) Grip on high grip side = $4693 \text{ N} \times 1 = 4693 \text{ N}$ Grip on low grip side = $4693 \text{ N} \times 0.1 = 469.3 \text{ N}$ As the torque in the left wheel is the same as the torque in the right wheel Maximum traction force = $(4693 \times 2) = 9386 \text{ N}$ Tractive force – force down slope = acceleration force $9386 \text{ N} - 2619 \text{ N} = -1680 \text{ N}$ The resulting acceleration = $-1680 \text{ N} / 1800 \text{ kg} = -0.9333 \text{ m/s}^2$ (Sliding backwards) The vehicle has an optional torque biasing (torsen) differential with a torque biasing ration of 3 to 1.	4
3c	What is the resulting vehicle acceleration ? Grip on high grip side = $4693 \text{ N} \times 1 = 4693 \text{ N}$ Grip on low grip side = $4693 \text{ N} \times 0.1 = 469.3 \text{ N}$ As the torque in the left wheel is three times the torque in the right wheel Drive force on the left side = $469.3 \text{ N} \times 3 = 1407.9 \text{ N}$ Drive force on the right side = 469.3 N Total drive force = $1407.9 \text{ N} + 469.3 \text{ N} = 1877.2 \text{ N}$ Tractive force – force down slope = acceleration force $1877.2 \text{ N} - 2619 \text{ N} = -741.8 \text{ N}$ The resulting acceleration = $-741.8 \text{ N} / 1800 \text{ kg} = -0.412 \text{ m/s}^2$ (just moving)	3

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3d	As an alternative to the torsen differential the vehicle fitted with the open differential can be fitted with a brake based traction control system. Where the wheel on the low friction side is applied allowing drive force to reach the high friction. Therefore all of the available rear axle grip can be utilised. What is the resulting vehicle acceleration? Grip on high grip side = $4693\text{ N} \times 1 = 4693\text{ N}$ Grip on low grip side = $4693\text{ N} \times 0.1 = 469.5\text{ N}$ As the torque in the left wheel an utilise all the available grip. Drive force on the left side = 4693 N Drive force on the right side = 469.3 N Total drive force = $4693\text{ N} + 469.3\text{ N} = 5162\text{ N}$ Tractive force – force down slope = acceleration force $5162\text{ N} - 2619\text{ N} = 2543\text{ N}$ The resulting acceleration = $-3543\text{ N} / 1800\text{ kg} = 1.413\text{ m/s}^2$ The vehicle has asymmetric drive forces. The extra forces on the left hand will have two additional effects. - There is a moment created ($4693\text{ N} - 469.3\text{ N}$) $\times (1.5\text{m}/2) = 3168\text{ N}$ - The car will try to steer if there is ant kingpin offset - Both will try to steer the vehicle onto the right side (low grip) The driver will have to steer towards the high grip side (left side) to ensure that the vehicle proceeds in a straight line up the slope.	5
3e	a) If the vehicle is moving at 10 kph what is the rate of energy entering the brake disc? Grip on high grip side = 4693 Effective force transfer from low grip side to high grip side = $4693 - 469.3 = 4224$ $10\text{ kph} \times 1000 / (60 \times 60) = 2.78\text{ m/s}$ Power = force $\times$ velocity = $4224\text{ N} \times 2.78\text{ m/s}$ = 11.74 kw	4

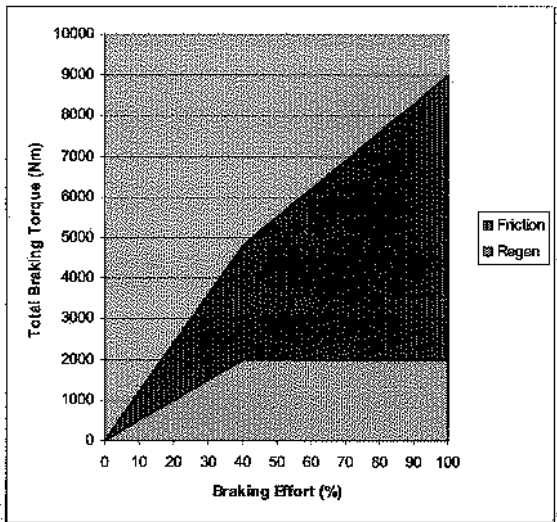
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Question No	Question	Marks
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4a

Parallel

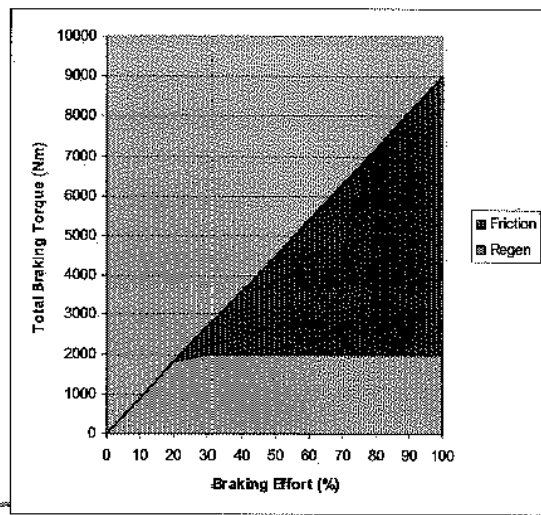
- ☐ Cheap
- ☐ Poor pedal feel
- ☐ Poor brake feel
- ☐ Lower energy recovery



3

Series

- ☐ Expensive
- ☐ Consistent pedal and brake feel
- ☐ Maximum energy recovery

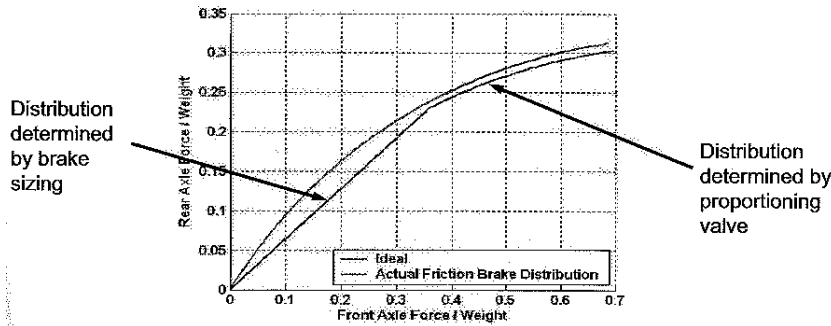


3

4b

The student should evaluate the application
 Rear wheel drive and regen
 Vehicle brakes at maximum deceleration
 Therefore capturing power at low speeds not required
 Assuming max deceleration both regen and friction brakes required
 If energy store full, rear braking reduced leading to a stable vehicle
 Could employ active brake balance to maintain balance for max deceleration if store full.
 Series braking much simpler and just as effective

2

4c	a) A small front engined front wheel drive petrol internal combustion small car has been hybridised with an additional electrically drive rear axle. Analyse the limitations to recovering energy from the rear axle. Use diagrams and numbers to illustrate your answer. Typical values: Mass 1200kg Weight distribution 63% front / 37% rear Wheel base 2.5m Height of centre of gravity 0.6m above the ground Rolling radius 0.26m The student should discuss 1. longitudinal weight transfer 2. Rear braking torque maximum limited by wheel lock, rear instability.  3. Rear brake or regen torque constant for decel 4. But power = Torque x omega therefore power that can be removed reduces as speed reduces. Therefore need to reduce regen power at lower speeds 5. In reality on the road most people brake at less that 0.3g therefore braking takes longer and there is more opportunity to use regen and avoid friction brakes. 6. Regen should be stopped before the vehicle is stationary to avoid driving the vehicle backwards.	6												
4d	During braking in a straight line, the vehicle decelerates from 144 kph to standstill. Determine the maximum energy recoverable (include effects of inertia from the engine, driveline and the wheels). Assume that the vehicle remains in top gear throughout the deceleration. <table><tr><td>Mass</td><td>600kg</td><td>Wheel rolling radius</td><td>0.264m</td></tr><tr><td>Inertia (engine and driveline)</td><td>0.15 kgm²</td><td>Final drive ratio</td><td>3.4 : 1</td></tr><tr><td>Inertia (per wheel assembly)</td><td>0.8 kgm²</td><td>Top gear ration</td><td>1 : 1</td></tr></table> assume total equivalent mass = actual mass + $4 \left(\frac{I_w}{r_w^2} \right) + I_e \cdot \left(\frac{G^2}{r_w^2} \right)$ $= 600 + 4 \left(\frac{0.8}{0.264^2} \right) + 0.15_e \cdot \left(\frac{3.4^2}{0.264^2} \right)$	Mass	600kg	Wheel rolling radius	0.264m	Inertia (engine and driveline)	0.15 kgm ²	Final drive ratio	3.4 : 1	Inertia (per wheel assembly)	0.8 kgm ²	Top gear ration	1 : 1	6
Mass	600kg	Wheel rolling radius	0.264m											
Inertia (engine and driveline)	0.15 kgm ²	Final drive ratio	3.4 : 1											
Inertia (per wheel assembly)	0.8 kgm ²	Top gear ration	1 : 1											

	$= 600 + 4.(11.478) + 0.15.(165.86)$ $= 600 + 45.914 + 24.879$ Total equivalent mass = 670.8 kg Velocity = 144 kph $= 144000/(60 \times 60) = 40 \text{ m/s}$ Kinetic energy of the vehicle = $\frac{1}{2} m.v^2$ $= \frac{1}{2} \times 670.8 \times 40^2$ $= 536.6 \text{ joules}$	
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