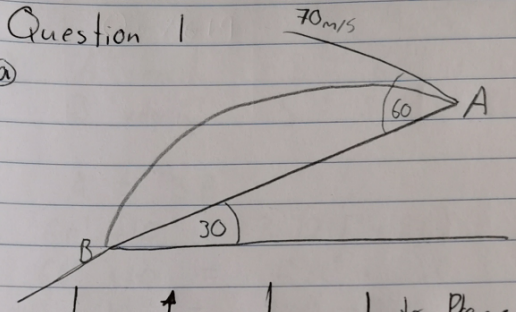
Dynamics Control and Instrumentation

Code : M4 X01405

Question 1

(a)



	↑	⊥ to Plane	→
S		0	
u	$70\sin(30)$	$70\sin(60)$	$70\cos(30)$
V			$70\cos(30)$
a	-9.81	$-9.81\cos(30)$	0
t			

⊥ to Plane

$S = ut + \frac{1}{2}at^2$

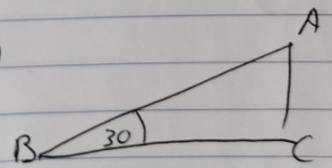
$0 = t(u + \frac{1}{2}at^2)$ either $t=0$ or $0 = u + \frac{1}{2}at$

$0 = u + \frac{1}{2}at$

$0 = 70\sin(60) + \frac{1}{2}(-9.81\cos 30)t$

$t = \frac{70\sin 60}{\frac{1}{2}(9.81\cos 30)} = 14.271\ s$

(b)



$B \rightarrow A$

$\cos 30 = \frac{BC}{BA}$ $\therefore BA = \frac{BC}{\cos 30}$

1.b)

$$BC = ut = 70\cos 30 \times 14.271 = 865.133\,m$$

(because $a = 0$)

$$BA = \frac{865.133}{\cos 30} = 998.97\,m$$

Question 2

A $S = 3t^2 + 10t - 8$
B $S = 7t^3 + 4t^2 + 4$
C $S = 10t^3 - 30t + 2$

a) A velocity $S = 6t^2 + 10$ m/s
B velocity $S = 21t^2 + 8t$ m/s

$A = 3B$

$6t^2 + 10 = 3(21t^2 + 8t)$

$6t^2 + 10 = 63t^2 + 24t$

$(-6t^2)$ $10 = 57t^2 + 24t$

(-10) $0 = 57t^2 + 24t - 10$

(-3) $2t^2 + \frac{10}{3} = 21t^2 + 8t$

$(-2t^2)$ $\frac{10}{3} = 19t^2 + 8t$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-24 \pm \sqrt{576 - 4(57)(-10)}}{114} = \frac{-24 \pm \sqrt{576 + 2280}}{114}$$

$$= \frac{-24 \pm \sqrt{-1704}}{114}$$

$$\therefore \frac{-24 + \sqrt{1704}}{114} \quad \text{or} \quad \frac{-24 - \sqrt{1704}}{114}$$

Questio 2

b) B velocity $S = 2t^2 + 8t$ m/s

C velocity $S = 30t^2 - 30$ m/s

B Acceleration $S = 42t + 8 m/s^2$

C Acceleration $S = 60t$ m/s^2

$\ddot{x}$ $B = C$

$42t + 8 = 60t$

$(-42t)$ $8 = 18t$

$t = \frac{8}{18} = 0.444$ s

(c) $t = 4$

~~A velocity $= 6t + 10$ m/s~~

~~$= 6(4) + 10 = 34$ m~~

A $S = 3t^2 + 10t - 8$

$= 3(4)^2 + 10(4) - 8 = 80_m$

B $S = 7t^3 + 4t^2 + 4$

$= 7(4)^3 + 4(4)^2 + 4 = 516_m$

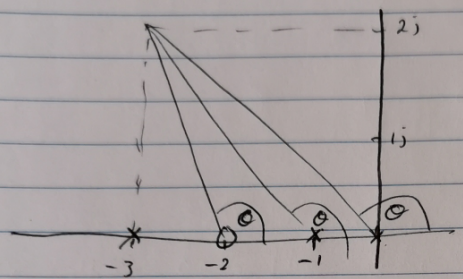
C $S = 10t^3 - 30t + 2$

$= 10(4)^3 - 30(4) + 2 = 522_m$

~~A accel~~

Question 3

a) $\frac{s+2}{s(s+1)(s+3)}$ find $s = -3 + 2j$



Zero $\angle = \sqrt{2^2 + 1^2} = \sqrt{5}$

Pole $0 = \sqrt{2^2 + 3^2} = \sqrt{13}$

Pole $-1 = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

Zero $\theta = 180 - \tan^{-1}\left(\frac{2}{1}\right) = 180 - 63.43 = 116.57^\circ$

Pole $0\ \theta = 180 - \tan^{-1}\left(\frac{2}{3}\right) = 180 - 33.69 = 146.31^\circ$

Pole $-1\ \theta = 180 - \tan^{-1}\left(\frac{2}{2}\right) = 180 - 45 = 135^\circ$

$\theta = 116.57 - (146.31 + 135)$

$= -164.74^\circ$

$M = \frac{\sqrt{5}}{\sqrt{13}(2\sqrt{2})} \angle \theta = -164.74^\circ$

$0.219 \angle -164.74^\circ$

Question 4

(a) $F = ma$

$f(t) - Kx - Kx - Kx - c\dot{x} - c\dot{x} = M\ddot{x}$

$f(t) - 3Kx - 2c\dot{x} = M\ddot{x}$

$f(t) = M\ddot{x} + 2c\dot{x} + 3Kx$

$F(s) = X(s)\left(Ms^2 + 2cs + 3k\right)$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + 2cs + 3K}$$

$$(\div m) = \frac{1/M}{s^2 + \frac{2c}{M}s + \frac{3K}{M}}$$

Question 4

$f = ma$

b) $f(t) - K_4x - K_3x - Kx - c\dot{x} - c\dot{x} = M\ddot{x}$

$f(t) - 2x - 2x - 2x - 2\dot{x} - 2\dot{x} = 6\ddot{x}$

$f(t) - 6x - 4\dot{x} = 6\ddot{x}$

$f(t) = 6\ddot{x} + 4\dot{x} + 6x$

$F(s) = X(s)(6s^2 + 4s + 6)$

$T.F = \frac{X(s)}{F(s)} = \frac{1}{6s^2 + 4s + 6}$

$(\div 6)$ $\frac{1/6}{s^2 + \frac{4}{6}s + 1}$

$\omega_n^2 = 1$

$\omega_n = \sqrt{1}$ R/s $\qquad \frac{\sqrt{1}}{2\pi} = 0.159$ Hz

c)

$\frac{4}{6}\zeta = 2\zeta\omega\zeta$

$\frac{4}{6} = 2\zeta\sqrt{1}$

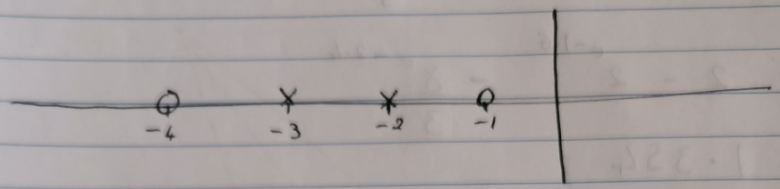
$(\div 2)$ $\frac{2}{6} = \zeta\sqrt{1}$

$\zeta = \frac{1/3}{\sqrt{1}} = 0.\dot{3}$

d) System is under damped as $0.\dot{3} < 1$

Question 5

a) $G_S = \frac{4(s+1)}{(s+2)(s+3)}$ Zeros -4, -1 Poles -2, -3



This shows the system is stable as all poles and zeros are on the same side. If one was on the opposite side, it would be unstable.

b) $t = 0.8$ $\frac{1}{s}$

Responce $\frac{1}{s}\left(\frac{4(s+1)}{(s+2)(s+3)}\right)$

$= \frac{4(s+1)}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{(s+2)} + \frac{C}{(s+3)}$

$\frac{4(s+1)}{s(s+2)(s+3)} = \frac{A(s+2)(s+3) + Bs(s+3) + Cs(s+2)}{s(s+2)(s+3)}$

Numerators must be equal

$4(s+1) = A(s+2)(s+3) + Bs(s+3) + Cs(s+2)$

Put $s = 0$
$4 = A(2)$
$A = 2$

Put $s = -2$
$4(-1) = B(-2)(1)$
$B = -2$

Put $s = -3$
$4(-2) = C(-3)(-1)$
$-8 = 3C$
$C = -8/3$

So responce $= \frac{2}{s} - \frac{2}{(s+2)} - \frac{8}{3(s+3)}$

Laplac^{-1} $= 2 - 2e^{-2t} - \frac{8}{3}e^{-3t}$

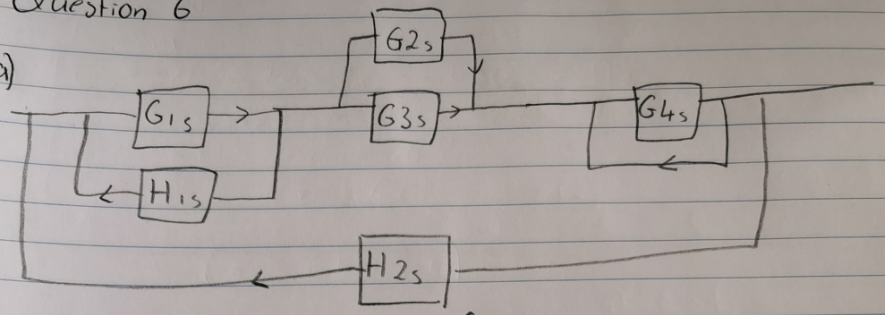
$t = 0.8$

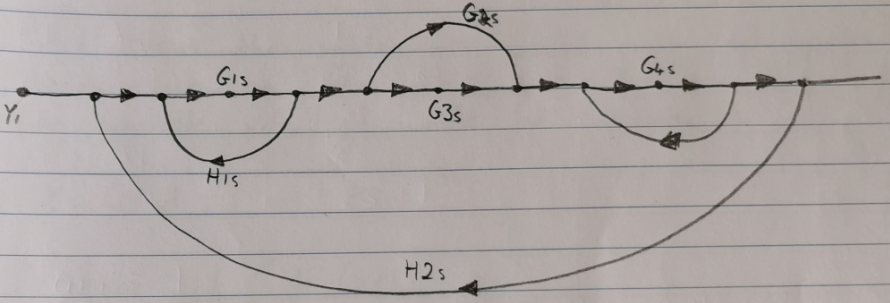
$= 2 - 2e^{-1.6} - \frac{8}{3}e^{-2.4}$

$= 1.354$

Question 6

a)





b)

$$\frac{G_1}{1 + G_1 H_1} \qquad G_2 + G_3 \qquad \frac{G_4}{1 + G_4}$$

$$H_2$$

$$\frac{G_1 (G_2 + G_3)\, G_4}{(1 + G_1 H_1)(1 + G_4)}$$

$$H_2$$

$$\frac{\dfrac{G_1 (G_2 + G_3)\, G_4}{(1 + G_1 H_1)(1 + G_4)}}{1 + \dfrac{G_1 (G_2 + G_3)\, G_4\, H_2}{(1 + G_1 H_1)(1 + G_4)}}$$