

1809799

by Thomas Grennan

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1. Executive Summary

Too long, 300

This report consists of 5 different topics based upon the principles of control engineering. The first task is based upon the transient response of a second order system, Values were chosen for the system and a single transfer function was derived. Calculations were then used to calculate key aspects of the response such as natural frequency, damping factor, damped frequency, percentage overshoot and more. The transfer function was then inserted into MATLAB using the step function to provide the systems step response, the response was then annotated at key points and comparisons were made between calculation and simulation.

Too descriptive

The second task was based upon the modelling of a control system. Laplace transforms were used to translate the systems equation into the frequency domain in order to produce an open-loop transfer function. Values were chosen for the transfer function, from this the systems poles and zeros were calculated and an argand diagram was plotted to visually predict the systems stability and response, this was then verified through the use of MATLABs step function and rlocus function in order to obtain a step response and argand diagram which proved the theory and predictions correct. A new transfer function was then derived for the closed-loop system and the gain value for a critically stable and critically damped response were calculated. A Simulink model of the system with a negative feedback response was then created, the overall system is constructed of three identical systems cascaded together, each system had a different gain value in order to display a stable, unstable and critically stable response, the response seen from the system was highly accurate proving the theory correct. The usefulness of the Root Locus method for control engineers was then discussed.

The third task was focused on the block reduction method, this method takes a complex control system and reduces it to a single transfer function through the use of mathematical equations. Firstly, the control system was reduced in stages by reducing one block at a time until two blocks remained and a single transfer function was derived. Values were then chosen for the transfer function, these values were then inserted into the reduced equations and simplified until a final transfer function was derived. The system was then analysed within MATLAB whereby a step response and argand diagram were obtained, these indicated that the system is unstable. A Simulink model was then created, this consisted of the original system cascaded with the reduced transfer function with a shared step input with both systems outputs connected to the same scope, Identical responses were obtained proving the success and accuracy of the method.

The fourth task was to analyse a systems response to a step input in the frequency domain, known as the frequency response method. Values for the system were chosen with an explanation of what the

values were for in terms of cut-off frequencies. Using MATLAB, the magnitude and phase over various frequencies were plotted and analysed. The phase-margin method was then used to determine the systems stability.

The final task is centred around digital control systems. Calculations were derived in order to find the minimum sampling time for a critically stable response, it was understood that this is the point along the perimeter of the z-plane, with the inside of the perimeter being the area of stability and the outer area being the area of instability. The calculated time for critical stability was entered into the zero-order-hold block in Simulink to prove that the response was critically stable. Values above and below this value were then inserted into the zero-order-hold block to show instability and stability respectively.

An overall conclusion was then made for each control system analysis method and suitable recommendations were made for further study.

*what were your
conclusions?*

2. Transient Response of a Second Order System

Values for K and A were chosen for the second order system, these are seen in table 1.

Transient Response of a Second Order System – Values for K and A	
K	20
A	4

Table 1 – Transient Response of Second Order System: Values for K and A

Figure 1 shows a model of the control system.

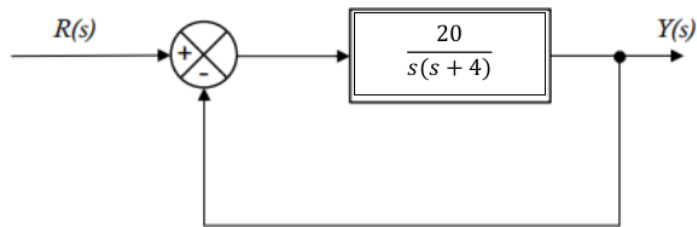


Figure 1 - Transient Response of Second Order System: Control System Model

From this control system, calculations were derived in to calculate the systems transfer function as seen below, Calculations are seen in appendix 8.1.

$$\frac{Y(S)}{U(S)} = \frac{20}{s(s + 4) + 1} = \frac{20}{s^2 + 4s + 1}$$

wrong denominator, this is not what you have in your appendix

Table 2 displays the calculated values of key aspects of the systems response to a step input, all calculations for these values are seen in appendix 8.1.

Transient Response of a Second Order System – Calculated Values	
Natural Frequency	4.47rad/s
Damping Factor	0.447
Damped Frequency	3.99rad/s
Rise time from 0 to 100% of final value	0.508 seconds
Peak time	0.786 seconds
Percentage overshoot	20.807%
Settling time within +/-2%	2.002 seconds
Number of Oscillations	1.274

Table 2 - Transient Response of Second Order System: Calculated Values

Figure 2 shows the transient response of the second order system from the MATLAB program seen in appendix 8.1. Data tips are used to identify the values at key points within the step response, these values can then be compared to the calculations.

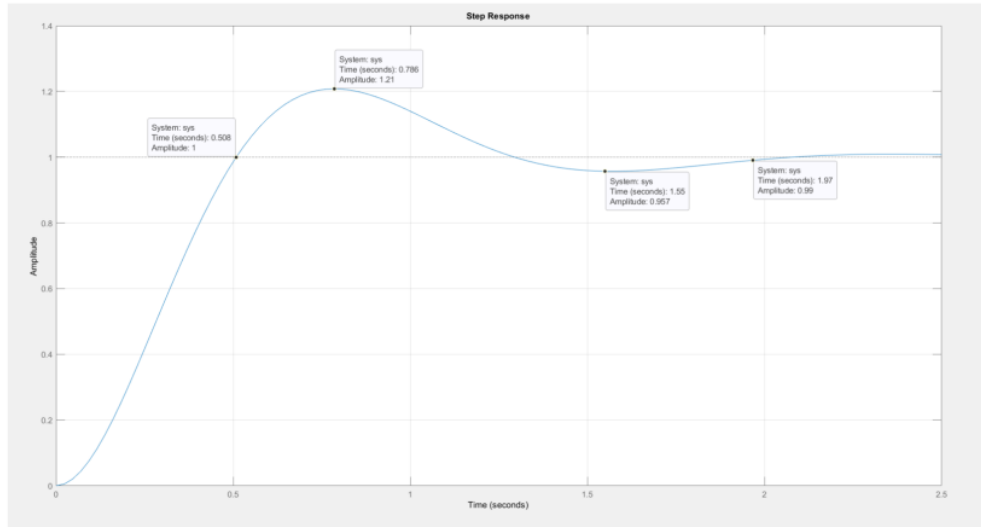


Figure 2 - Transient Response of Second Order System: Annotated Step Response

A comparison of the calculated and simulated values of this system are seen in table 3. The results seen show clear correlation, proving the accuracy between simulated and calculated methods. The damping factor was calculated as 0.447, from this the system is underdamped, this is seen by the step response in figure 2 which shows an oscillating waveform which decays and enters steady state. All simulated values are highly accurate when compared to the calculations with very little discrepancies, the largest seen when comparing the percentage overshoot which has a difference of 0.19 which is miniscule and likely due to cursor accuracy within MATLAB, the damped frequency also has a negligible error between calculation and simulation which is also likely due to cursor error.

Transient Response of a Second Order System – Calculated Values vs Simulated Values		
	Calculation	Simulation
Damping Factor	0.447	0.447
Damped Frequency	3.99rad/s	4.00 rad/s
Rise time from 0 to 100% of final value	0.508 seconds	0.508 seconds
Peak time	0.786 seconds	0.786 seconds
Percentage overshoot	20.807%	21%
Settling time within +/-2%	2.00 seconds	1.97 seconds
Number of Oscillations	1.274	~1

Table 3 - Transient Response of Second Order System: Calculated Values vs Simulated Values

excellent !

3. Control System Modelling

An open loop control system is represented by the differential equation seen below, whereby x is the input and y is the output, both being functions of time, this system must be transformed into the frequency domain.

$$\frac{d^2y}{dt^2} + a \frac{dy}{dt} + by = \frac{d^2x}{dt^2} - p \frac{dx}{dt} + qx$$

Assuming all conditions are zero, values were chosen for 'a', 'b', 'p' and 'q', the values are seen in table

4.

these value produce the poles and zeros but are not themselves the poles and zeros

Control System Modelling – Chosen Values			
Poles		Zeros	
a	3	p	1
b	1	q	3

Table 4 - Control System Modelling: Chosen Values

From the calculations seen in appendix 8.2.1, the systems transfer function was obtained, as seen below:

$$\frac{Y(s)}{X(s)} = G(s) = \frac{s^2 - 1s + 3}{s^2 + 3s + 1}$$

The systems 'Poles' and 'Zero' values were calculated in appendix 8.2, these values are seen in table

5.

Control System Modelling – Calculated Poles and Zeros	
Poles	-0.3819, -2.6180
Zeros	0.5 ± j 1.658

Table 5 - Control System Modelling: Calculated values for Poles and Zeros

The calculated poles and zeros in table 5 were then manually plotted onto an argand diagram as seen in figure 3. The position of the poles is within the left-hand plane, the rules of stability specifies that the system will be stable, if these poles were in the right-hand plane the system is unstable. The damping factor is also determined by the area whereby the poles reside, in this case they are spaced out along the real axis, this determines that the system will be over-damped. This theory can be confirmed by MATLAB and by calculating the natural frequency and damping factor.

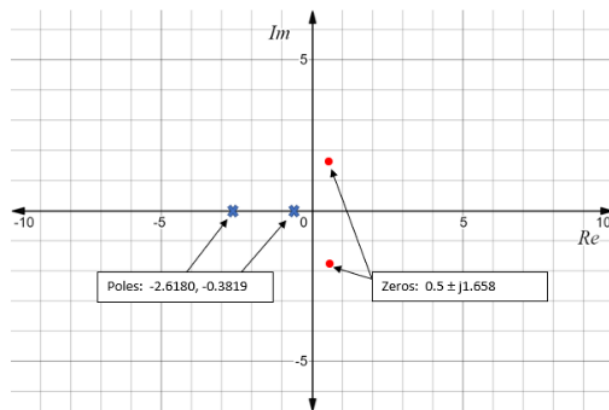


Figure 3 - Control System Modelling: Manually plotted argand diagram

The systems damping factor was calculated as 1.5, for this the natural frequency needed to be calculated in order to calculate the damping factor, these equations are seen in appendix 8.2. The damping factor rules state that a damping factor of greater than 1 is overdamped, This was then further confirmed by performing a step-response in MATLAB shown in figure 4, this response is slow to reach steady state but has no oscillations or overshoot which are the characteristics of an overdamped response.

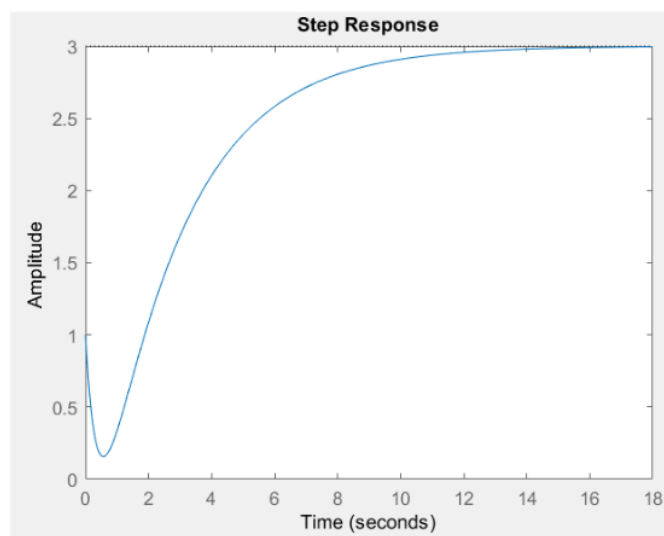


Figure 4 - Control System Modelling: Overdamped Step Response

The systems transfer function was then inputted into MATLAB using the 'Pole' and 'Zero' functions, this is displayed in appendix 8.2. From this, simulated values for the systems poles and zeroes were produced. The calculated values are compared to the simulated in values in table 6, the results attained through MATLAB are identical to the calculations, this consistency proves the accuracy of both methods.

Control System Modelling – Calculated Poles vs Simulated Poles				
	Calculated		Simulated	
Poles	-0.3467	-8.6533	-0.3467	-8.6533
Zeros	$1 + j 2.236$	$1 - j 2.236$	$1 + j 2.236$	$1 - j 2.236$

Table 6 - Control System Modelling: Calculated Poles vs Simulated Poles

The 'Root-Locus' function was performed in MATLAB, this produced a Root-Locus diagram whereby the systems poles and zeros are seen as data tips, these values are rounded to 2d.p but are still accurate to the calculated values. It is clearly seen that this response is a replica of the manually plotted graphs, showing the (Figure 5)

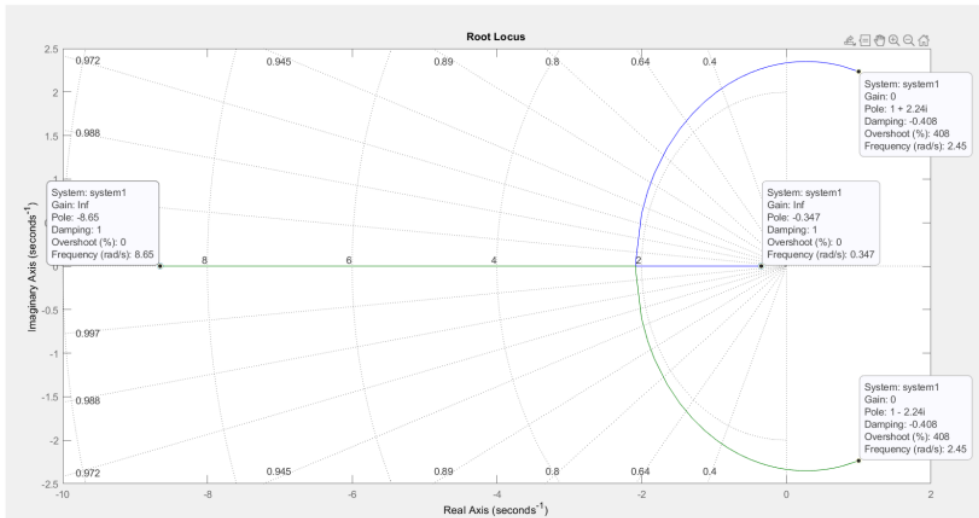


Figure 5 - Control System Modelling: MATLAB Argand Diagram for poles and zeroes.

A new transfer function was derived for the closed-loop system, the equation is seen below with the calculations seen in appendix 8.2.

$$T(s) = s^2 + \frac{(3 - 1k) \text{ where } k \text{ is the numerator}}{(1 + k)s + (1 + k)}$$

Using this equation, the point of critical stability is calculated:

$$T(S) = \frac{(3 - 1k)}{(1 + k)} s = 0 \quad \begin{matrix} w \\ hy \\ ? \end{matrix}$$

Therefore:

$$3 - 1k = 0, \text{ therefore, } K = 3$$

This is confirmed using MATLABs 'Root-Locus' function, the point of critical stability is at the point of zero on the real axis, Figure 6 shows that at this point the gain is 3 confirming the calculation. When a negative feedback is added, the poles move toward the zeros, by using variable values of gain the response can be changed for three types of damping, these being overdamped, underdamped and critically damped and three types of stability being critical stability (No Damping), stability and instability

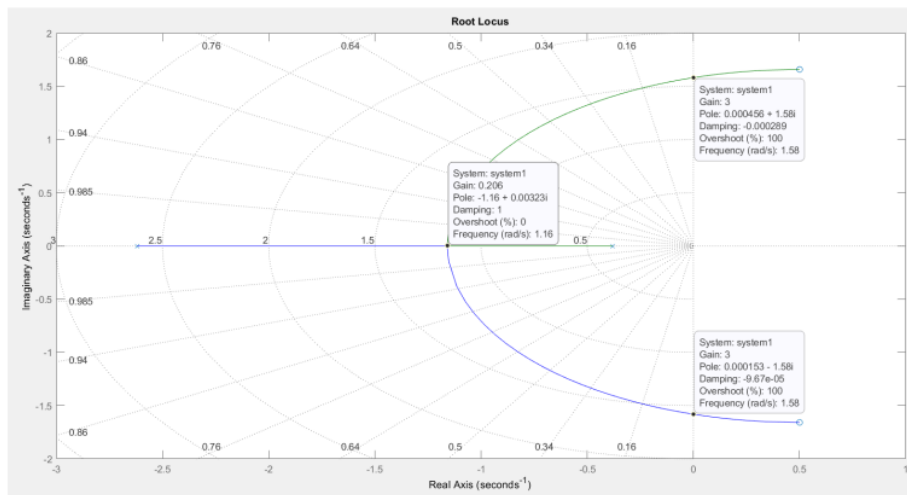


Figure 6 - Control System Modelling: MATLAB Argand Diagram Critical Stability and Damping

Figure 6 also displays the point of critical damping at a gain of 0.206, Critical damping is calculated using the transfer function above, Critical damping is the best response for a control system as it is the quickest response with no oscillations or overshoot, at this point the damping factor is 1.

$$T(s) = s^2 + \frac{(3 - 1(0.206))}{(1 + (0.206))} s + \frac{(1 + 3(0.206))}{(1 + (0.206))}$$

$$T(s) = s^2 + 2.3167s + 1.3416$$

This is compared to the general transfer function of the denominator ' $s^2 + 2\zeta\omega_n s + \omega_n^2$ ':

$$\omega n^2 = 1.3416, \text{ therefore, } \omega n = \sqrt{1.3416} = 1.1582$$

And:

$$2\zeta\omega n S = 2.3167, \text{ therefore, } \zeta = \frac{2.3167}{2\sqrt{1.1582}} = 1.07$$

From this calculation it proves the accuracy of this calculation, the damping factor was calculated as 1.07 and the natural frequency at 1.158. As seen in figure 6 the data pointer shows that the damping factor at critical damping is 1 with a frequency of 1.16. These are highly accurate with the minor errors due to rounding, in turn proving MATLABs root locus functions accuracy.

Control System Modelling - Simulink Gain Values	
Gain Value	Expected Response
3	Critical Stability
1.5	Stability
4.5	Instability

Table 7 - Control System Modelling: Simulink Gain Values

A Simulink model was created for the control system, it is designed as a closed-loop system with negative feedback, the negative feedback values will be gain values for critical stability, stability and instability. The chosen gain values and their expected response are seen in table 7, as the point of critical stability is seen in the MATLAB produced argand diagram at a gain of 3, values were chosen above and below this value for instability and stability. Figure 8 shows the constructed Simulink model with the three gain values added as negative feedback.

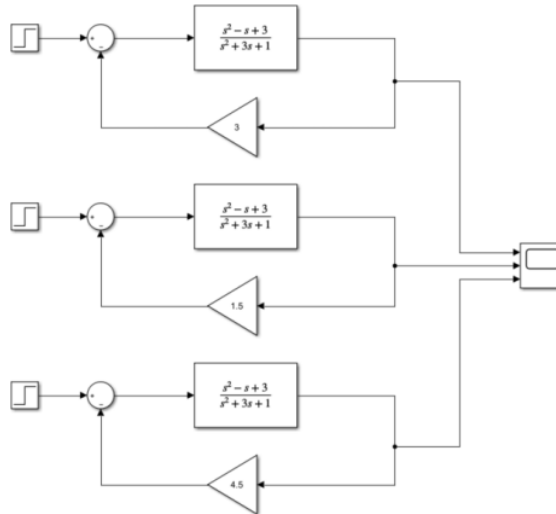


Figure 7 - Control System Modelling: Simulink Model of Cascaded Circuits

The step response of the Simulink circuit is seen in Figure 8, a comparison table between the expected result and simulated result is seen in table 8. The system step response for various gains is seen in figure 8. At a gain value of 3 it was predicted that the system response would be critically stable, therefore, it was expected that the response would oscillate at a constant amplitude which will never enter steady-state, this is clearly proven as seen by the blue waveform which oscillates at a constant amplitude and frequency.

A stable response is expected at a gain value of less than 3, therefore a value of 1.5 is chosen. Stable responses tend to have an overshoot and oscillation which quickly dies away as it enters steady state. As seen in the orange waveform, the response has an overshoot and oscillation which quickly enter a steady-state.

The point of instability is expected when the value of gain is greater than 3, a value of 4.5 was chosen to display this waveform. The yellow waveform clearly displays an unstable response, this response is seen to start oscillating from its origin and constantly increases in amplitude over time, this is expected of an unstable response.

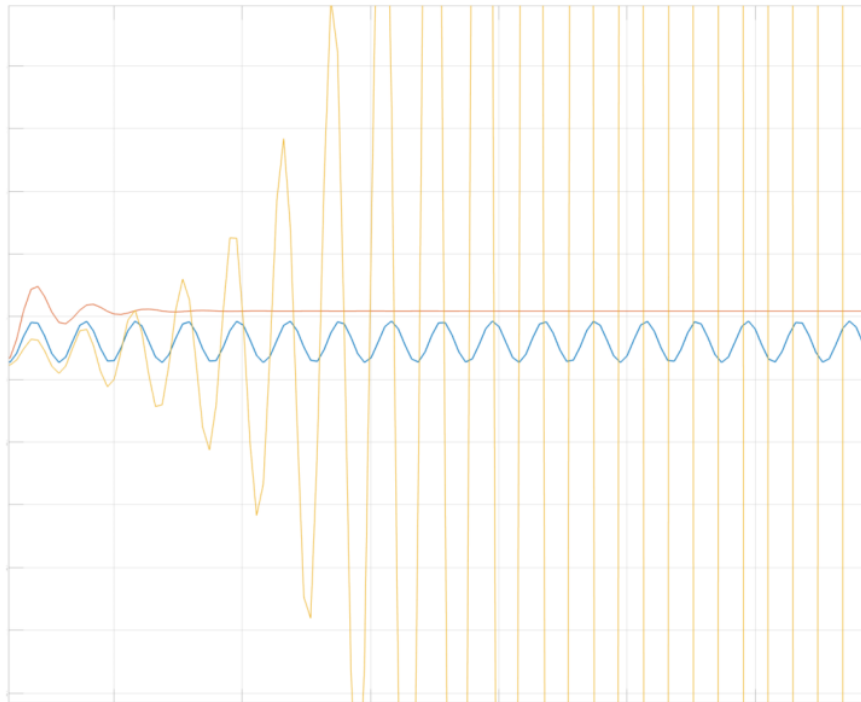


Figure 8 - Control System Modelling: Simulink Responses, Critically Stable, Stable and Unstable

Control System Modelling - Simulink Gain Values		
Gain Value	Expected Response	Simulink Response
3 (Blue)	Critical Stability	Critical Stability
1.5 (Orange)	Stability	Stability
4.5 (Yellow)	Instability	Instability

Table 8 - Control System Modelling: Gain Values, Predicted Response vs Simulink Response

The root-locus diagrams have proved to be a useful method of control-system analysis. It has proved to be a successful method of determining the stability of a system, especially with the use of MATLAB which has the ability to quickly produce a Root-Locus from a transfer function. Root-Locus diagrams are also highly useful due to their ability to find key characteristics of a system, such as the gain at which critical stability, stability and instability occur. Root-Locus' are highly useful for control engineers designing the closed-loop dynamics of a control system.

4. Block Diagram Reduction Method

Figure 9 displays a control system which is constructed of 6 individual transfer blocks which are connected together, some connected by summing points. In order to simplify the system algebraic equations are used to reduce the system down into one transfer function.

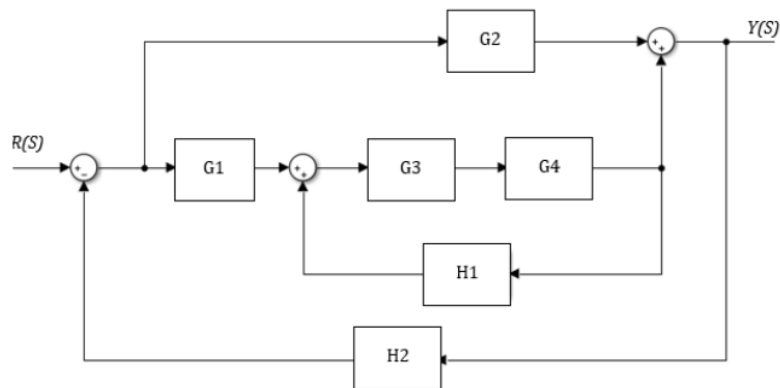


Figure 9 – Block Diagram Reduction Method: Block Diagram Model

Table 9 displays the values chosen for each transfer block.

Block Diagram Reduction Method – Chosen Values	
$G1$	4
$G2$	3
$G3$	$\frac{2}{s+1}$
$G4$	$\frac{1}{s}$
$H1$	$\frac{1}{s+1}$
$H2$	1

Table 9 - Block Diagram Reduction Method: Chosen calculation values

From the equations seen in appendix 8.3, the transfer function was derived:

$$T(s) = \frac{3s^3 + 6s^2 + 11s + 2}{4s^3 + 8s^2 + 12s + 0}$$

The 'tf2zp' function is used for the reduced transfer function to find the systems poles, zeroes and gain. As seen in figure 10, one of the poles derived is at $0.00 + 0.00i$, therefore, it is expected that this system will be critically stable.

```

z =

-0.8991 + 1.5797i
-0.8991 - 1.5797i
-0.2018 + 0.0000i

p =

0.0000 + 0.0000i
-1.0000 + 1.4142i
-1.0000 - 1.4142i

k =

0.7500

```

Figure 10 - Block Diagram Reduction Method: MATLAB calculated Poles, Zeroes and Gain

This is visually proven using the rlocus function which produces a root locus diagram which plots the poles and zeros. The complex conjugates seen at poles $-1 + j1.4142$ and $-1 - j1.412$ would cause a stable response. However, due to the pole location at the point of critical stability at $0.00 + j0.00$ the system will be critically stable (Figure 11)

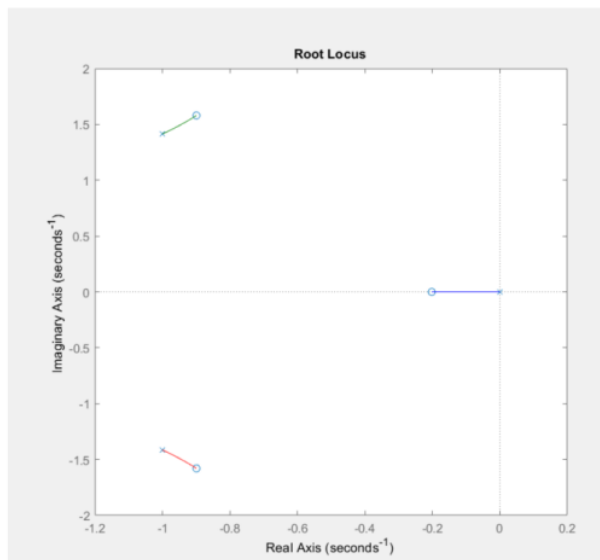


Figure 11 - Block Diagram Reduction Method: MATLAB plotted argand diagram

The theory that this system is critically stable is further confirmed using MATLABs step response, As seen in figure 12 the step response shows that the gradient is constantly increasing and never enters steady state, this happens at a starting amplitude of 0.75, this constantly increasing gradient is the expected response of a critically stable system. It would be expected to enter an oscillating state, however, this cannot be seen on the step response diagram.

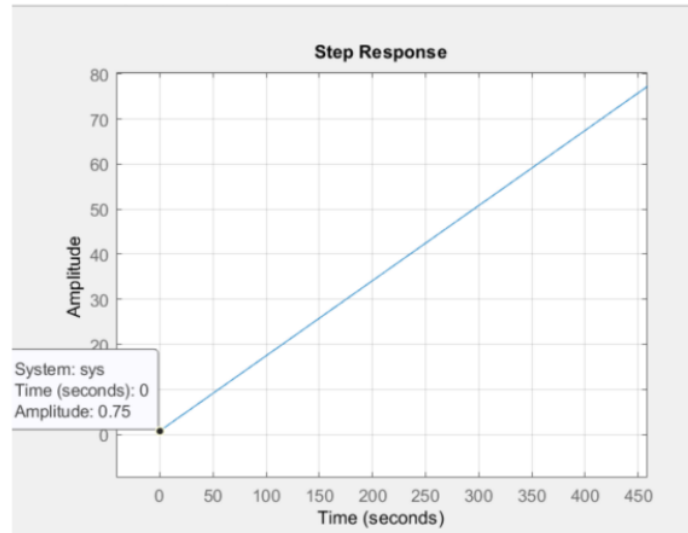


Figure 12 - Block Diagram Reduction Method: Step Response

Figure 13 shows the Simulink model of the control system, this model consists of the original 6 block system, whilst also having the reduced model connected to the same scope in order to prove they create the same step response.

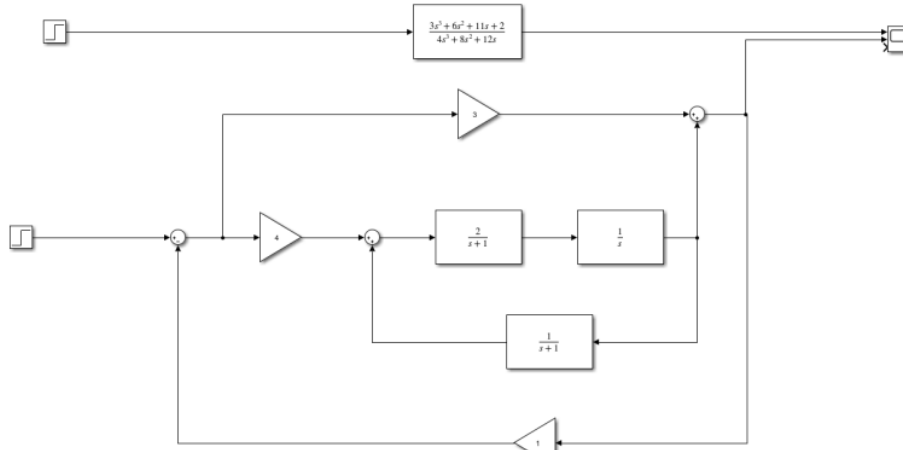


Figure 13 - Block Diagram Reduction Method: Simulink model

Figure 14 displays the step-response of both control systems, as only one response is visible it is clear to see that the original control system and the reduced system produce the same critically stable exponentially rising response starting at an amplitude of 0.75 identical to the step response, in turn proving the calculations correct.

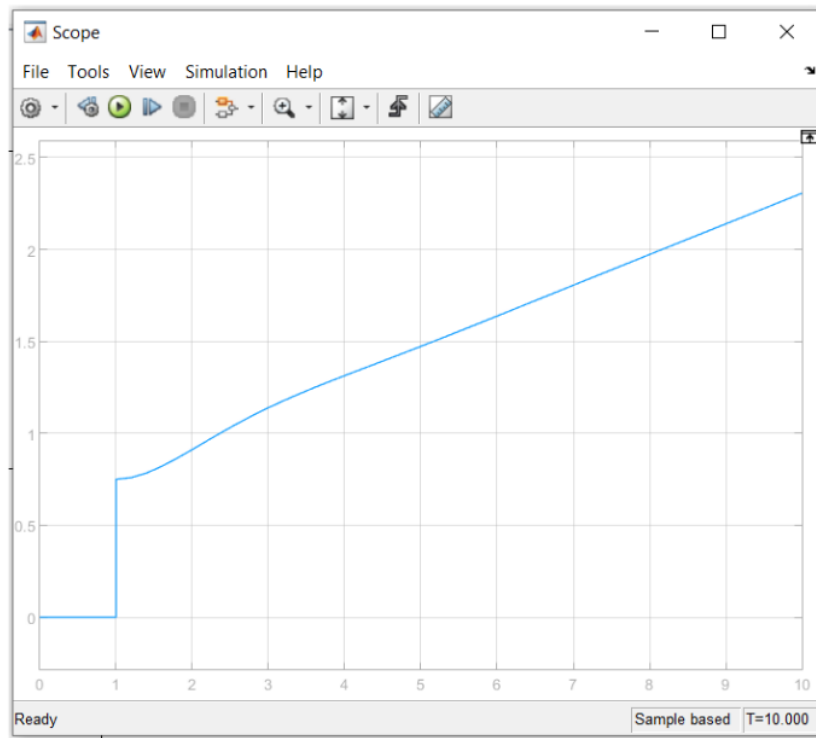


Figure 14 – Block Diagram Reduction Method: Simulink Response of both models

In order to validate the result, the scope was split to display both responses seen in figure 15. It clearly displays both step responses increasing at the same rate, both starting at an amplitude of 0.75 with the original control system response seen in red with the reduced transfer function seen in blue. This proves that the block reduction method is a viable method in order to reduce a complex control system into a single transfer function.

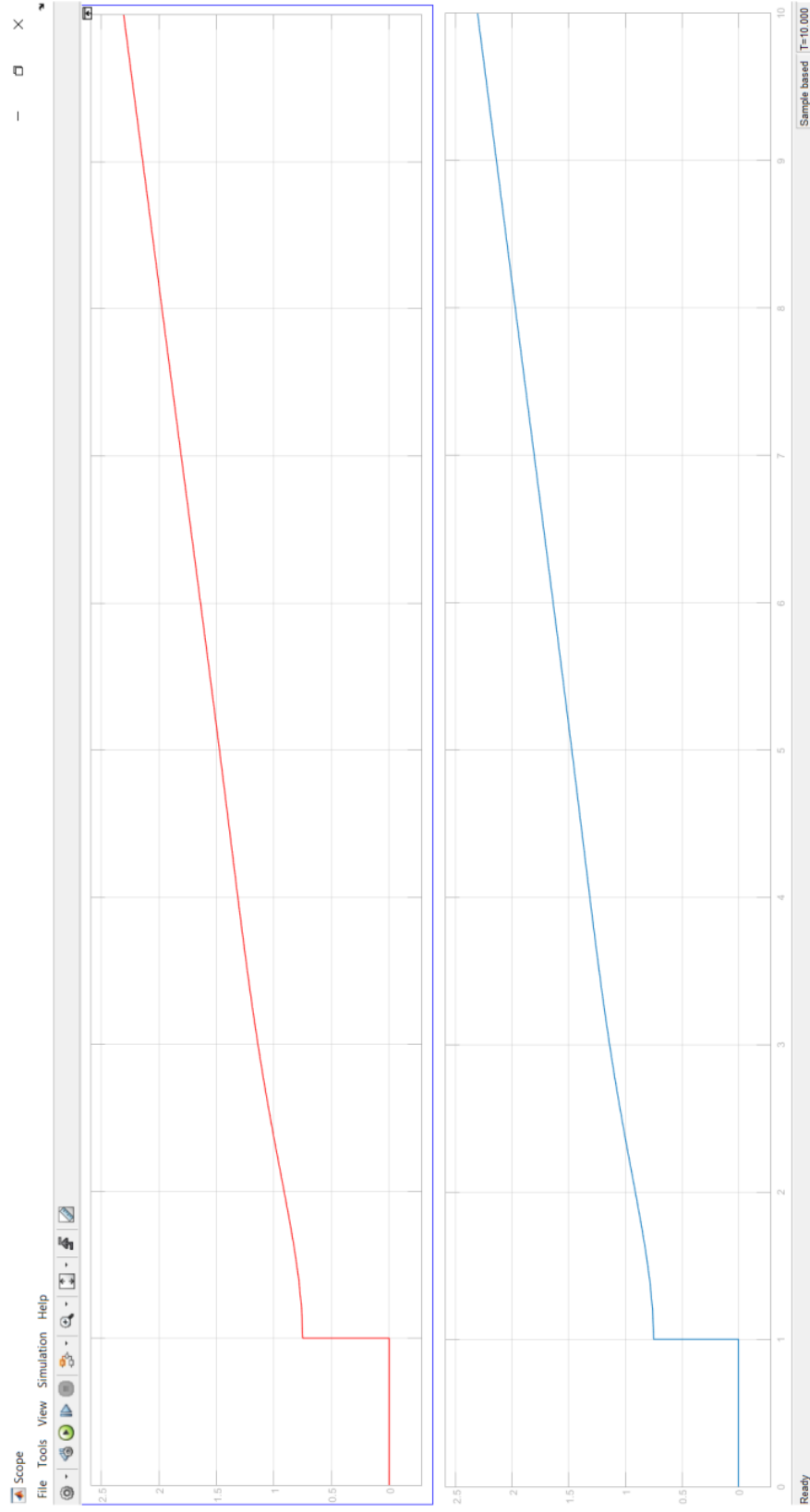


Figure 15 - Block Diagram Reduction Method: Simulink response of both models with split scope

5. Frequency Response Method

A control system has the following transfer function:

$$G(S) = \frac{k(s + p)}{(s + q)(s + r)}$$

The frequency response is defined as the steady state response to a sinusoidal input over a frequency range. In the bode frequency response, 'p', 'q' and 'r' all relate to the frequency curves' phase shift and also to the turning points of the magnitude curve. 'p', 'q' and 'r' are also referred to as ' $\omega c1$ ', ' $\omega c2$ ' and ' $\omega c3$ ' respectively, these are the systems cut-off frequencies. When a ' ω ' is present within the numerator of a transfer function at the frequency of ' ω ' the magnitude increases at a gradient of 20dB/decade in a second order system, however, when a ' ω ' is present in the systems denominator the opposite effect occurs whereby the magnitude falls by 20dB/decade.

As this transfer function is a second order system with one zero and two real poles, it is expected to have a response with one zero and two real poles, therefore, the system is expected to have one roll-up gradient and two roll-off gradients, a cut-off frequency of +3dB is expected for the roll-up gradient whilst a cut-off error of -3dB is expected for both roll-off gradients.

Values for this transfer function were chosen as:

Frequency Response Method – Chosen Calculation Values	
k_1	2000
$p (\omega c1)$	2
$q (\omega c2)$	20
$r (\omega c3)$	200

Table 10 – Frequency Response Method: Chosen Calculation Values

Using the chosen variable values seen in table 10, the systems gain (K) can be calculated:

$$K = \frac{k_1 \times p}{q \times r} = \frac{2000 \times 2}{20 \times 200} = 1$$

This is then calculated in logs for the bode diagram:

$$20 \log_{(10)}(1) = 0$$

The transfer function can also be derived, full calculations are seen in appendix 8.4:

$$G(s) = \frac{2000(s + 2)}{(s + 20)(s + 200)} = \frac{2000s + 4000}{s^2 + 220s + 4000}$$

The transfer function is written in MATLAB, the bode function is used to display a bode graph (Figure 16).

```

Editor - C:\Users\Tom Grennan\A\
Task_5.m x Task5_expected.m
1 - num=[2000 4000];
2 - den=[1 220 4000];
3 - bode(num,den);
4 - grid on

```

Figure 16 - Frequency Response Method: MATLAB code for bode response

Figure 17 shows the Bode Diagram response obtained from the MATLAB program, Asymptotes have been manually drawn in red to display the expected response of the system and the key aspects of the response have been annotated.

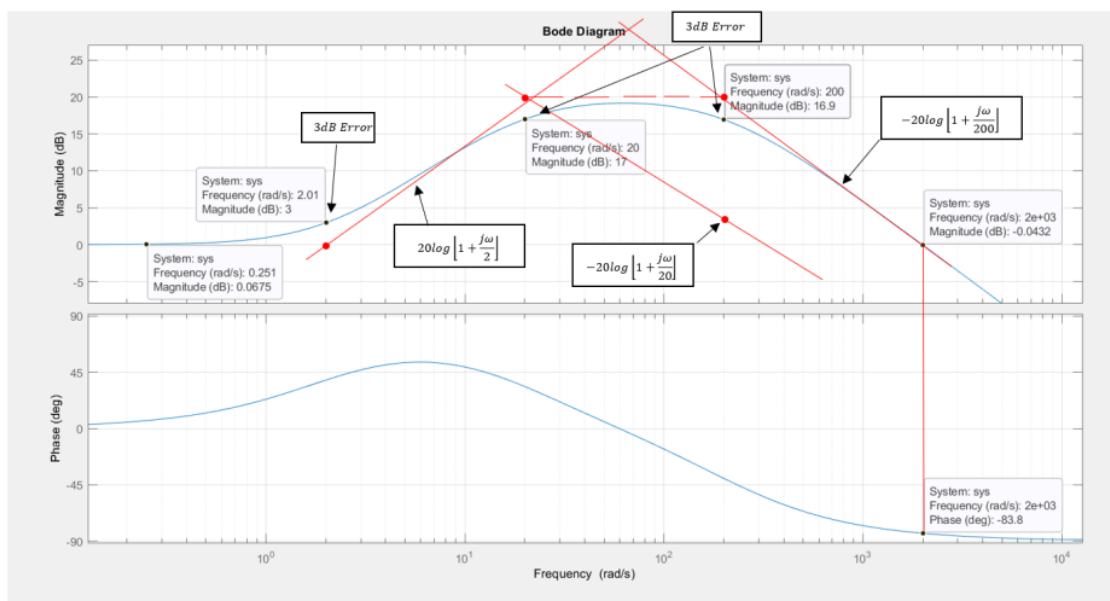


Figure 17 - Frequency Response Method: Annotated bode response

5.1 Analysis of the response

As seen from the bode response in figure 17 and as expected due to the position of the cut-off frequencies within the transfer function the response has one roll-up gradient with two roll-off gradients. It is clearly seen that the Bode diagrams magnitude graph correlates with the calculated gain of 0, this is seen from the initial straight line response along the magnitude of 0dB.

The manually drawn asymptotes predict that there is a roll-up of 20dB/decade from 2 rad/s ($\omega c1$) to 20rad/s ($\omega c2$) as expected for a second order system. The MATLAB response as anticipated inherits the +3dB error at this cut off frequency, this is a positive error as this is the systems numerator, this response is of the form $-20\log\left[1 + \frac{j\omega}{2}\right]$.

A second asymptote is drawn starting at 20rad/s ($\omega c2$), as this is a decade later the asymptotes origin is at 20dB which is expected, as this is a part of the systems denominator this is predicted to have a roll-off gradient of 20dB per decade, this response is of the form $-20\log\left[1 + \frac{j\omega}{20}\right]$. It is seen in the MATLAB response that the origin of the response starts at 17dB due to the -3dB cut-off that is expected.

The third asymptote is drawn at 200rad/s ($\omega c3$), this asymptote originates at 20dB as per prediction, as this is a cut-off frequency the expected -3dB cut-off is seen by the MATLAB marker displaying 17dB, thus proving the cut-off theory correct, this response is of the form $-20\log\left[1 + \frac{j\omega}{200}\right]$.

From the manually drawn asymptotes it was predicted that the bode diagrams peak magnitude would be at 20dB, as seen on the MATLAB diagram the peak was 19.2, an error of 0.8. The reason that the magnitude did not settle at 20dB before it rolled-off is due to the effects of all three cut-off frequencies preventing it from reaching full magnitude.

5.2 Determining the Systems Relative Stability

The stability of a system is determined, this can be done using two possible methods: Gain-margin and Phase-margin. For this instance, as the phase does not fall below -180° , the gain-margin method cannot be used. Therefore, the phase-margin is used. The Phase-Margin method is utilised by measuring the frequency at which the gain is at 0dB, the measured frequency is then plotted on the phase diagram to obtain a phase reading. The phase-reading is then subtracted from -180° as this is the phase margin of stability, in order for the system to be stable the value from the subtraction must be greater than 45° .

As seen in figure 17, the phase-shift at 0dB is at -83.8° , in order to find stability:

$$-180^\circ - (-83.8^\circ) = -96.2^\circ$$

As the phase margin is greater than 45° away from -180° the system is proved stable.

6. Digital Control Systems

A closed-loop Simulink model was created for the Digital Control System as seen in figure 18, a zero-order hold block is used to set the sampling period for the system.

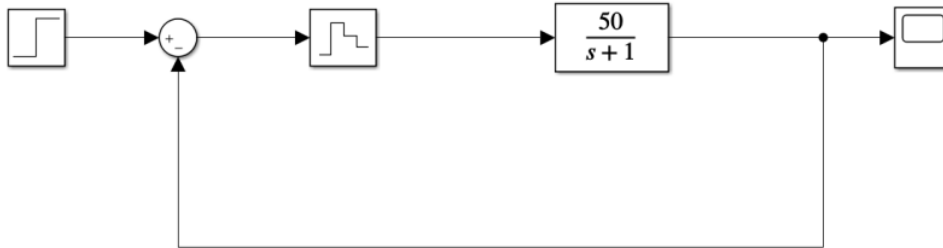


Figure 18 – Digital Control Systems: Digital control system model

The calculations for calculating the minimum sampling period for a stable system are seen in appendix 8.5. It was calculated in order for the system to be stable the sampling time must be 40.00533461 milliseconds or less, to prove this, sampling times for greater than the minimum sampling time and lower than the minimum sampling time will also be used as variables within the model to analyse their response, table 11 shows the values chosen.

Digital Control Systems – Sampling Times	
Minimum Sampling Time	40.00533461ms
Greater than Minimum	50ms
Lower than Minimum	10ms

Table 11- Digital Control Systems: Sampling times

The stability of a digital control system can be visually understood with the use of a 'z-plane'. This is a circle with a radius of 1, anything within the circle is stable, outside the circle is unstable and anything that lies along the line of the circle is at the point of critical stability. As seen in figure 19 50ms is in the unstable region, 10ms is in the stable region whilst 40ms is along the outer region where it is critically stable.

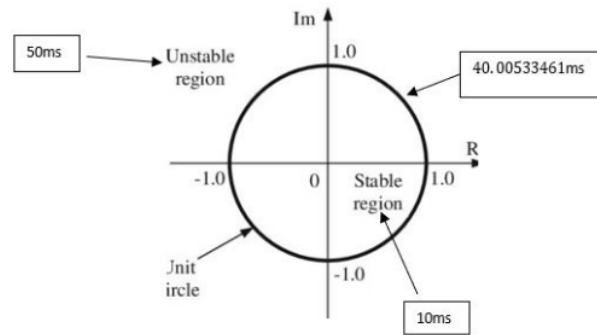


Figure 19 - Digital Control Systems – Z-Plane showing stability of sampling times

The various sampling times were then inserted into the Simulink model, the responses that these sampling times produced are seen in figures 20 - 22.

40.00533461ms – Critically Stable Region

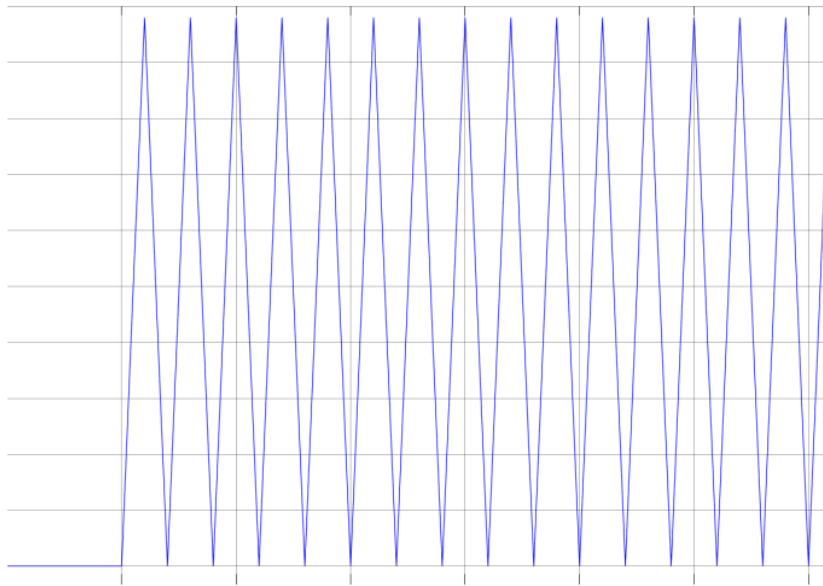


Figure 20 - Digital Control Systems: Critically Stable Response at 40.00533461ms

50ms – Unstable Region

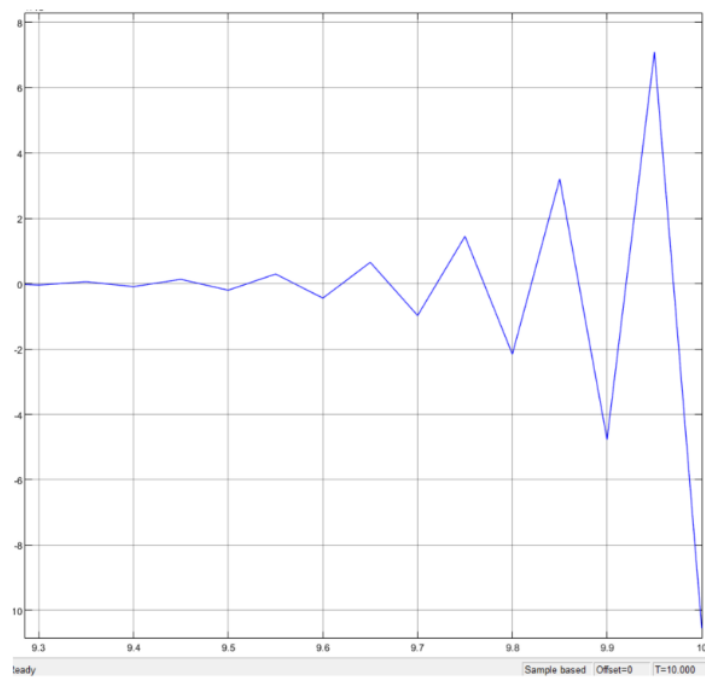


Figure 21 - Digital Control Systems: Unstable Response at 50ms

10ms – Stable Region

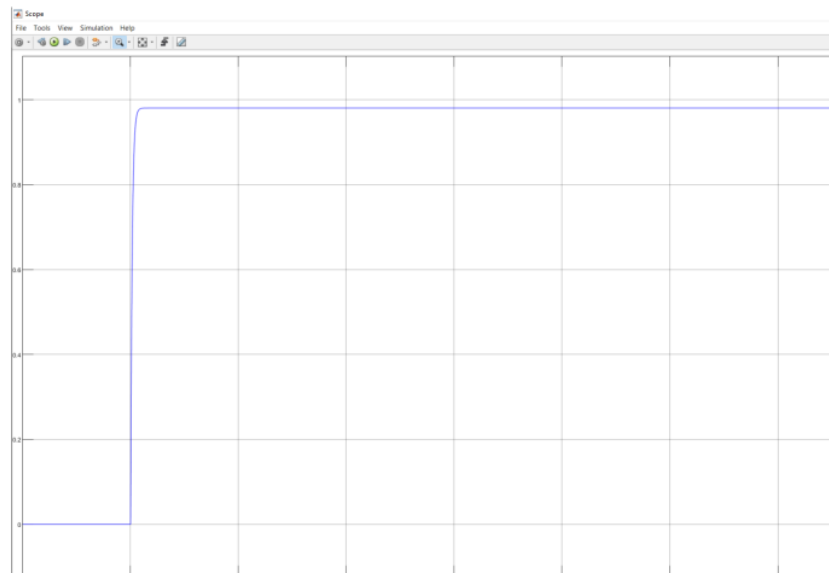


Figure 22 - Digital Control Systems: Stable Response at 10ms

The results of the various responses are compared with the simulated response in table 12.

Digital Control Systems – Results		
Sample Time (ms)	Expected Response	Simulated Response
40.00533461	Critically Stable	Critically Stable
50	Unstable	Unstable
10	Stable	Stable

Table 12 - Digital Control Systems: Results

As seen in figure 20, when the system was at the minimum sampling rate of 40ms it is clearly seen that it is oscillating at a constant frequency and amplitude as expected of a critically stable system, this is due to this sampling rate being along the z-planes area of critical stability, proven by the critically stable response.

When the sampling rate was increased to 50ms which is 10ms greater than the maximum time for a stable response it is clearly seen that the response in figure 21 is unstable as expected as the oscillations increase excitedly with increasing amplitude, this is due to it being within the region of instability of the z-plane.

A sampling time of 10ms produced a stable step response which quickly enters a steady-state as expected of a response of this type, this is due to the sampling time being well within the z-planes region of stability (Figure 22).

7. Overall Conclusions and Recommendations

In conclusion this report has been highly successful. Throughout the report extensive knowledge was obtained on the principles and applications of various methods of control system analysis, in turn greatly improving control system analysis skills on a wide range of applications. The calculations and simulation procedures provided extensive experience and understanding of control system calculation, modelling and analysis. Overall, the calculations and simulations were highly accurate with no real discrepancies when comparing results, all minor discrepancies that were obtained in the results are negligible and the error is accountable due to rounding errors for both MATLAB and mathematical calculations

Throughout the report, Mathematical skills were greatly improved with various topics being covered throughout from basic algebra seen in the first task compared to more complex Laplace transforms seen in the control system modelling. Algebraic skills were vastly improved from deriving the simplest transfer function from a complex control system for the block reduction method.

A key aspect of this report was centred around the stability of control systems, previous knowledge of system stability was built-upon, especially from the analysis of Root Locus and argand diagrams for poles and zeroes, this was found particularly useful due to the visual representation of the control systems stability. It was understood that the Root Locus plots the poles and zeros to depict the systems response and stability, this includes principles of how the position of poles move when the gain is varied. These diagrams provide invaluable information to control engineers trying to obtain a particular system response. Principles on the effect of negative feedback was further understood using argand diagrams, knowledge of how this feedback is applied was also learnt from the modelling of control systems with a great understanding developed on the affect this has on the systems poles and zeroes. Computational skills on MATLAB and Simulink were improved by using the application to produce various diagrams, responses and system models to solidify the theory derived from equations, throughout this report it was also found how useful the MATLAB and Simulink programs are to controls engineers when trialling and designing control systems.

Key concepts about frequency response systems were understood, such as that the frequency response is the steady-state response to a sinusoidal input over a range of frequencies and how the stability of a control system is found in the frequency domain through the use of phase and gain-margin methods. New analysis methods were also learned through the use of Asymptotes which help predict the frequency response of a control system.

New topics that were covered such as the digital control system task opened a new area of control engineering that the author was not familiar with, such as the Z-Plane rules for stability whereby the point of critical stability is at the edge of the Z-planes circumference, with the closer to the circles centre bring more system stability, whereas the further outside of the circumference had an adverse effect. This provided an understanding of how the stability of a digital control system can be controlled.

excellent deductions

8. Appendix

8.1 Transient Response of a Second Order System

Using the formula:

$$\frac{Y(S)}{U(S)} = \frac{G(s)}{1 + G(s)H(s)}$$

Since:

$$G(s) = \frac{20}{s(s+4)} \text{ and } H(s) = 1$$

Then:

$$\frac{Y(S)}{U(S)} = \frac{\frac{20}{s(s+4)}}{1 + \frac{20}{s(s+4)} \times 1} = \frac{20}{s^2 + 4s + 20}$$

This is compared with the standard transfer function for a 2nd order system:

$$T.F = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The Natural Frequency is found as:

$$\begin{aligned}\omega_n^2 &= 20 \\ \omega_n &= 4.47 \text{ rad/s}\end{aligned}$$

The Damping Factor is then calculated:

$$\begin{aligned}2\zeta\omega_n &= 4 \\ \zeta\omega_n &= \frac{4}{2} = 2 \\ \zeta &= \frac{2}{\omega_n} = \frac{2}{4.47} = 0.447\end{aligned}$$

Using the damping factor, the Damped Frequency is calculated:

$$\begin{aligned}\omega &= \omega_n \sqrt{1 - \zeta^2} \\ \omega &= 4.47 \sqrt{1 - 0.447^2} = 3.99 \text{ rad/s}\end{aligned}$$

The rise time from 0 to 100% of the final value is calculated:

$$\frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)}{\omega n \sqrt{1-\zeta^2}} = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-0.447^2}}{0.447}\right)}{4.47 \sqrt{1-0.447^2}} = 0.508 \text{ seconds}$$

The peak time is calculated:

$$t_p = \frac{\pi}{\omega n \sqrt{1-\zeta^2}} = \frac{\pi}{4.47 \sqrt{1-0.447^2}} = 0.786 \text{ seconds}$$

The percentage of overshoot is calculated:

$$100e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} = 100e^{-\left(\frac{0.447 \times \pi}{\sqrt{1-0.447^2}}\right)} = 20.807\%$$

The settling time within 2% is calculated:

$$t_s = 2\% = \frac{4}{\zeta\omega n} = \frac{4}{0.447 \times 4.47} = 2.002 \text{ Seconds}$$

The number of oscillations during settling time is calculated:

$$\pm 2\% = n = \frac{2}{\pi} \sqrt{\frac{1}{\zeta^2} - 1} = \frac{2}{\pi} \sqrt{\frac{1}{0.447^2} - 1} = 1.274$$

Figure 23 shows the code written in MATLAB for the derived transfer function along with the step response of this system.

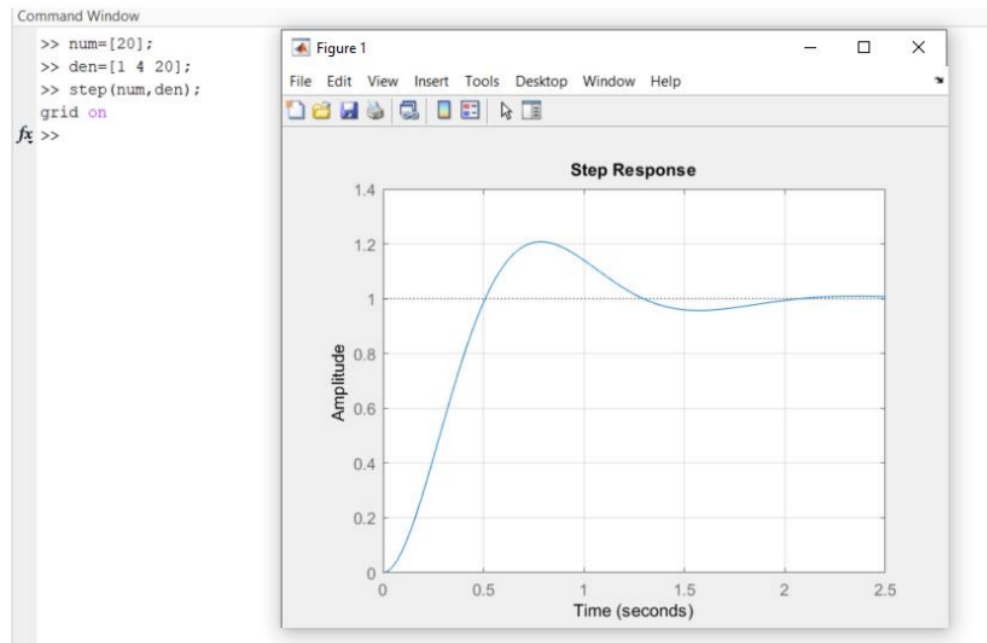


Figure 23 – Appendix: Transient Response of a Second Order System: MATLAB code and response

8.2 Control System Modelling

The Laplace Transform for dy is seen as:

$$\mathcal{L} \left\{ \frac{d^2 y}{dt^2} + a \frac{dy}{dt} + by \right\} = s^2 Y(s) + as Y(s) + bY(s), \text{ where } Y(0) = 0$$

The Laplace Transform for dx is seen as:

$$\mathcal{L} \left\{ \frac{d^2 x}{dt^2} - p \frac{dx}{dt} + qx \right\} = s^2 X(s) - ps X(s) + qX(s), \text{ where } X(0) = 0$$

With the chosen values entered these equations become:

$$\mathcal{L} \left\{ \frac{d^2 y}{dt^2} + 3 \frac{dy}{dt} + 1y \right\} = s^2 Y(s) + 3s Y(s) + Y(s), \text{ where } Y(0) = 0$$

And:

$$\mathcal{L} \left\{ \frac{d^2 x}{dt^2} - 1 \frac{dx}{dt} + 3x \right\} = s^2 X(s) - 1s X(s) + 3X(s), \text{ where } X(0) = 0$$

The equation becomes:

$$s^2 Y(s) + 3s Y(s) + Y(s) = s^2 X(s) - 1s X(s) + 3X(s)$$

This is factorised to become:

$$Y(s)[s^2 + 3s + 1] = X(s)[s^2 - 1s + 3]$$

∴

$$\frac{Y(s)}{X(s)} = \frac{s^2 - 1s + 3}{s^2 + 3s + 1}$$

∴

$$G(s) = \frac{s^2 - 1s + 3}{s^2 + 3s + 1}$$

The systems poles are found:

$$s = \frac{-b \pm \sqrt{(b)^2 - 4(ac)}}{2(a)} = \frac{-3 \pm \sqrt{(3)^2 - 4(1 \times 1)}}{2(1)} = -0.3819, -2.6180$$

The systems Zeros are found:

$$s = \frac{-b \pm \sqrt{(b)^2 - 4(ac)}}{2(a)} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1 \times 3)}}{2(1)} = 0.5 \pm j 1.658$$

The poles and zeros are plotted onto an argand diagram as seen in figure 24. It is clearly seen that the poles lay along the real-axis within the left-hand plane, this provides the indication that the system will be stable. Due to the poles being along the real-axis it is expected that the system response would be overdamped.

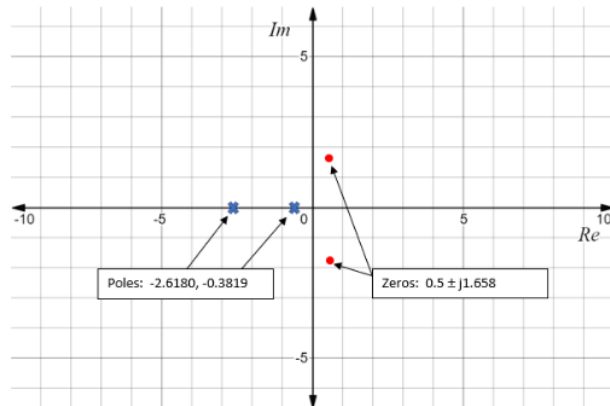


Figure 24 – Appenaix - Control System Modelling: Manually Plotted Argand Diagram

This theory is confirmed by comparing the systems denominator to the standard form equation:

$$s^2 + 3s + 1 = s^2 + 2\zeta\omega_n s + \omega_n^2$$

From this, the natural frequency is calculated:

$$\omega n^2 = 1 \text{ rad/s},$$

Therefore:

$$\omega n = \sqrt{1} = 1$$

The damping factor is also calculated:

$$2\zeta\omega n = 3$$

∴

$$\zeta = \frac{3}{2 \times 1} = 1.5$$

From the calculation seen above, a damping factor of 1.5 is overdamped, this coincides with the root locus diagram seen in figure 24. To further confirm this theory, a step response is performed in MATLAB for the calculated transfer function.

The MATLAB code written for this function is seen in figure 25, with the response seen in figure 26. The response seen is overdamped, even though there is a drop at the start of the response, as it enters steady state it does not oscillate.

```

1 - s = tf('s');
2 - system1 = (s^2 - 1*s + 3)/(s^2 + 3*s + 1)
3 - pole(system1)
4 - zero(system1)
5 - grid on
6 - hold off
7 - step(system1)

```

Figure 25 – Appendix - Control System Modelling: Step Response MATLAB Code

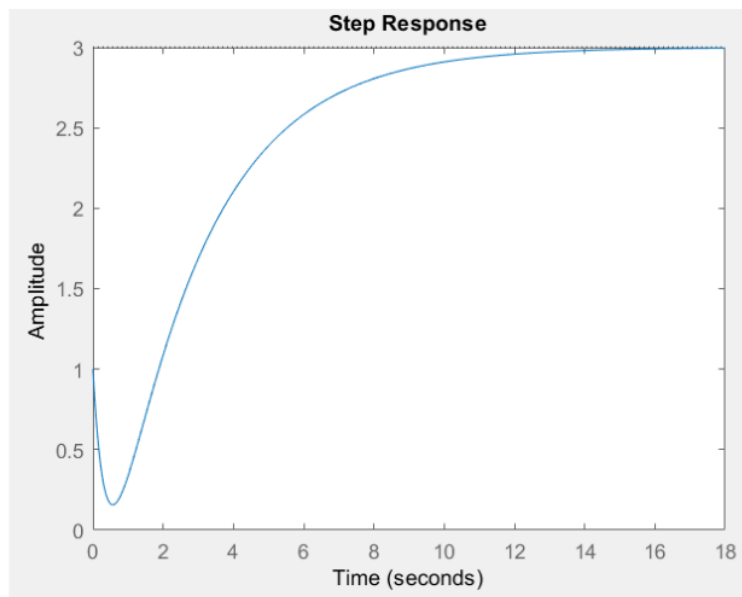


Figure 26 -- Appendix - Control System Modelling: Overdamped Step Response

A root locus response was then performed for the transfer function within MATLAB. The MATLAB code for this is seen in figure 27.

```
task_3_step_input.m x +
1 - s = tf('s');
2 - system1 = (s^2 - 1*s + 3)/(s^2 + 3*s + 1)
3 - pole(system1)
4 - zero(system1)
5 - rlocus(system1)
6 - grid on
7 - hold off
```

Figure 27 -- Appendix - Control System Modelling: Root Locus Response MATLAB Code

From this root locus diagram the calculated values for the poles and zeros can be verified, these are seen in figure 28. It is seen that the systems poles are in the left-hand plane along the real axis at -0.382 and -2.62 , whilst the zeros are within the right-hand plane at $0.5 \pm j 1.658$. These are exactly correct to the calculated values but have been rounded to 2d.p.

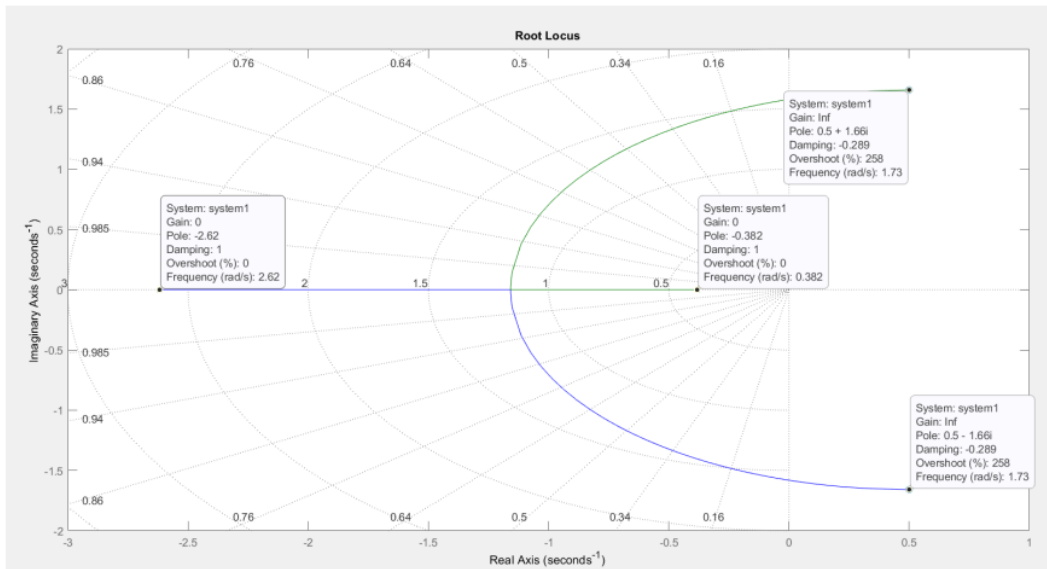


Figure 28 - Appendix - Control System Modelling: Root Locus Response displaying calculated poles and zeroes

The root-locus diagrams can be used to find other key aspects of the control system, such as the gain value at critical damping, critical stability, stability and instability. Figure 29 shows the root-locus response of this control system with data-pointers at the point of critical damping. Here it is seen that the point of critical damping is at a gain of 0.206, the point of critical damping is when the damping factor is equal to 1 ($\zeta = 1$), this is seen at a gain of 0.206 in figure 29.

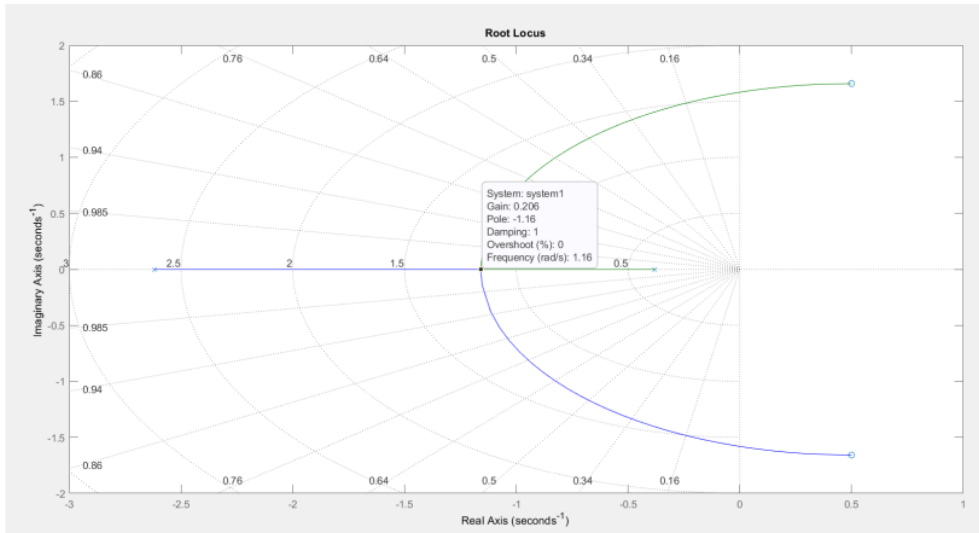


Figure 29 – Appendix - Control System Modelling: Root Locus Response displaying point of critical damping

This can be mathematically proven using the new calculated transfer function by entering 0.206 as k.

$$T(s) = s^2 + \frac{(3 - 1(0.206))}{(1 + (0.206))}s + \frac{(1 + 3(0.206))}{(1 + (0.206))}$$

$$T(s) = s^2 + 2.3167s + 1.3416$$

This is compared to the general transfer function of the denominator ' $s^2 + 2\zeta\omega_n s + \omega_n^2$ ':

$$\omega_n^2 = 1.3416, \text{ therefore, } \omega_n = \sqrt{1.3416} = 1.1582$$

And:

$$2\zeta\omega_n s = 2.3167, \text{ therefore, } \zeta = \frac{2.3167}{2\sqrt{1.1582}} = 1.07$$

From this calculation it proves the accuracy of this calculation, the damping factor was calculated as 1.07 and the natural frequency at 1.158. As seen in figure 29 the data pointer shows that the damping factor at critical damping is 1 with a frequency of 1.16. These are highly accurate with the minor errors due to rounding, in turn proving MATLAB's root locus accuracy.

The root locus diagram in MATLAB can also be used to find the point of critical stability, Figure 30 displays the value of critical stability on the root locus diagram. The point of critical stability is at the point of 0 on the real axis, at this point the gain can be seen at a value of 3. From this diagram the points of stability and instability can be determined, the system is stable when its gain value is less than 3 and the system is unstable when its gain value is greater than 3.

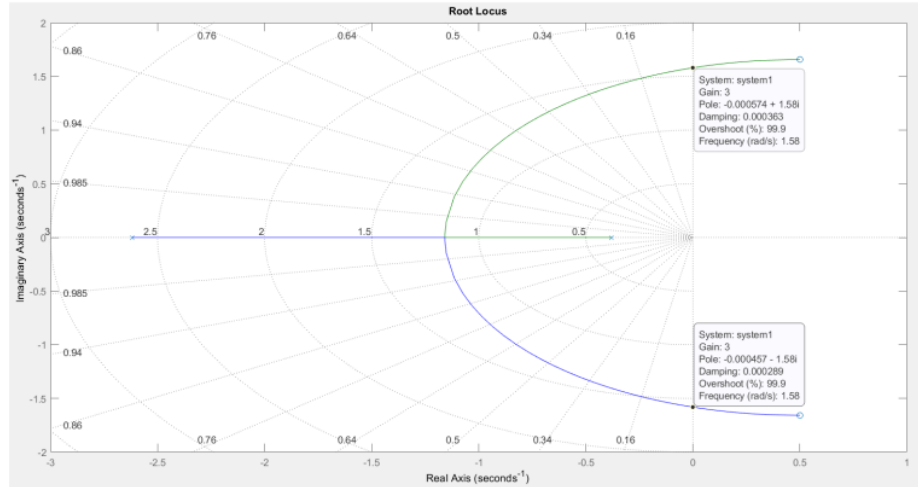


Figure 30 – Appendix - Control System Modelling: Root Locus Response displaying point of critical damping

The point of critical stability can also be mathematically derived, this is seen below. In order to add negative feedback to the system a new transfer function $T(s)$ for the closed loop system is derived:

$$T(s) = \frac{G(s)}{1 + G(s) + H(s)} = \frac{\frac{s^2 - 1s + 3}{s^2 + 3s + 1}}{1 + K \times \left(\frac{s^2 - 1s + 3}{s^2 + 3s + 1}\right)}$$

∴

$$T(s) = \frac{s^2 - 1s + 3}{(s^2 + 3s + 1) + K \times (s^2 - 1s + 3)} = \frac{s^2 - 1s + 3}{s^2 + 3s + 1 + Ks^2 - 1Ks + 3K}$$

∴

$$T(s) = \frac{s^2 - 1s + 3}{(1 + K)s^2 + (3 - 1K)s + (1 + 3K)}$$

Factor by (1+K):

$$T(s) = \frac{1}{(1+k)} \frac{s^2 - 1s + 3}{s^2 + \frac{(3-1K)}{(1+k)}s + \frac{(1+3K)}{(1+k)}}$$

Using the derived equation seen below, values of K can be theoretically calculated as the equations variable in order to prove the validity of the root locus. The numerator is ignored and the denominator becomes the transfer function:

$$T(s) = s^2 + \frac{(3-1k)}{(1+k)}s + \frac{(1+3k)}{(1+k)}$$

∴

$$s^2 + \frac{(3-1k)}{(1+k)}s + \frac{(1+3k)}{(1+k)} = 0, \text{ therefore, } \frac{(3-1k)}{(1+k)}s = 0$$

Therefore:

$$3 - 1k = 0, \text{ therefore, } K = 3$$

These figures of critical stability, stability and instability can be confirmed by monitoring their response to a step-input, this is utilised by using a closed-loop control system with negative feedback in Simulink. Firstly, the original transfer function was created in Simulink with no negative feedback, due to this it is expected that the step response will be overdamped, this is confirmed in figure 31.

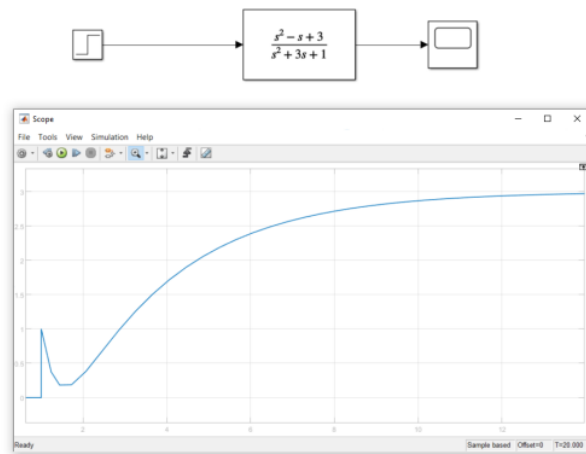


Figure 31 – Appendix - Control System Modelling: Simulink Overdamped Response

Negative feedback is then added for the control system using a summing point, three systems are cascaded with each of their negative feedback outputs being connected to the same scope. Values of gain were applied as negative feedback, one value of gain for a critically stable response (3), one value of gain for a stable response (1.5) and one value of gain for an unstable response (4.5), this system is seen in figure 7.

In order to further solidify the theory of the system, another control system was created in Simulink. The gain value applied to the negative feedback is 0.206, this is the value calculated in order to obtain a damping factor of 1, this was also seen as the point of critical damping on the root-locus diagram seen in figure 32. Figure 32 clearly displays a critically damped response; this is seen as the step-response is quick to reach steady-state with no oscillations or overshoot.

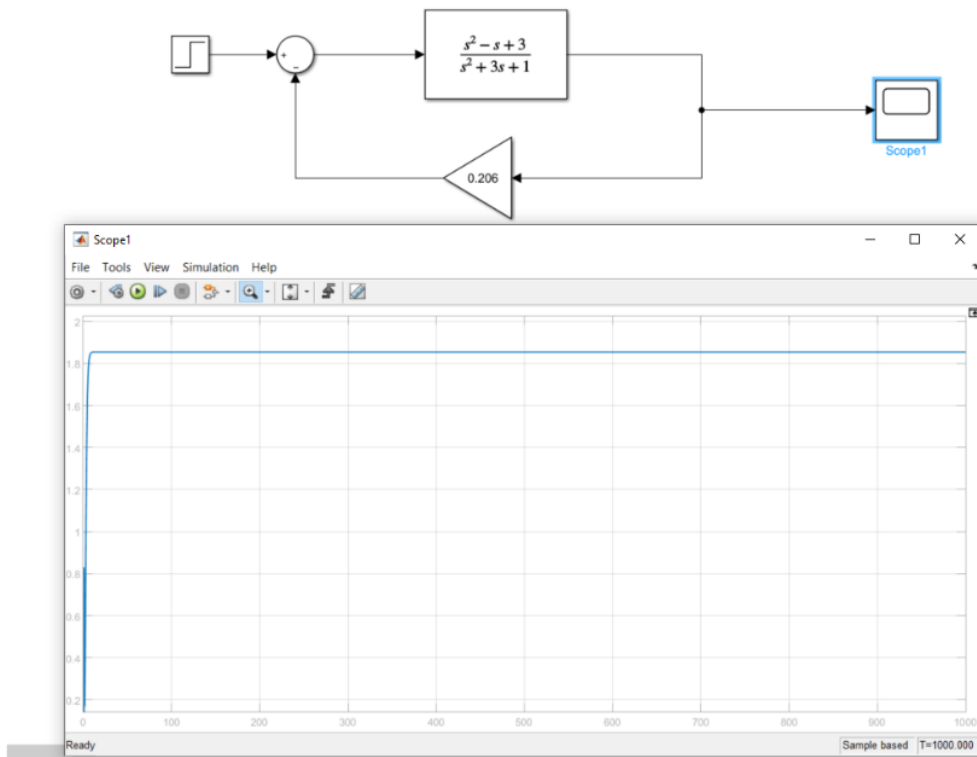


Figure 32 - Appendix - Control System Modelling: Simulink Critical Damping Response

8.3 Block Diagram Reduction Method

The first stage of the block reduction method is to reduce the control system down to a single transfer function, this is done most efficiently by reducing a block at a time in order to reduce potential errors, the original block diagram is seen in figure 33.

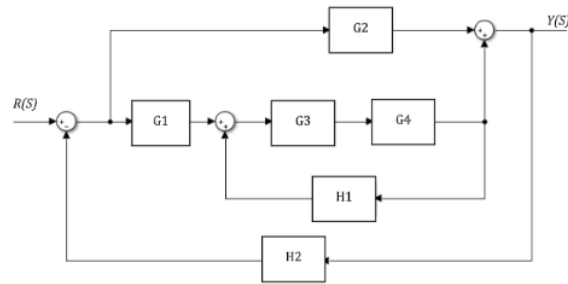


Figure 33 - Appendix – Block Diagram Reduction Method: Stage 1 Reduction

Firstly, as blocks G3 and G4 are in series they can be multiplied to become G5, therefore $G5 = G3 \times G4$. The reduced diagram is seen in figure 34.

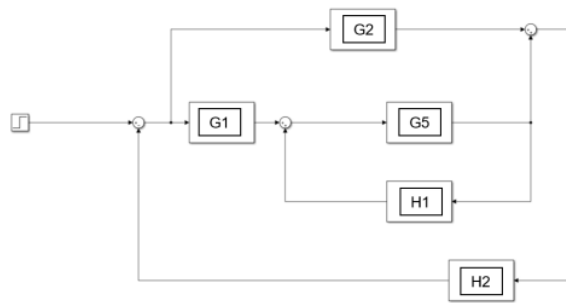


Figure 34 - Appendix – Block Diagram Reduction Method: Stage 2 Reduction

As H1 and G5 are in parallel, they can be reduced down to become G6, therefore $G6 = \frac{G5}{1 - G5 \times H1}$, The reduced diagram is seen in figure 35.

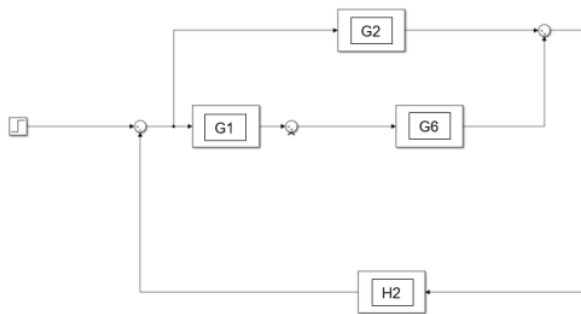


Figure 35 - Appendix – Block Diagram Reduction Method: Stage 3 Reduction

In order to create G7 a common denominator is created, this creates G7 as seen in figure 36. $G7 = \frac{G1 \times G6 + G2}{G1}$.

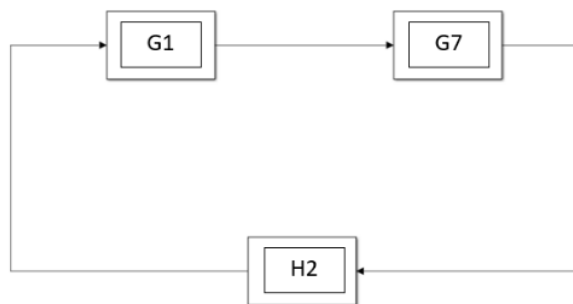


Figure 36 - Appendix – Block Diagram Reduction Method: Stage 4 Reduction

Figure 37 shows the system reduced into its most basic form by creating G8, as G1 and G7 are in series they are multiplied. Therefore, $G8 = G1 \times G7$

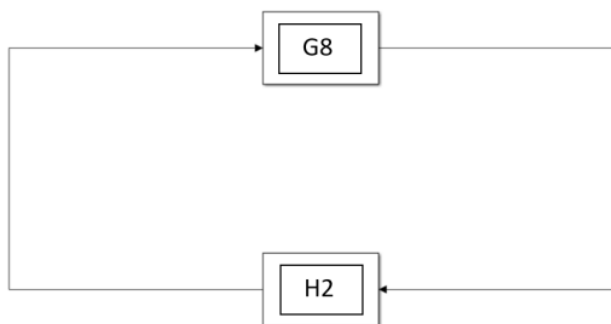


Figure 37 - Appendix – Block Diagram Reduction Method: Stage 5 Reduction

As the block diagram has been reduced down to one block, the full equation for the transfer function can be derived.

$$T = \frac{G8}{1 + G8 \times H2}$$

As $G8 = G1 \times G7$:

$$T = \frac{G1 \times G7}{1 + G1 \times G7 \times H2}$$

As $G7 = G6 + \frac{G2}{G1}$, a common denominator is created as $G7 = \frac{G1 \times G6 + G2}{G1}$, therefore:

$$T = \frac{\frac{G1(G1 \times G6 + G2)}{G1}}{1 + G1 \times H2 \left(\frac{G1 \times G6 + G2}{G1} \right)}$$

This can be further simplified by cancelling out $G1$:

$$T = \frac{G1 \times G6 + G2}{1 + H2(G1 \times G6 + G2)} = \frac{G1 \times G6 + G2}{1 + H2 \times G1 \times G6 + H2 \times G2}$$

As $G6 = \frac{G5}{1 - G5 \times H1}$:

$$T(s) = \frac{G1 \times \left(\frac{G5}{1 - G5 \times H1} \right) + G2}{1 + H2 \times G1 \left(\frac{G5}{1 - G5 \times H1} \right) + H2 \times G2}$$

This is simplified by reducing the fractions by multiplying by $1 - G5 \times H1$:

$$T = \frac{G1 \times G5 + G2(1 - G5 \times H1)}{(1 - G5 \times H1) + H2 \times G1 \times G5 + H2 \times G2(1 - G5 \times H1)}$$

The equation is then reduced to its final form, this is as $G5 = G3 \times G4$:

$$T = \frac{G1 \times G3 \times G4 + G2 (1 - G3 \times G4 \times H1)}{(1 - G3 \times G4 \times H1) + H2 \times G1 \times G3 \times G4 + H2 \times G2 (1 - G3 \times G4 \times H1)}$$

The simplest form:

$$T = \frac{G1 \times G3 \times G4 + G2 - G2 \times G3 \times G4 \times H1}{1 - G3 \times G4 \times H1 + H2 \times G1 \times G3 \times G4 + H2 \times G2 - H2 \times G2 \times G3 \times G4 \times H1}$$

The values chosen for this system are seen in table 13.

Block Diagram Reduction Method – Chosen Values	
$G1$	4
$G2$	3
$G3$	$\frac{2}{s+1}$
$G4$	$\frac{1}{s}$
$H1$	$\frac{1}{s+1}$
$H2$	1

Table 13 - Appendix – Block Diagram Reduction Method: Chosen Values

These values are substituted into the equation:

$$T = \frac{4 \times \frac{2}{s+1} \times \frac{1}{s} + 3 - 3 \times \frac{2}{s+1} \times \frac{1}{s} \times \frac{1}{s+1}}{1 - \frac{2}{s+1} \times \frac{1}{s} \times \frac{1}{s+1} + 1 \times 4 \times \frac{2}{s+1} \times \frac{1}{s} + 1 \times 3 - 1 \times 3 \times \frac{2}{s+1} \times \frac{1}{s} \times \frac{1}{s+1}}$$

This is simplified as:

$$T = \frac{\frac{8}{s(s+1)} + 3 - \frac{6}{s(s+1)^2}}{1 - \frac{2}{s(s+1)^2} + \frac{8}{s(s+1)} + 3 - \frac{6}{s(s+1)^2}}$$

This can be further simplified to:

$$T = \frac{8(s+1) + 3s(s+1)^2 - 6}{s(s+1)^2 - 2 + 8(s+1) + 3(s+1)^2 - 6}$$

The brackets are then expanded:

$$T = \frac{8s + 8 + 3s^3 + 3s^2 + 3s^2 + 3s - 6}{s^3 + s^2 + s^2 + s - 2 + 8s + 8 + 3s^3 + 3s^2 + 3s^2 + 3s - 6}$$

This equation is then further simplified to find the final transfer function:

$$T(s) = \frac{3s^3 + 6s^2 + 11s + 2}{4s^3 + 8s^2 + 12s + 0}$$

8.4 Frequency Response Method

A control system has the following transfer function:

$$G(S) = \frac{k(s + p)}{(s + q)(s + r)}$$

Values for 'p', 'q' and 'r' are chosen as:

$$p = 3, q = 30 \text{ and } r = 300$$

The overall gain is calculated using the value of K in the equation:

$$K = \frac{k \times p}{q \times r}$$

Therefore, for an overall gain of 1:

$$1 = \frac{k \times 2}{20 \times 200}$$

Therefore:

$$2k = 4000$$

∴

$$\frac{k}{2} = 2000$$

The transfer function is now written as:

$$G(s) = \frac{2000(s + 2)}{(s + 20)(s + 200)} = \frac{2000s + 4000}{s^2 + 220s + 4000}$$

8.5 Digital Control System

The Laplace transform for the zero-order-hold block was firstly derived:

$$\mathcal{L}\{ZOH\} = \frac{1 - e^{-sT}}{s}$$

The open loop transfer becomes:

$$G(s) = \frac{1 - e^{-sT}}{s} \times k \times \frac{50}{(s+1)}$$

Since $Z = e^{sT}$:

$$G(Z) = Z\{G(s)\} = Z\left\{\left(\frac{1 - e^{-sT}}{s}\right) \times \frac{50k}{(s+1)}\right\} = (1 - Z^{-1}) \times k \times Z\left\{\frac{50}{(s+1)}\right\}$$

Partial fractions are used in order to find $Z\left\{\frac{50}{(s+1)}\right\}$:

$$\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$\therefore$

$$\frac{1}{s(s+1)} = \frac{A(s+1) + B(s)}{s(s+1)}$$

$\therefore$

$$A(s+1) + B(s) = 1$$

When $s = 0$:

$$A(0+1) + B(0) = 1$$

$\therefore$

$$A = 1$$

When $s = -1$:

$$A(-1+1) + B(-1) = 1$$

$\therefore$

$$-B = 1, \quad \text{therefore } B = -1$$

$\therefore$

$$\frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$$

∴

$$Z\left\{\frac{1}{s(s+1)}\right\} = Z\left\{\frac{1}{s} - \frac{1}{s+1}\right\}$$

From the Z-transform table:

$$1\left\{\frac{Z}{Z-1} - \frac{Z}{Z-e^{-1T}}\right\}$$

∴

$$G(Z) = (1 - Z^{-1}) \times 50 \times Z \left[\frac{Z - e^{-t} - Z + 1}{(Z-1)(Z-e^{-t})} \right]$$

∴

$$G(Z) = \frac{50(1 - e^{-t})}{Z - e^{-t}}$$

The closed-loop transfer function is:

$$T(Z) = \frac{G(Z)}{1 + G(Z)} = \frac{50(1 - e^{-t})}{\frac{(Z - e^{-t})}{1 + \frac{50(1 - e^{-t})}{Z - e^{-t}}}}$$

∴

$$T(Z) = \frac{50(1 - e^{-t})}{Z - e^{-t} + 50(1 - e^{-t})} = \frac{50(1 - e^{-t})}{Z + 50 - 51e^{-t}}$$

The poles are found when:

$$Z + 50 - 51e^{-t} = 0$$

When:

$$Z = 51e^{-t} - 50$$

For stability:

$$-1 < Z < +1$$

For Z = +1:

$$51e^{-t} - 50 = 1$$

$$e^{-t} = \frac{51}{51} = 1$$

Taking logs base e:

$$-T = \ln(1)$$

$$-T = 0$$

$$T = 0$$

For Z = -1

$$51e^{-t} - 50 = -1$$

∴

$$e^{-t} = \frac{50-1}{51} = 0.9607843137254901$$

Take natural $\log_e$:

$$\ln(0.9607843137254901) = -0.04000533461$$

$$-T = -0.04000533461$$

$$T = 0.04000533461$$

Therefore:

$$T = 40ms, \text{ which is the minimum sampling period}$$

Therefore, the minimum sampling frequency for a stable response is calculated as:

$$\text{Minimum Sampling Period} = \frac{1}{0.04000533461} = 25Hz$$

A closed-loop Simulink model was created for the Digital Control System , a zero-order hold block is used to set the sampling period for the system, this system and zero-order hold parameters are seen in figure 38.

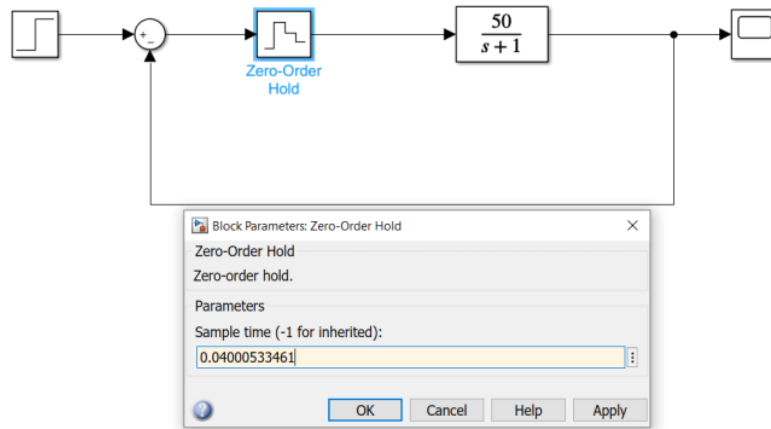


Figure 38 - Appendix – Digital Control Systems: Simulink Model with ZOH value for critical stability

Figure 39, shows the response at the sample time for critical stability, the waveforms cannot be seen but it clearly shows a critically stable response as the amplitude has stayed at a constant peak.

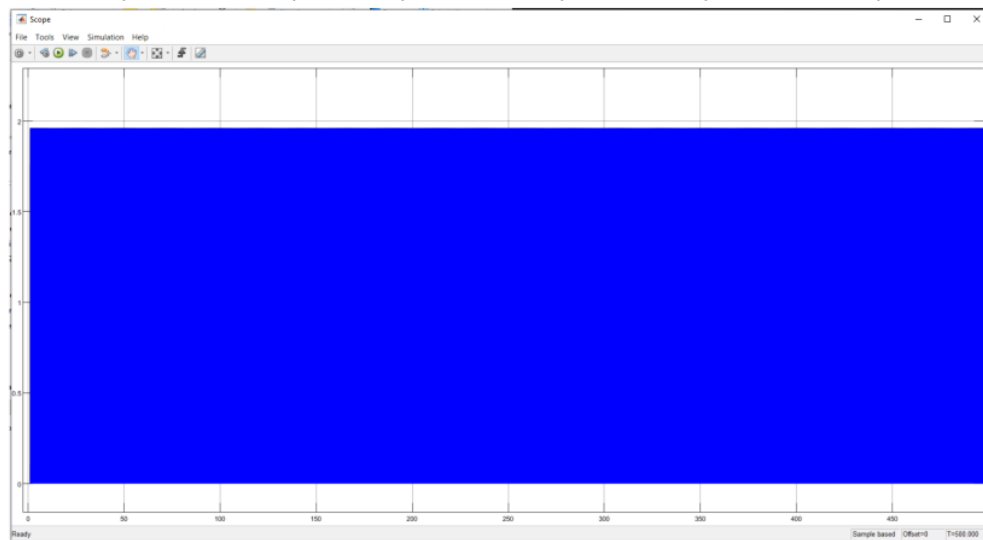


Figure 39 - Appendix – Digital Control Systems: Critical Stable Response

In order to achieve instability, the minimum sampling time was increased to 50ms (0.05s). As this is greater than the minimum sampling time for critical stability the response will be unstable (Figure 40).

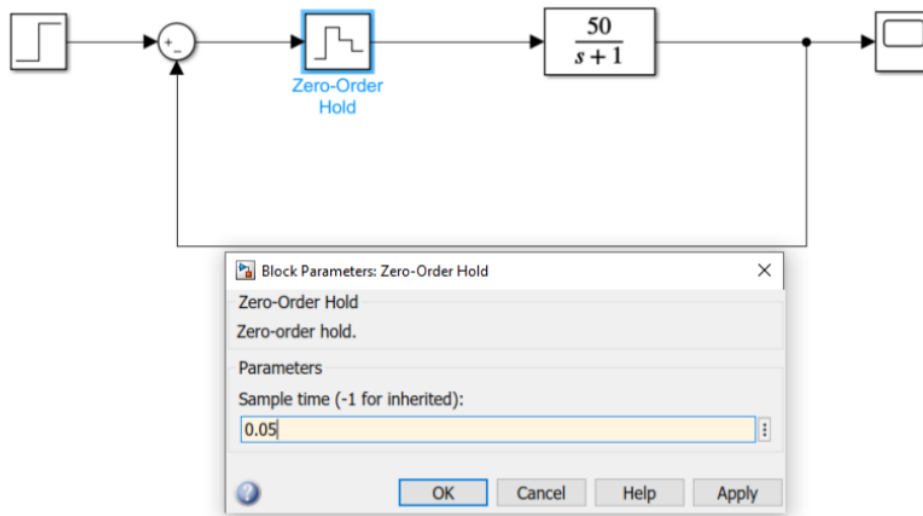


Figure 40 - Appendix – Digital Control Systems: Simulink Model with ZOH value for instability

Figure 41, shows the response at the sample time for instability, the waveforms can be seen to be quickly increasing in amplitude uncontrollably. This is the expected outcome of an unstable response.



Figure 41 - Appendix – Digital Control Systems: Unstable Response

In order to achieve stability, the minimum sampling time was decreased to 10ms (0.01s). As this is lower than the minimum sampling time for critical stability the response will be stable (Figure 42).

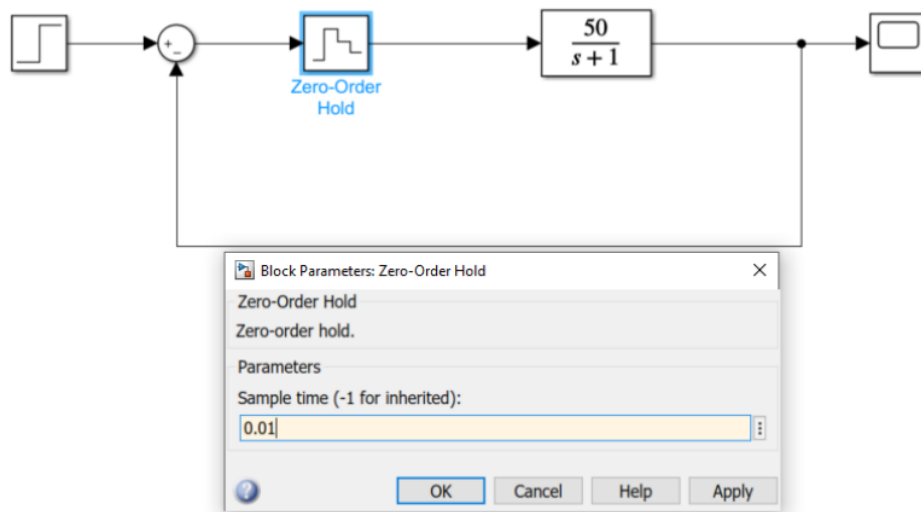


Figure 42 - Appendix – Digital Control Systems: Simulink Model with ZOH value for stability

Figure 43, shows the response at the sample time for stability, the waveforms can be seen to be quickly increasing in amplitude and reaching steady state. This is the expected outcome of a stable response.

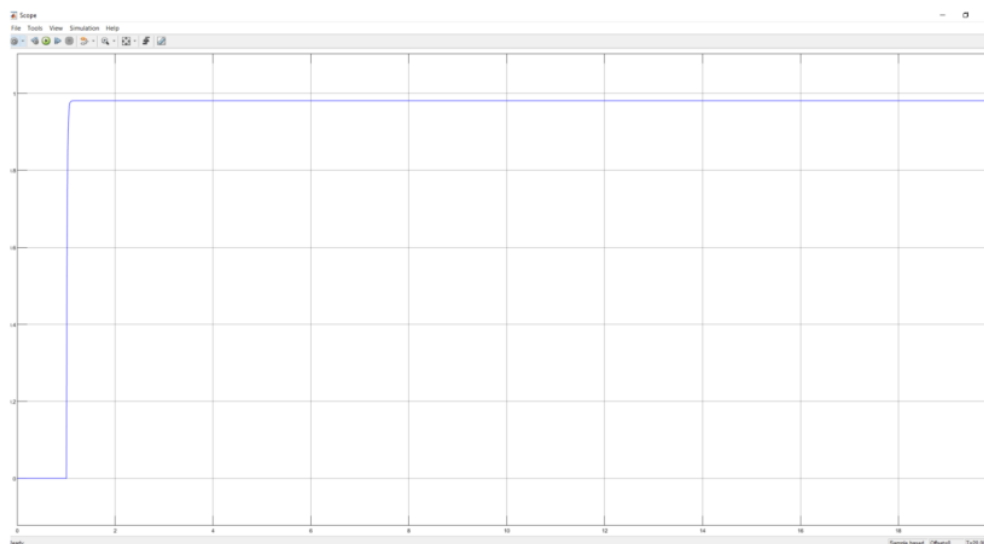


Figure 43 - Appendix – Digital Control Systems: Stable Response

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GRADEMARK REPORT

FINAL GRADE

GENERAL COMMENTS

Instructor

77/100

PAGE 1

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PAGE 2

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PAGE 5

Text Comment. Too long, 300 words is enough

Text Comment. Too descriptive

PAGE 6

Text Comment. what were your conclusions?

PAGE 7

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PAGE 8

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PAGE 9

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PAGE 11

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1. ENGAGEMENT WITH LITERATURE



N/A

2. KNOWLEDGE AND UNDERSTANDING



Excellent, detailed knowledge and highly critical understanding of the well-established concepts and principles of the subject and the main methods of enquiry. (16/20)

16

3. COGNITIVE AND INTELLECTUAL



Excellent critical evaluation and analysis of concepts and principles leading to logical, evidence-based, reasoned arguments and judgements. Explicit recognition of other stances and a strong awareness of the limits of their knowledge. A capacity for independent thought and ability to 'see beyond the question', suggesting some grasp of the broader field and wider concepts. (24/30)

24

4. PRACTICAL APPLICATION



Exceptional levels of application and deployment skills using established methods, materials, tools and/or techniques. Consistent, accurate and logical application of theory to practice, making highly developed links between the two. Can identify routine and non-routine problems and propose quite sophisticated, creative solutions outside the context in which they were first studied. (27/30)

27

5. TRANSFERABLE SKILLS



Can communicate well, confidently and consistently in a suitable medium for the audience. Work is coherent, fluent, well-structured and organised. Can work very well autonomously and/or as part of a team, with very good contribution to group activities. Demonstrates very good skills for employment requiring the exercise of personal responsibility and decision-making, with just occasional minor weakness. Excellent presentation of results Executive summary is far too long and descriptive. Word count has been exceeded. (10/20)

10