

12 Pages

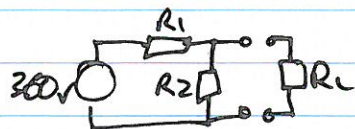
1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

Graham, Jay

List the numbers of the questions you have attempted	Office Use
Q1	19
Q2	20
Q3	7
Q4	10
Office Use	56 %

1 a
i

$$V_{ab} = \frac{R_2}{R_1 + R_2} \times V$$



$$V_{ab} = \frac{150}{30 + 150} \times 360$$

$$V_{ab} = 300V$$

$$\therefore V_{\text{Thevenin}} = 300V$$

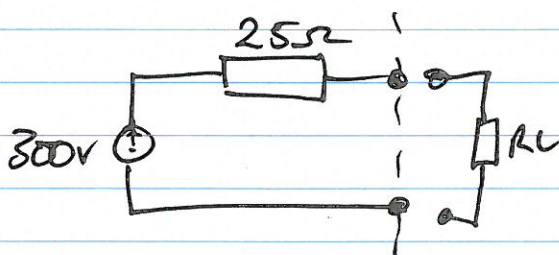
$$R_{TH} = \frac{R_1 \times R_2}{R_1 + R_2}$$

$$= \frac{150 \times 30}{150 + 30}$$

$$= \frac{4500}{180}$$

$$R_{TH} = 25 \Omega$$

Thevenin equivalent circuit



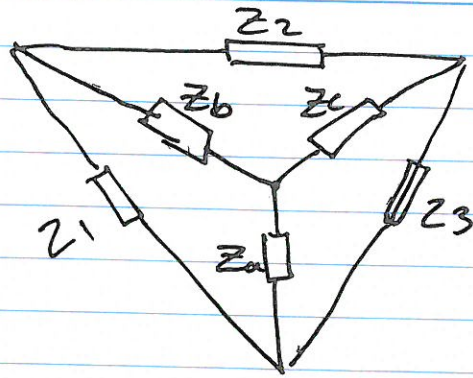
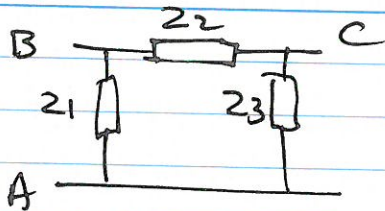
For Maximum power transfer, R_L must equal R_{TH} ,
therefore R_L must equal 25 Ω

ii - Max power delivered $P = \frac{V^2}{R}$

- Total power delivered $= \frac{300^2}{25 + 25} = 1800W$

$\therefore$ Power delivered through $R_L = \frac{1800}{2} = 900W$

1 b



$$Z_1 = 5 + j15 \, \Omega$$

$$Z_2 = 15 - j5 \, \Omega$$

$$Z_3 = 10 + j30 \, \Omega$$

Convert to polar $a + jb \rightarrow \sqrt{a^2 + b^2} \angle \tan^{-1} \frac{b}{a}$

$$\therefore Z_1 = 15.81 \, \Omega \angle 71.57^\circ$$

$$Z_2 = 15.81 \, \Omega \angle -18.43^\circ$$

$$Z_3 = 31.62 \, \Omega \angle 71.57^\circ$$

$$Z_b = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3}$$

$$Z_b = \frac{(15.81 \angle 71.57^\circ)(15.81 \angle -18.43^\circ)}{5 + j15 + 15 - j5 + 10 + j30}$$

$$= \frac{249.96 \angle 53.14^\circ}{30 \angle 40^\circ} \Rightarrow \frac{249.96 \angle 53.14^\circ}{30 \angle 40^\circ}$$

$$Z_b = 8.33 \, \Omega \angle 13.14^\circ \text{ Polar}$$

$$Z_b = 8.11 + j1.89 \, \Omega \text{ Rec}$$

$$Z_a = \frac{Z_1 Z_3}{Z_1 + Z_2 + Z_3} \Rightarrow \frac{(15.81 \angle 71.57^\circ)(31.62 \angle 71.57^\circ)}{30 \angle 40^\circ}$$

$$Z_a = \frac{499.91 \angle 143.14^\circ}{30 \angle 40^\circ}$$

$$Z_a = 16.66 \, \Omega \angle 103.14^\circ \text{ Polar}$$

$$Z_a = -3.79 + j16.22 \text{ Rec}$$

$$Z_c = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3}$$

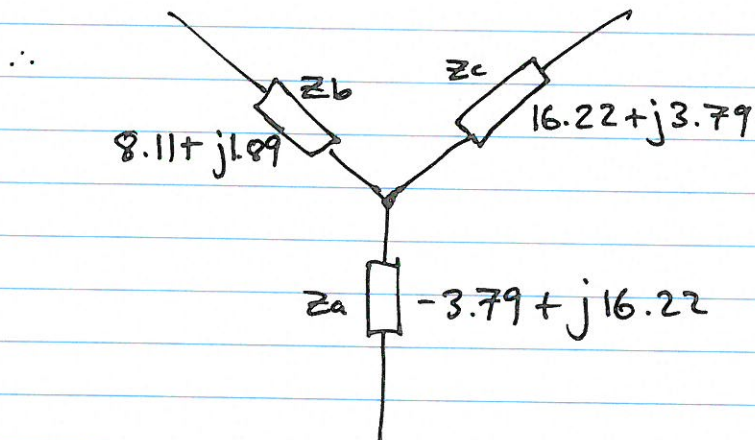
$$Z_c = \frac{(15.81 \angle -18.43)(31.62 \angle 71.57)}{30 \angle 40^\circ +}$$

$$= \frac{499.91 \angle 53.14}{30 \angle 40^\circ +}$$

$$Z_c = 16.66 \Omega \angle 13.14^\circ \quad \text{Polar}$$

$$Z_c = 16.22 + j3.79$$

(3)
✓



2

$$f(x) = \begin{cases} 0 & -\pi \leq x < 0 \\ 1 & 0 \leq x < \pi \end{cases}$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx] \quad \text{--- eq (1)}$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$\therefore a_0 = \frac{1}{2\pi} \int_{-\pi}^0 0 dx + \frac{1}{2\pi} \int_0^{\pi} 1 dx$$

$$a_0 = \frac{1}{2\pi} [x]_0^{\pi}$$

$$= \frac{1}{2\pi} [0 - \pi]$$

$$= \frac{-\pi}{2\pi}$$

$$a_0 = -0.5$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$\therefore a_n = \frac{1}{\pi} \int_{-\pi}^0 0 \cos nx dx + \frac{1}{\pi} \int_0^{\pi} 1 \cos nx dx$$

$$\therefore a_n = \frac{1}{\pi} \int_0^{\pi} \cos nx dx$$

6.
✓

$$a_n = \frac{1}{\pi} \left[\frac{\sin nx}{n} \right]_0^{\pi}$$

When $n=1$

$$a_n = \frac{1}{n\pi} [\sin 0 - \sin \pi]$$

$$= \frac{1}{\pi} [0 - 0]$$

$$\therefore a_n = 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$\therefore b_n = \frac{1}{\pi} \int_{-\pi}^0 0 \sin nx \, dx + \frac{1}{\pi} \int_0^{\pi} \sin nx \, dx$$

When $n=1$ (odd)

$$b_n = \frac{1}{\pi} \left[-\frac{\cos nx}{n} \right]_0^{\pi}$$

$$= -\frac{1}{n\pi} [\cos 0 - \cos \pi]$$

$$= -\frac{1}{\pi} [1 - (-1)]$$

$$= -\frac{2}{\pi}$$

When $n=2$ (even)

$$b_n = \frac{-1}{2\pi} [\cos 0 - \cos 2\pi]$$

$$b_n = \frac{-1}{2\pi} [1 - 1]$$

$$= \frac{-1}{2\pi} [\emptyset]$$

$\therefore b_n = \emptyset$ when $n = \text{even}$ ✓

So put values a_0, a_n, b_n into eq 1.

$$f(x) = -0.5 + \sum_{n=1}^{\infty} \left[\emptyset + -\frac{2}{2\pi} \sin nx \right]$$

for b_n ~~$-\frac{1}{2\pi} \sin x + -\frac{1}{2\pi} \sin 3x + -\frac{1}{2\pi} \sin 5x$~~

$$b_n = -\frac{2}{\pi} \sin x + -\frac{2}{3\pi} \sin 3x + -\frac{2}{5\pi} \sin 5x \dots$$

$$\therefore f(x) = -0.5 - \frac{2}{\pi} \left(\sin x + \sin \frac{1}{3}x + \sin \frac{1}{5}x \right)$$

3 a i $e^{4t} + 5 \Rightarrow \frac{1}{s+4} + \frac{5}{s}$

ii $\cos(2t) + 7\sin(2t)$

$\Downarrow$

$$\frac{s}{s^2+2^2} + \frac{7 \times 2^4}{s^2+2^2} \Rightarrow \frac{s}{s^2+4} + \frac{14}{s^2+4}$$

$$= \frac{s+14}{s^2+4}$$

iii $(t^2+4t+2)e^{3t}$

expand: $t^2e^{3t} + 4te^{3t} + 2e^{3t}$

$$\frac{n!}{(s-a)^{n+1}} = \frac{2}{(s-3)^3}$$

$$\frac{4 \times 1!}{(s-3)^{1+1}}$$

$$\frac{2 \times 1}{s-3}$$

$$= \frac{2}{(s-3)^3} + \frac{4}{(s-3)^2} + \frac{2}{s-3}$$

iv

$$6e^{5t} \cos(2t) - e^{7t}$$

↓

↓

$$e^{-at} \cos \omega t$$

$$\frac{1}{s-7}$$

↓

$$\frac{6 \times s - 5}{(s-5)^2 + 2^2}$$

$$\frac{6s - 30}{(s-5)^2 + 4} - \frac{1}{s-7}$$

✓

2.

✓

v

$$v(t) = 5e^{-2t} \cdot \sin(4t)$$

$$V(s) = \frac{5 \times 1}{s+2} \cdot \frac{4}{s^2+4^2}$$

✗

$$V(s) = \frac{6}{s+2} \cdot \frac{4}{s^2+16}$$

b

$$5 \frac{dx}{dt} + 3x = 4 \quad \text{when } x=0, t=0$$

$$\Downarrow$$

$$5X(s) + 3X(s) = \frac{4}{s} \quad \frac{5}{s} + \frac{3}{s} = \frac{8}{s}$$

$$X(s) (8) = \frac{4}{s}$$

$$\therefore X(s) = \frac{4}{8s}$$

$$X(s) = \frac{1}{4s} \quad \text{No equiv? on table}$$

$$\left(\frac{8}{s} \right) \left(\frac{4}{s} \right) = \frac{4s}{8s^2}$$

$$= \frac{s}{4s^2}$$

$$= \frac{s}{(s-2)(s+2)}$$

$$\begin{aligned} & \underline{s^2 - s + 4s - 4} \\ & -2s + 2s \end{aligned}$$

$$= \frac{s}{(s+2)(s-2)}$$

$$\text{wL} \Rightarrow \frac{1}{a-b} (ae^{-at} - be^{-bt})$$

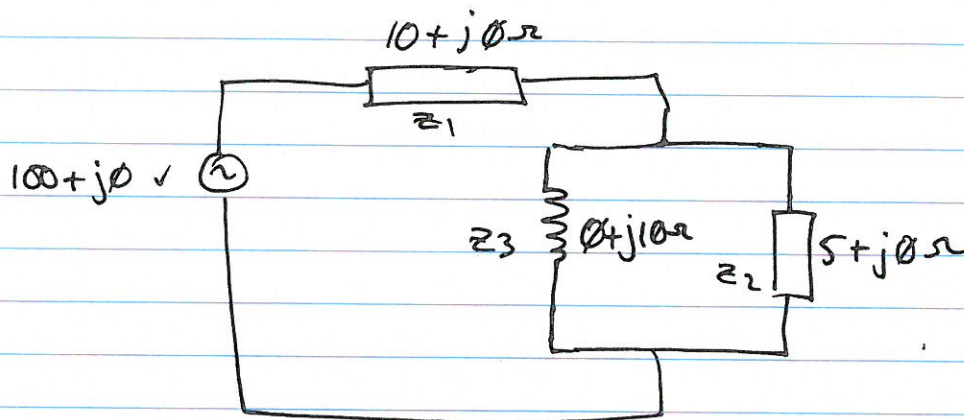
$$\frac{1}{2+2} (2e^{-2 \times 5} + 2e^{+2 \times 5})$$

$$\frac{1}{4} (9.079 \times 10^{-5} + 44052.9)$$

$$x = 11013.23 \quad \text{when } t = 5s$$

nb. I hope the answer wasn't on the transform page where only the top half was printed.

4



$$Z_T = \frac{Z_3 \times Z_2}{Z_3 + Z_2} + Z_1$$

$$= \frac{(0 + j10)(5 + j0)}{0 + j10 + 5 + j0} + 10 + j0$$

$$= \frac{0 + j50}{5 + j10} + 10 + j0$$

$$Z_T = \frac{0 + j50 \times 5 - j10}{5 + j10 \times 5 - j10} = \frac{0 + j250 - j10}{25 + 100} = \frac{50 + j250}{125} = 0.4 + j2.50$$

~~ARITHMETIC ERROR~~
14

$Z_T = 10.4 + j2.50 \Omega$

$$I_T = \frac{V}{Z_T} = \frac{100 + j0}{10.4 + j2.50}$$

ERROR CARRIED

$$\frac{100 + j0}{10.4 + j2.50} \times \frac{10.4 - j2.50}{10.4 - j2.50} = \frac{1040 - j2500}{108.16 - (-1.6250)}$$

$$= \frac{1040 - j2500}{62.608.16}$$

$$I_T = 0.017 - j25000$$

$$V_{in Z_1}$$

$$= \frac{Z_1}{Z_1 + 0.4 + j250}$$

$$= \frac{(10 + j0)(100 + j0)}{10 + j0 + 0.4 + j250}$$

$$= \frac{1000 + j0}{10.4 + j250} = \frac{10900 + j25000 + 250}{108.16 + j62500}$$

$$= \frac{10650 - j25000}{62608.16}$$

$$= 0.17 - j25000$$

$$\therefore I_{Z_1} = \frac{V_{Z_1}}{Z_1} = \frac{0.17 - j25000}{10 + j0}$$

$$= \frac{0.17 - j25000}{10 + j0} \times \frac{10 - j0}{10 - j0} = \frac{1.7 - j250000}{100}$$

$$= 0.017 - j250000$$

REPEAT OF PREVIOUS CALL.

$$I_{Z_2} = \frac{Z_2}{Z_1 // Z_3} \times I_T$$

0.

