

4) a) Convert system d.e. into standard form

$$\frac{d^2x}{dt^2} + \frac{c}{m} \frac{dx}{dt} + \frac{k}{m} x = \frac{f}{m}$$

$$= 2\zeta \omega_n = \frac{c}{m} \quad \text{and} \quad \omega_n^2 = \frac{k}{m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{400}{14.5}} = 5.25 \text{ Hz}$$

$$\text{Now, } \zeta = \frac{c}{2\omega_n m} = \frac{200}{2 \times 5.25 \times 14.5} = 1.31$$

Hence the system is overdamped.

$$\begin{aligned} \text{b) \% peak overshoot} &= \left(\exp \left(\frac{-1.31\pi}{\sqrt{1-1.31^2}} \right) \right) \times 100\% \\ &= 23.9\% \end{aligned}$$

$$\text{Rise time} = t_r = \frac{\pi}{2\omega_n \sqrt{1-\zeta^2}} = 0.308 \text{ s}$$

$$\text{Peak time} = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = 0.603 \text{ s}$$

$$5\% \text{ settling time} = 7.27 \text{ s}$$