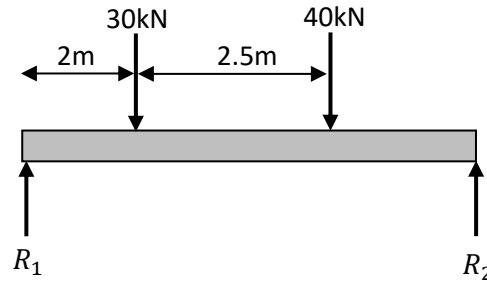


Beam is 7 m long with an $(E \times I)$ value of 200 MN.m². Determine slope and deflection values in the middle.



Implementing the conditions for static equilibrium to determine support reactions R_A and R_B :

- Condition 1: the algebraic sum of forces is zero that is,

$$\text{sum of the } \textbf{upward} \text{ component of forces} = \text{sum of the } \textbf{downward} \text{ component of forces}$$

Thus,

$$R_1 + R_2 = 30 + 40$$

- Condition 2: the sum of the anticlockwise moments about a point or axis equals the sum of the clockwise moments about the same point or axis. If we choose left tip of the beam,

$$30 \times 2 + 40 \times 4.5 = 7 \times R_2$$

$$\therefore R_2 = \frac{30 \times 2 + 40 \times 4.5}{7} = 34.29 \text{ kN}$$

And substituting R_2 into condition 1 gives,

$$R_1 + 34.29 = 30 + 40$$

$$\therefore R_1 = 70 - 34.29 = 35.71 \text{ kN}$$

Taking the origin at the left hand end of the beam and writing down the bending moment at a section located at the extreme right hand part of the beam in terms of all loads to the left of that section gives,

$$M_{xx} = EI \frac{d^2y}{dx^2} = 35.71x - 30[x - 2] - 40[x - 4.5]$$

It must be emphasised that all loads in the right-hand side of the above bending moment equation are in units of kN (i.e. newton $\times 10^3$). In subsequent workings, therefore, it is convenient to carry through this factor as a denominator on the left-hand side of the equation in order that the expressions are dimensionally correct.

Integrating gives,

$$\frac{EI}{10^3} \frac{dy}{dx} = \frac{35.71}{2} x^2 - \frac{30}{2} [x - 2]^2 - \frac{40}{2} [x - 4.5]^2 + A$$

$$\therefore \frac{EI}{10^3} \frac{dy}{dx} = \frac{35.71}{2} x^2 - 15[x - 2]^2 - 20[x - 4.5]^2 + A$$

Integrating a second time gives,

$$\frac{EI}{10^3} y = \frac{35.71}{6} x^3 - \frac{15}{3} [x - 2]^3 - \frac{20}{3} [x - 4.5]^3 + Ax + B$$

$$\therefore \frac{EI}{10^3} y = \frac{35.71}{6} x^3 - 5[x - 2]^3 - \frac{20}{3} [x - 4.5]^3 + Ax + B$$

Determination of the constant of integration A and B:

- At $x = 0, y = 0$. Therefore,

$$\frac{EI}{10^3} \times 0 = \frac{35.71}{6} \times 0^3 + A \times 0 + B \quad \therefore \quad 0 = 0 + 0 + B \quad \therefore \quad B = 0$$

- At $x = 7, y = 0$. Therefore,

$$\frac{EI}{10^3} \times 0 = \frac{35.71}{6} \times 7^3 - 5(7 - 2)^3 - \frac{20}{3} (7 - 4.5)^3 + A \times 7$$

$$\therefore \quad 0 = 1312.5 + A \times 7 \quad \therefore \quad A = \frac{-1312.5}{7} = -187.5$$

Therefore the slope equation is,

$$\frac{dy}{dx} = \frac{10^3}{EI} \left(\frac{35.71}{2} x^2 - 15[x - 2]^2 - 20[x - 4.5]^2 - 187.5 \right)$$

And the deflection equation is,

$$y = \frac{10^3}{EI} \left(\frac{35.71}{6} x^3 - 5[x - 2]^3 - \frac{20}{3} [x - 4.5]^3 - 187.5x \right)$$

Hence the slope and deflection at mid-span, that is $x = 3.5$, are:

$$\frac{dy}{dx} = \frac{10^3}{200 \times 10^6} \left(\frac{35.71}{2} \times 3.5^2 - 15(3.5 - 2)^2 - 187.5 \right) = -1.25 \times 10^{-5}$$

and,

$$y = \frac{10^3}{200 \times 10^6} \left(\frac{35.71}{6} \times 3.5^3 - 5(3.5 - 2)^3 - 187.5 \times 3.5 \right) = -2.0896 \times 10^{-3} \text{ m} = -2.0896 \text{ mm}$$