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CONNECTING ROD ACCELERATIONS, FORCES AND DISPLACEMENTS

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1 Introduction

The following coursework is intended to analyse the physical principles and properties acting upon the mechanical link that transforms the vertical oscillation of the piston into the rotational motion of a crankshaft: the connecting rod (McCune & Weber, 2001)

Furthermore, the analysis will cover the differences between a standard Ford 1.6 litre engine and a modified version of it.

The starting point of the analysis and the creation of the modified engine is the pressure data of the production engine that provides the cylinder's pressures from 1000 to 6000 rpm with 500 rpm steps. On the other hand, the intended capability of the modified engine was to rotate at higher speeds, in this case up to 9000 rpm, thanks to an external aid such as a turbine or a volumetric compressor. In future chapters it will be discussed how higher regimes were reached.

2 Value tables

MASS [g]		[kg]
Conrod	267.4	0.2674
conrod cap	103.8	0.1038
conrod bolts	35.19	0.03519
conrod assembly	406.4	0.4064
Gaudgeon pin	80.22	0.08022
Piston	257.7	0.2577
Piston ring	5.49	0.00549
Circlip	0.77	0.00077
Piston Assembly	350.4	0.3504

Table 1 Masses

material	Youngs modul [Gpa]	Uts [N/mm^2]
forged steel	210	1500
powder forged steel	170	750
titanium	70	950

Table 2 Material properties

FORD 1.6			
# CYLINDERS	4		
V_swept	1596	cc	
BORE	0.079	m	
STROKE	0.0814	m	
COMP RATIO	10.1:1		
V_clearance	175	cc	
E: forged steel	2.1E+11	Pa	
rod LENGTH	0.134	m	
crank RADIUS	0.0407	m	
rod min A-sect	1.24E-04	m^2	
UTS	1500000000	N/m^2	
piston ring h	0.0012	m	
Area_piston	0.00490167	m^2	
k	194485075	N/m	

Table 3 Engine's features

3 Methodology

3.1 Task A

Peak gas force in the production engine was calculated per each given rpm from:

$$Force_{peak} = Pressure * Area$$

Where pressure represents the average pressure between all four cylinders values and area is the piston's surface.

Peak pressure value for the modified engine will be discussed in the following chapters.

3.2 Task B

Inertia is the force generated by a rotating mass and its values, over a vertical reference, and was calculated as follows:

$$Inertia_{vertical} = Mass_{piston} * acceleration_{piston} \text{ (Taylor, 1985)}$$

in which the acceleration of the piston was calculated per each degree of crankshaft rotation as follows:

$$\frac{d^2s}{dt^2} = R\omega^2 \cos \vartheta + \frac{R}{L}R\omega^2 \cos 2\vartheta \text{ (Nigus, 2016)}.$$

However, inertia loads on the rod must keep into account the rod angle, therefore the previous equation becomes:

$$Inertia_{rod} = Mass_{piston} * acceleration_{piston} * \cos \varphi$$

Where $\cos \varphi$ is the parameter that links the vertical load to the moving con-rod.

It must be highlighted that the original formula takes into account the reciprocating mass (piston mass plus approximately 2/3 conrod mass) (Nigus, 2016) but this assignment had considered only the mass of the piston's assembly.

3.3 Task C

The highest force acting on the rod was calculated as follows:

$$F_{rod} = \frac{(F_{gas} - Inertia_{vertical})}{\cos \varphi}$$

3.4 Task D

The maximum tensile force acting on the rod it is represented by the negative peak value within the force previously calculated.

3.5 Task E

From task C and D it was possible to calculate the maximum compression and tensile stresses in the rod following its definition:

$$\sigma = \frac{F}{A}$$

Where F is the maximum force acting on the rod either in compression or tensile according to the value researched.

3.6 Task F

From the stresses it was possible to calculate the nominal static safety factor for compression and tensile load, as follows:

$$Safety\ factor = \frac{UTS}{\sigma}$$

Where the UTS is a given property of the material and σ are either stress in compression or tensile according to the value researched.

3.7 Task G

To calculate the maximum compression distance and stretch in the conrod, taking into consideration the already calculated stresses, it was necessary to combine the definition of elastic force and stress, as follows:

$$1) F = k * x$$

$$\rightarrow \frac{F}{A} = \frac{k x}{A} \quad \rightarrow \sigma = \frac{k x}{A} \quad \rightarrow x = \frac{\sigma A}{k}$$

$$2) \sigma = \frac{F}{A}$$

3.8 Task H

Following the previous task, the distance and stretch of a titanium con-rod was calculated.

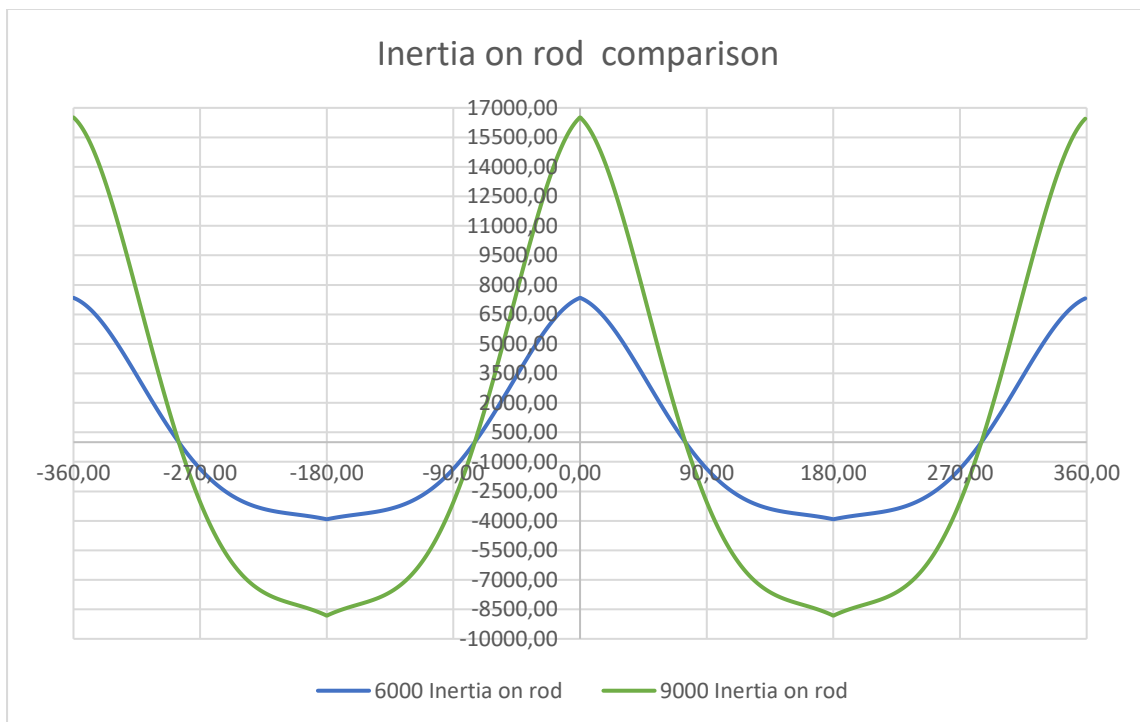
4 Results

4.1 Task A

Peak gas forces	
Production Engine	Modified Engine
46862.91 N	56922.500 N

Table 4 Peak Force

4.2 Task B



Graph 1 Inertia Loads on con-rod

4.3 Task C

NET COMPRESSIVE FORCE ON CON-ROD	
PRODUCTION ENGINE	MODIFIED ENGINE
43175.09 N	45036.6 N

Table 5 Net compressive force on con-rod

4.4 Task D

NET TENSILE FORCE ON CON-ROD	
PRODUCTION ENGINE	MODIFIED ENGINE
6178.09 N	15194.9 N

Table 6 Net tensile force on con-rod

4.5 Task E

MAXIMUM STRESSES IN CON-ROD			
PRODUCTION ENGINE		MODIFIED ENGINE	
COMPRESSION σ	TENSILE σ	COMPRESSION σ	TENSILE σ
347.91 Pa	49.78 Pa	337.70 Pa	120.11 Pa

Table 7 Maximum stresses in con-rod

4.6 Task F

NOMINAL STATIC SAFETY FACTOR			
PRODUCTION ENGINE		MODIFIED ENGINE	
Compressive load	Tensile load	Compressive load	Tensile load
4.31	30.13	4.44	12.49

Table 8 Nominal static safety factor

4.7 Task G

MAXIMUM DISTANCES IN THE CON-ROD			
PRODUCTION ENGINE		MODIFIED ENGINE	
Compression	Stretch	Compression	Stretch
0.222 mm	0.032 mm	0.215 mm	0.077 mm
DIFFERENCE;		+0.007 mm	+0.045 mm

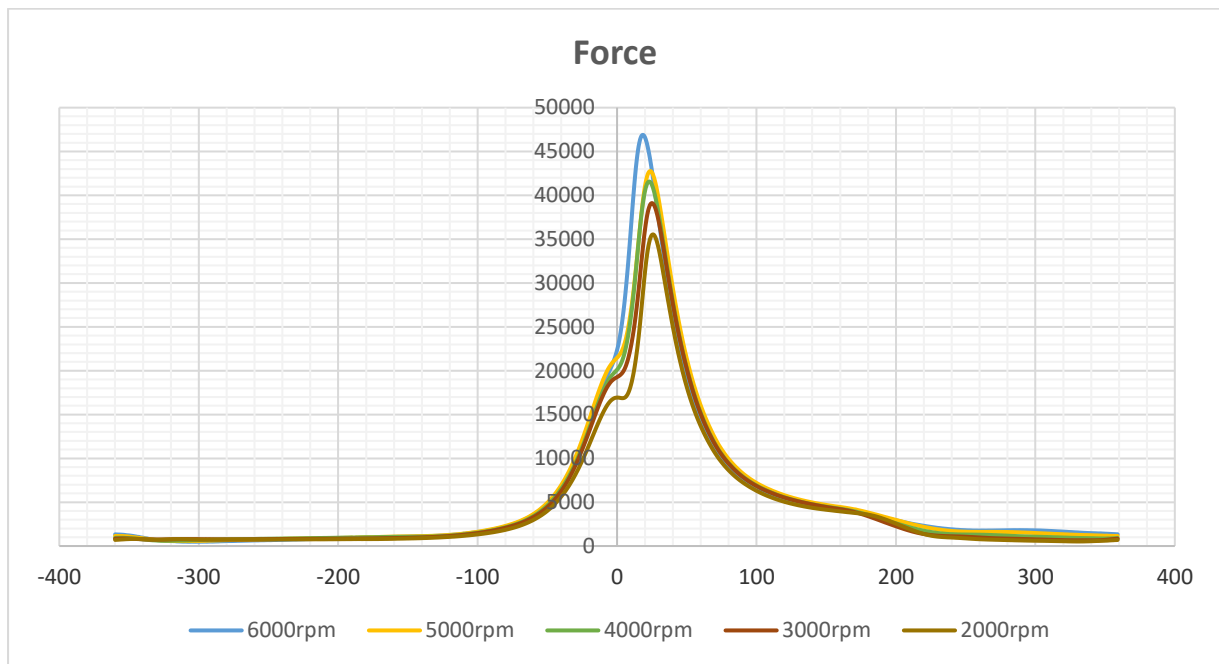
Table 9 Maximum distance in con-rod

4.8 Task H

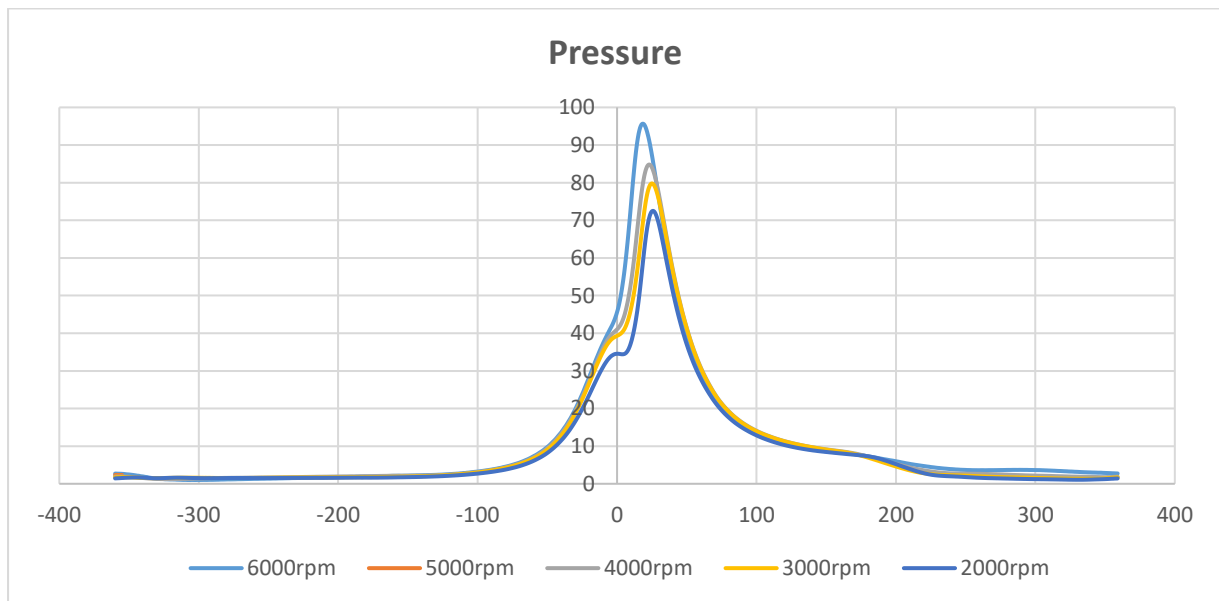
Table 10 Maximum distance in Titanium con-rod

MAXIMUM DISTANCES IN THE CON-ROD			
Forged steel		Titanium	
Compression	Stretch	Compression	Stretch
0.222 mm	0.032 mm	0.666 mm	0.095 mm
DIFFERENCE;		+0.444 mm	+0.063 mm

5 Analysis



Graph 2 Maximum forces per rpm



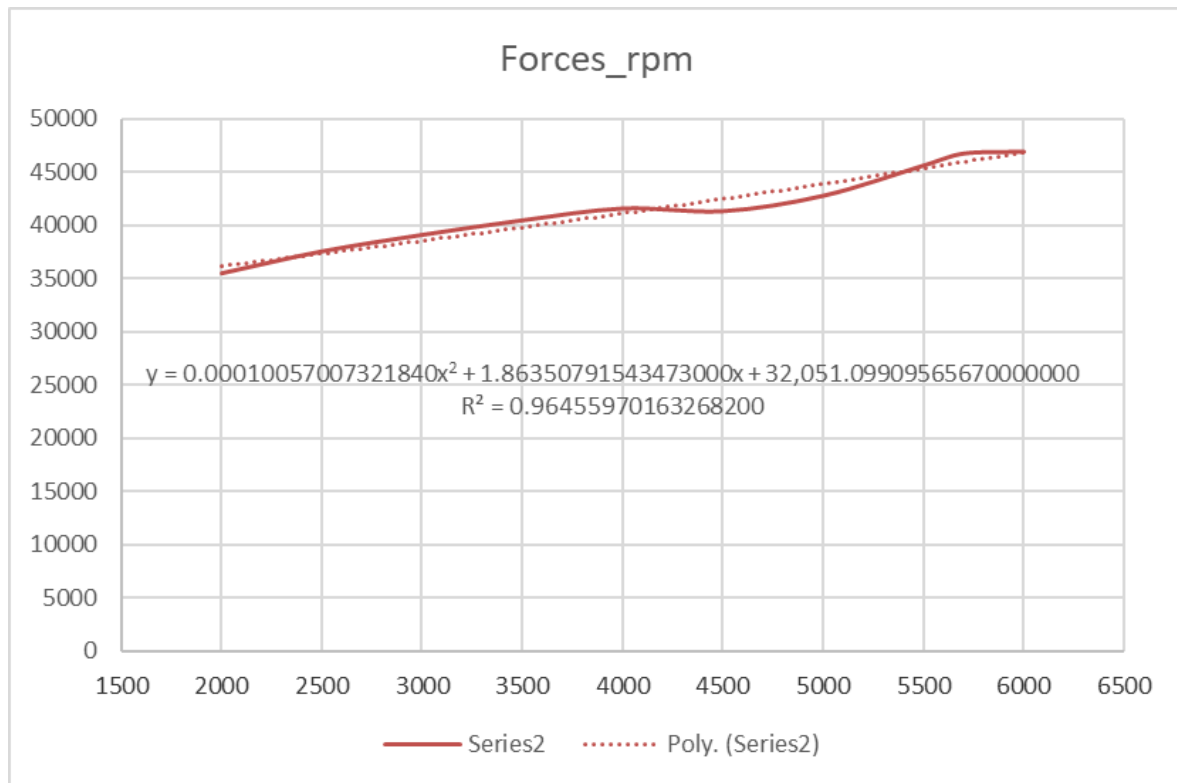
Graph 3 Maximum pressures per rpm

The force graph is equivalent to the pressure graph and highlights how with the increase of engine speeds, the force acting upon the piston increases. It is possible to notice how per each higher engine speeds the force peak is reached always a couple of degrees earlier than the previous, confirmed also by the highlighted green cells in the screenshot bellow.

	A	B	C	D	E	F	G	H	J
1	Crank Angle		3000_Pavrg	3500_Pavrg	4000_Pavrg	4500_Pavrg	5000_Pavrg	5500_Pavrg	6000_Pavrg
2	deg	radians	Pa \ N/m^2	Pa \ N/m^2	Pa \ N/m^2	Pa \ N/m^2	Pa \ N/m^2	Pa \ N/m^2	Pa \ N/m^2
3			7979125	8256150	8481425	8430625	8723500	9298825	9560600
376	12	0.2094395	5040075	5488475	5916700	5640050	6002525	7110100	8265450
377	13	0.2268928	5274200	5777475	6240400	5913650	6298975	7510750	8636900
378	14	0.2443461	5536225	6092775	6581700	6211650	6614950	7900475	8953475
379	15	0.2617994	5825550	6429250	6925550	6528250	6943750	8259125	9204625
380	16	0.2792527	6137650	6772150	7261400	6856050	7275100	8576975	9390000
381	17	0.296706	6461925	7107300	7572525	7181625	7595900	8844150	9506750
382	18	0.3141593	6784275	7418750	7847700	7488925	7892575	9048750	9560600
383	19	0.3316126	7087075	7691050	8077350	7764125	8151750	9191450	9559950
384	20	0.3490659	7357825	7913650	8256925	7997625	8367925	9272875	9509300
385	21	0.3665191	7585550	8081075	8382650	8183025	8534375	9298825	9413675
386	22	0.3839724	7761975	8192675	8456400	8316450	8648200	9275400	9282800
387	23	0.4014257	7885150	8249775	8481425	8398225	8710100	9204450	9122725
388	24	0.418879	7956650	8256150	8461525	8430625	8723500	9095350	8935750
389	25	0.4363323	7979125	8219275	8402150	8418100	8691725	8953575	8731400
390	26	0.4537856	7957925	8144825	8309100	8363850	8620775	8785450	8510375
391	27	0.4712389	7898275	8038675	8185600	8274400	8513625	8598125	8282300

Table 11 Maximum pressure over degrees

Furthermore, by plotting a graph of all peak forces (see graph 4) related to their own engine speeds, it was possible to identify a trendline and its equation, displayed in the image. By substituting the unknown value in the equation with the new higher rpm, it was possible to achieve maximum force values per it (see Table 12). Furthermore, a crucial step to predict reasonable peak force values for higher rpm is the choice of the trendline itself. A linear relation was firstly used but the resultant values were too high, even for a supercharged engine. A second order polynomial relation was then tried which was the one adopted; a further attempt was done with a third order polynomial trendline, but the more precise curve fit was resulting in lower values of peak force, as if the engine was made to run faster but with no aids to cope with the necessity of more fluid to exploit the new scenario.



Graph 4 Forces trendline

F_max		P_[bar]
9000	56922.5	116.1
8000	53359	108.9
7000	49995.5	102.0
6000	46862.91	95.6
5700	46690.86	95.3
5500	45579.77	93.0
5000	42759.72	87.2
4500	41324.14	84.3
4000	41573.15	84.8
3500	40468.92	82.6
3000	39111.04	79.8
2500	37575.47	76.7
2000	35531.47	72.5

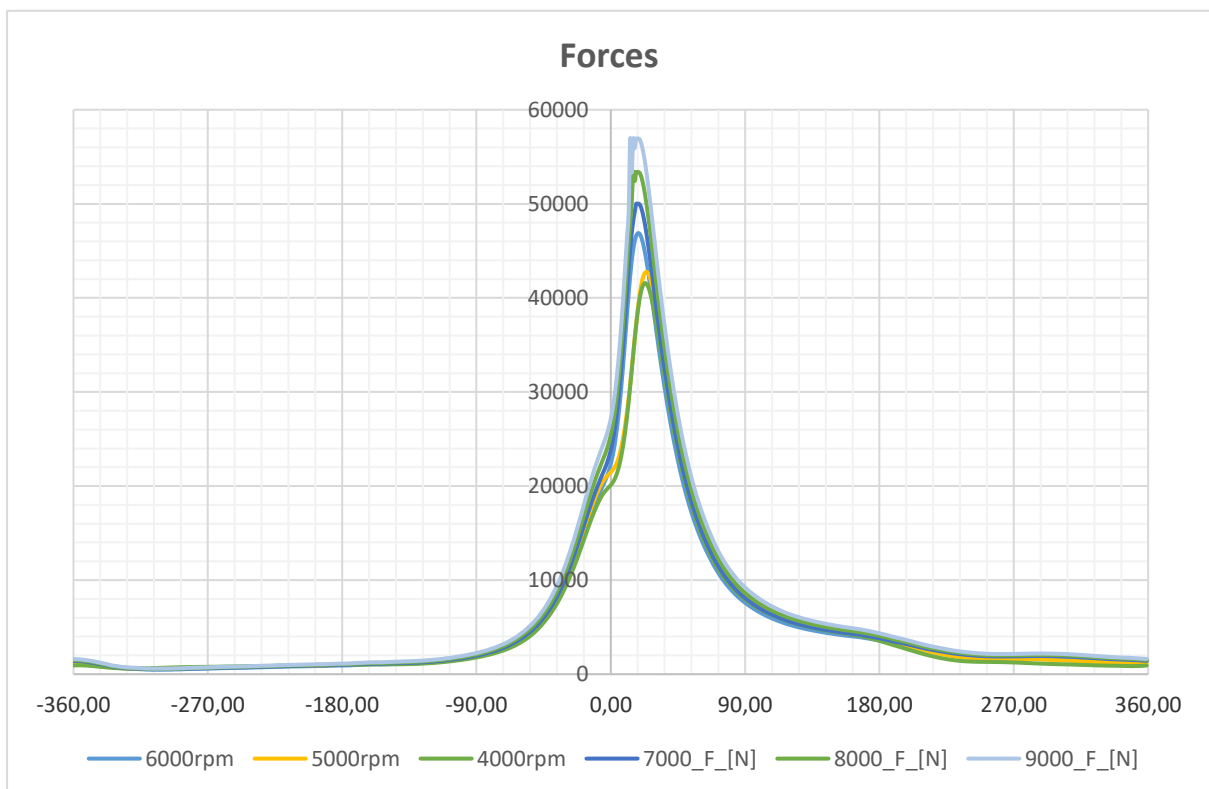
Table 12 modified engine calculated peak force

Following the identified trend of anticipation in the degree, where the maximum pressure is reached, it was assumed it would be the same for the new peak forces calculated i.e. two degrees per each new higher rpm.

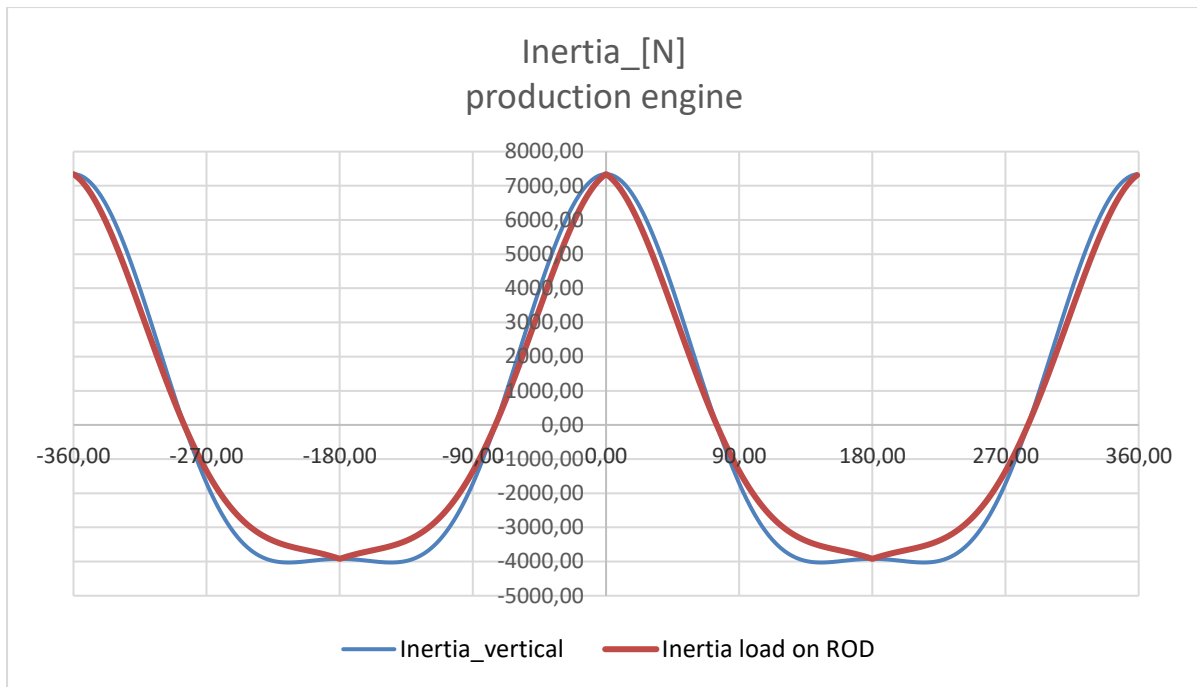
K	L	M	N	O
Crank Angle	6000Force_avrg [N]	7000_F_[N]	8000_F_[N]	9000_F_[N]
deg	46862.906	49995.500	53359.000	56922.500
		7%	7%	7%
0	22355.659	23850.044	25454.580	27154.526
1	23028.658	24568.030	26220.870	27971.991
2	23860.471	25455.447	27167.989	28982.362
3	24852.447	26513.732	28297.471	30187.275
4	26008.261	27746.807	29613.503	31591.196
5	27324.237	29150.750	31111.898	33189.659
6	28836.769	30764.390	32834.097	35026.872
7	30528.581	32569.292	34760.425	37081.848
8	32372.834	34536.826	36860.328	39321.989
9	34341.467	36637.054	39101.851	41713.209
10	36414.016	38848.144	41461.694	44230.651
11	38511.440	41085.773	43849.862	46778.309
12	40514.508	43222.738	46130.593	49211.355
13	42335.233	45165.171	48203.706	51622.500
14	43886.979	46820.645	49970.553	53307.761
15	45118.034	48133.990	51359.000	54922.500
16	46026.681	49103.377	52406.858	55906.771
17	46598.951	49995.500	53359.000	56922.500
18	46862.906	49995.500	53359.000	56922.500
19	46859.720	49992.101	53355.372	56918.630
20	46611.450	49727.236	53072.688	56617.067

Table 13 Modified engine force per degree

Moreover, to trace force variation per each degree of crank rotation, the percentage difference between peak force at 6000rpm and the new value for 7000rpm was calculated.

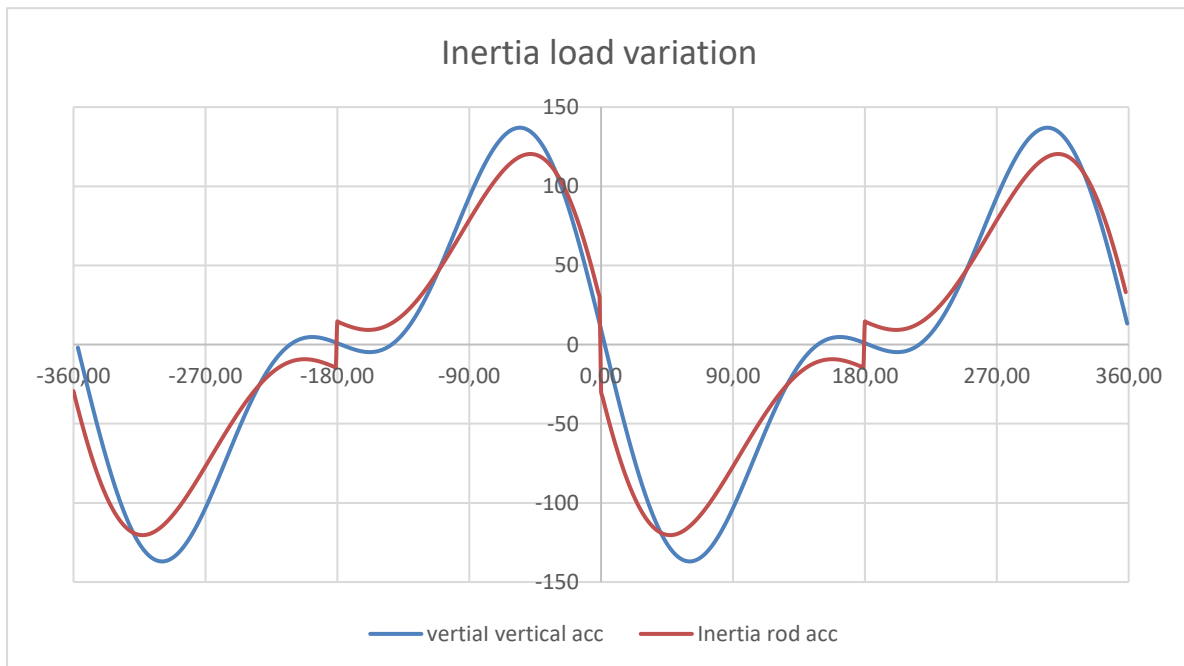


Graph 5 Modified engine forces



Graph 6 Inertia loads comparisons

The absolute inertia's maximum value is reached when the piston is at top dead centre (TDC) but peak values are reached every 180 degrees during the turning points. Vertical inertial loads on the contrary, during the piston's travel inversion, from expansion to compression cycle, reaches two pikes 35 degrees before and after bottom dead centre (BDC). The graph highlights how at BDC the con-rod experiences the minimum force acting upon it and that either vertical and con-rod inertial loads, at turning points, have equal values. However these values are reached differently in time (see Graph 7).



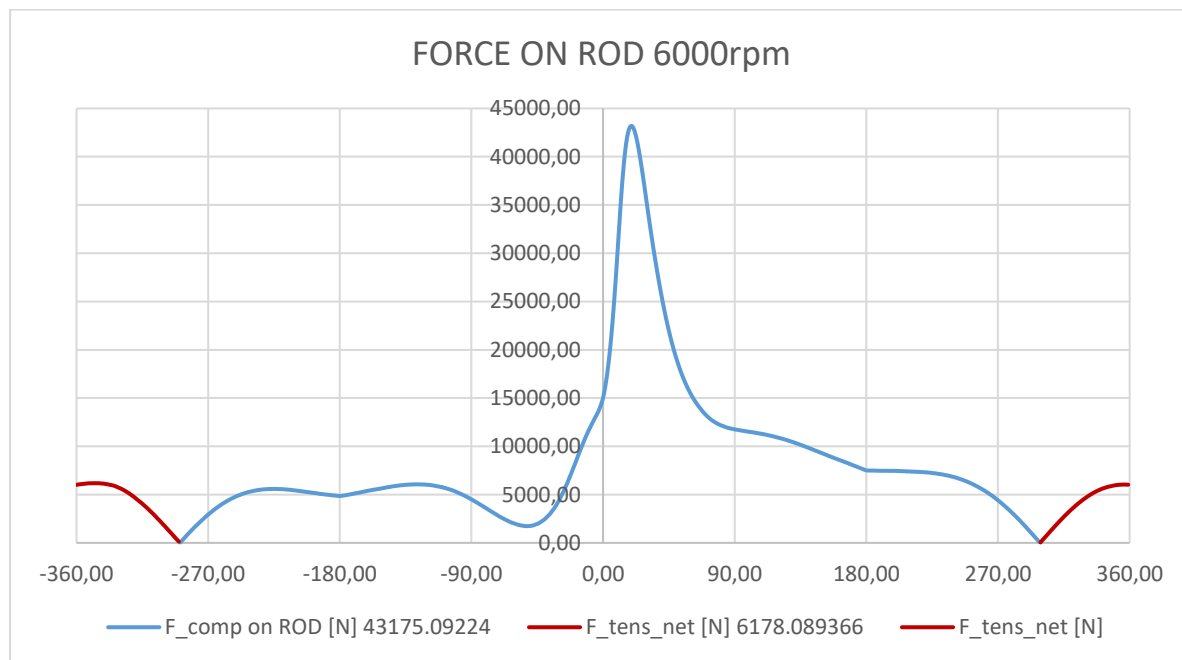
Graph 7 Inertias variation

P	Q	R	S	T	Y	Z	AA	AB	AC	AD	F
		CA	S distance	ϕ rod angl	Force_avrg [N]	acc_piston	Inertia_vertical	Inertia_tensile	Inertia load on ROD	Inertia ten on ROD	
		rads	mm	degrees	46862.90561	m/s^2	7340.173915	4027.06033	7340.173915	3920.081433	
69.00	-291.00	-5.08	31.61	36.41	514.06	2131.41	746.85	-746.85	601.08	-601.08	
70.00	-290.00	-5.06	32.35	36.53	512.59	1756.98	615.65	-615.65	494.72	-494.72	
71.00	-289.00	-5.04	33.09	36.64	518.84	1385.43	485.46	-485.46	389.52	-389.52	
72.00	-288.00	-5.03	33.84	36.75	523.62	1016.98	356.35	-356.35	285.52	-285.52	
73.00	-287.00	-5.01	34.58	36.85	529.99	651.82	228.40	-228.40	182.76	-182.76	
74.00	-286.00	-4.99	35.32	36.95	531.59	290.16	101.67	-101.67	81.25	-81.25	
75.00	-285.00	-4.97	36.06	37.04	542.37	-67.81	-23.76	23.76	-18.96	18.96	
76.00	-284.00	-4.96	36.81	37.13	550.21	-421.89	-147.83	147.83	-117.87	117.87	
77.00	-283.00	-4.94	37.55	37.20	551.81	-771.91	-270.48	270.48	-215.43	215.43	
78.00	-282.00	-4.92	38.29	37.28	556.58	-1117.68	-391.64	391.64	-311.63	311.63	
79.00	-281.00	-4.90	39.03	37.35	561.36	-1459.04	-511.25	511.25	-406.44	406.44	
80.00	-280.00	-4.89	39.77	37.41	569.21	-1795.82	-629.26	629.26	-499.85	499.85	
81.00	-279.00	-4.87	40.50	37.46	573.99	-2127.87	-745.60	745.60	-591.83	591.83	
82.00	-278.00	-4.85	41.24	37.51	572.39	-2455.02	-860.24	860.24	-682.37	682.37	
83.00	-277.00	-4.83	41.97	37.56	573.99	-2777.14	-973.11	973.11	-771.45	771.45	
84.00	-276.00	-4.82	42.71	37.59	577.05	-3094.09	-1084.17	1084.17	-859.05	859.05	

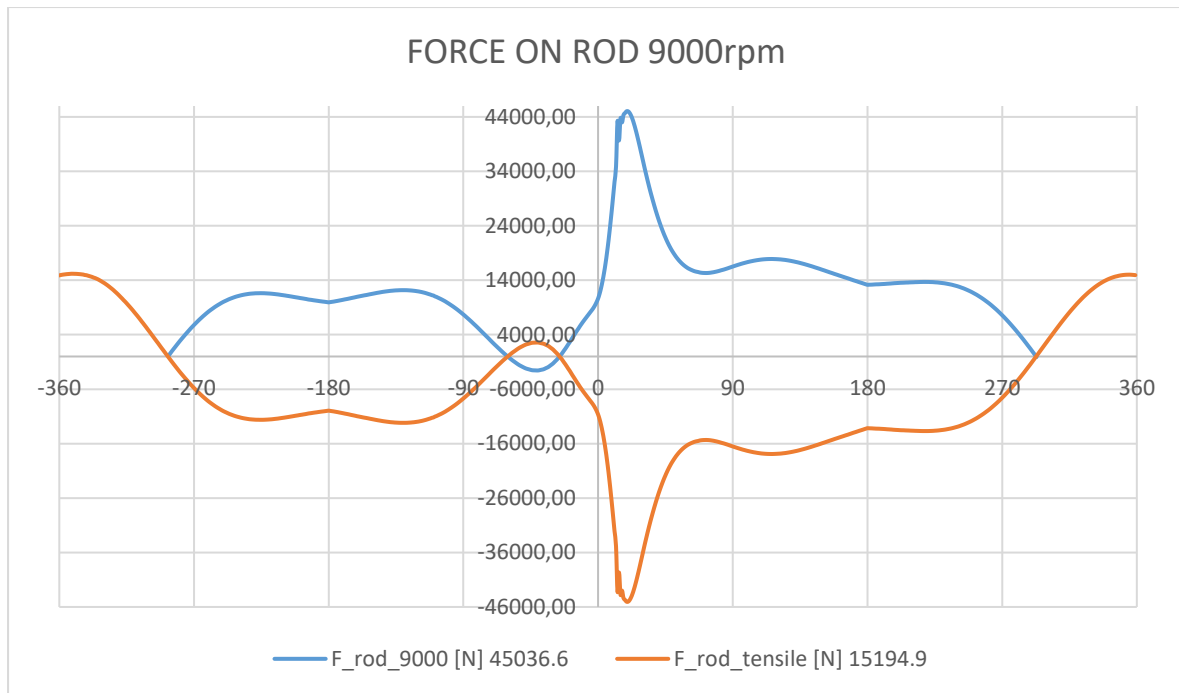
Table 14 Inertias recurring point

The green cells in table 14 shows how either vertical loads and the one on the con-rod have equal angle degrees when shifting from tensile to compressive and vice versa.

Due to higher pressures generated by the modified engine, it is intuitive why either compressive and tensile rod force have higher values than the one developed in the production engine.



Graph 8 Compressive and tensile force on the rod at 6000rpm



Graph 9 Compressive and tensile force on the rod at 9000rpm

Graph 8 and 9 and the following table, highlights that tensile forces on the con-rod occur only during the first 70 and last 61 degrees relatively of the intake and exhaust stroke: this is due to the vacuum effect generated by the fluid first entering in the chamber and then exiting it. Therefore, during the intake phase, as soon as that vacuum effect ends the tensile forces becomes negligible, because of a negative magnitude, and are substituted by the compressive forces. On the other hand, the compressive forces are generated by the fluid entering the cylinder and starts building pressure upon the piston, even before the compression stroke as we have previously seen, due to the speed achieved. The opposite of this principle occurs when dealing with the last 61 degrees of the exhaust stroke, where the gas leaving the chamber are firstly pushed out by the piston and then pulled out by the vacuum effect.

P	Q	R	S	T	Y	Z	AE	AF	
		CA	S distance	φ rod angl	Force_avrg [N]	acc_piston	F_comp on ROD [N]	F_tens_net [N]	F_si
		rads	mm	degrees	46862.90561	m/s^2	43175.09224	6178.089366	
60.00	-300.00	-5.24	25.07	35.05	495.44	5593.72	-1788.99	1788.99	
61.00	-299.00	-5.22	25.78	35.22	495.44	5203.63	-1625.53	1625.53	
62.00	-298.00	-5.20	26.50	35.39	500.22	4814.32	-1455.74	1455.74	
63.00	-297.00	-5.18	27.22	35.56	501.69	4426.04	-1289.65	1289.65	
64.00	-296.00	-5.17	27.95	35.71	504.75	4039.03	-1121.40	1121.40	
65.00	-295.00	-5.15	28.68	35.86	503.28	3653.53	-958.67	958.67	
66.00	-294.00	-5.13	29.41	36.01	501.56	3269.79	-796.32	796.32	
67.00	-293.00	-5.11	30.14	36.15	509.28	2888.04	-622.52	622.52	
68.00	-292.00	-5.10	30.88	36.28	514.06	2508.50	-452.67	452.67	
69.00	-291.00	-5.08	31.61	36.41	514.06	2131.41	-289.23	289.23	
70.00	-290.00	-5.06	32.35	36.53	512.59	1756.98	-128.24	128.24	
71.00	-289.00	-5.04	33.09	36.64	518.84	1385.43	41.61	-41.61	
72.00	-288.00	-5.03	33.84	36.75	523.62	1016.98	208.76	-208.76	
73.00	-287.00	-5.01	34.58	36.85	529.99	651.82	376.91	-376.91	
74.00	-286.00	-4.99	35.32	36.95	531.59	290.16	537.96	-537.96	
75.00	-285.00	-4.97	36.06	37.04	542.37	-67.81	709.25	-709.25	
76.00	-284.00	-4.96	36.81	37.13	550.21	-421.89	875.49	-875.49	
77.00	-283.00	-4.94	37.55	37.20	551.81	-771.91	1032.39	-1032.39	
78.00	-282.00	-4.92	38.29	37.28	556.58	-1117.68	1191.67	-1191.67	
79.00	-281.00	-4.90	39.03	37.35	561.36	-1459.04	1349.20	-1349.20	
80.00	-280.00	-4.89	39.77	37.41	569.21	-1795.82	1508.74	-1508.74	
81.00	-279.00	-4.87	40.50	37.46	573.99	-2127.87	1662.46	-1662.46	

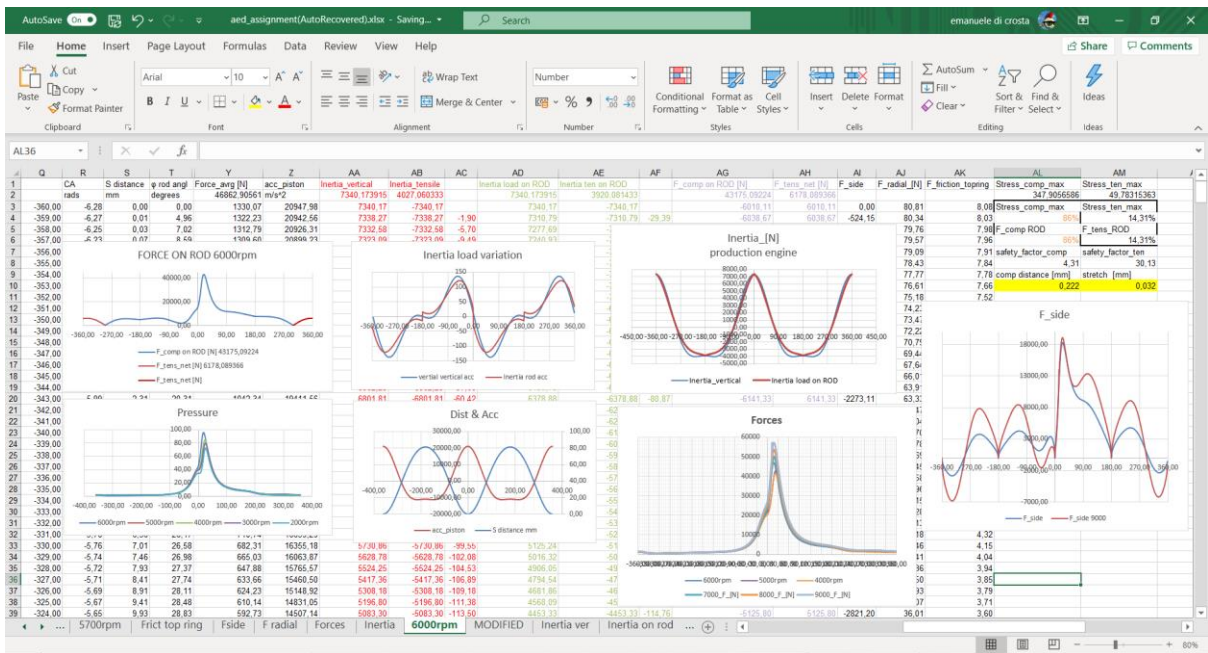
Graph 10 Tensile force

Directly related to the force on the rod, by formulae, are the stresses upon it, therefore, in the modified engine, both compression and tensile stresses are higher than the production one; moreover, due to this relationship, from previous values it is also possible to assume which stress will be higher. By simple mathematical operation, as tensile force is 14.31% of the compressive once, as well the tensile stress is 14.31% of the compressive one.

On the contrary, even though the nominal static safety factor is directly related to stresses, for compressive loads, it tends to be 0.13 higher for the modified engine and 17.64 less for tensile load (see table 9). Therefore, it can be stated that the standard con-rod would be able to accept higher engine speeds.

Confirmation of the previous statement can be found in the values of compression distance and stretch (see table 9). In fact, the modified engine will experience a 0.045 mm stretch and only 0.007 mm in compression.

A similar scenario occurs if the standard con-rod is substituted with a Titanium one, but it is a predictable result when referring to Young's Module and ultimate tensile strength values: in fact, the conrod experiences a 0.063 mm in stretch but an astonishing 0.444 mm in compression distance (see table 10). The latter value intuitively suggests that the known lightness of this material is not a viable solution, because even just with standard engine's features it leads to failures e.g. compression loss if the deflection doesn't cause major problems, and goes against the main task of a connecting rod: transfer the force generated by the combustion of the air-fuel mixture to the crankshaft (Nigus, 2016).



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