

FLUID FLOW

I. Energy relationships and the Bernoulli's equation

The total energy of a fluid in motion consists of the following components:

- Pressure energy,
- Kinetic energy,
- Potential energy.

Each of these energies may be considered with reference to an arbitrary base level (datum). It is also convenient to make calculations on unit mass of fluid.

1. Pressure energy:

If one pushes a mass against a spring and compress it, or against any force, then work is done. Similarly if one pushes fluid against the pressure of the fluid then work is done. When a fluid flows through a pipe against the pressure in the pipe then work is done. Thus, if at some section of a pipe the pressure is P , then the force opposing the motion of the fluid is given by:

$$P = \frac{\text{force}}{\text{area}} = \frac{F}{A}$$

$$\Rightarrow F = P \times A$$

Where A is the cross-sectional area at the section concerned.

The work done in moving a distance L against the force resulting from the pressure is:

$$\begin{aligned}\text{work done} &= \text{force} \times \text{distance} \\ &= F \times L \\ &= P \times A \times L\end{aligned}$$

The work done per unit mass of the fluid moved is:

$$\frac{\text{work done}}{\text{mass } m} = \frac{P \times A \times L}{m}$$

Where the mass m of the fluid moved is: $m = \rho \times V$

The volume moved is: $A \times L$

$$\Rightarrow m = \rho \times A \times L$$

Therefore:

$$\text{work done per unit mass} = \frac{P \times A \times L}{\rho \times A \times L} = \frac{P}{\rho}$$

This quantity is referred to as the **pressure energy of unit mass** or **specific pressure energy**.

Thus:

$$\text{specific pressure energy} = \frac{P}{\rho}$$

N / m² or bar

J / kg

kg / m³

2. **Kinetic Energy KE:**

If a mass m of fluid has a linear velocity v , then the kinetic energy in the fluid is:

$$KE \text{ in the fluid} = \frac{1}{2} \cdot m \cdot v^2$$

For unit mass of fluid:

$$\text{specific KE} = \frac{1}{2} \cdot v^2$$

J / kg

3. **Potential Energy PE:**

If a mass of fluid is stored at a height h above the datum, then the potential energy in the fluid is:

$$PE \text{ stored in the fluid} = m \cdot g \cdot h$$

For unit mass of fluid:

$$\text{specific PE} = g \cdot h$$

J / kg

4. **Total energy E:**

Summing these components, the total energy E per unit mass of fluid is given by:

$$E = \frac{P}{\rho} + \frac{v^2}{2} + g \cdot h$$

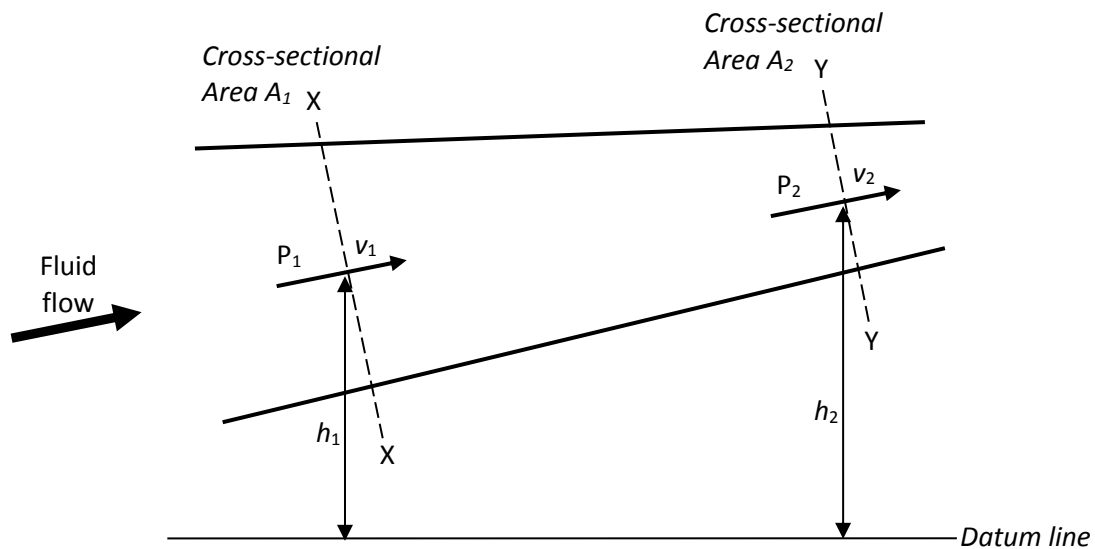
J / kg

h is the height of the fluid above some datum line.

5. Bernoulli's equation:

According to the principle of the conservation of energy (*the total amount of energy in a isolated system remains constant over time*), the total energy of a fluid will remain constant, provided we take into account all the various forms of energy involved.

Thus if a fluid flows through a pipe then the total energy at one section of the pipe must equal the total energy at another further section further down the pipe. Hence for the flow through the pipe shown as follows:



Total energy at section XX = Total energy at section YY

At section XX:

$$\text{effective pressure energy} = \frac{P_1}{\rho}$$

$$\text{effective KE} = \frac{1}{2} v_1^2$$

$$\text{effective PE} = g \cdot h_1$$

At section YY:

$$\text{effective pressure energy} = \frac{P_2}{\rho}$$

$$\text{effective KE} = \frac{1}{2} v_2^2$$

$$\text{effective PE} = g \cdot h_2$$

Hence:

$$\frac{P_1}{\rho} + \frac{v_1^2}{2} + g \cdot h_1 = \frac{P_2}{\rho} + \frac{v_2^2}{2} + g \cdot h_2$$

The above equation when expressed in that form is known as the **Bernoulli's equation**.

The latter equation assumes that no energy is lost, or gained, by the fluid in its movement between section XX and YY. With steady flow the loss of energy between XX and YY may be neglected, with turbulent flow the loss can be significant. The result of any energy loss is that an energy term representing the energy lost between the 2 sections needs to be added to the energy per unit mass at section YY. This energy 'lost' will probably have appeared as a rise in temperature of the fluid and the pipe walls.

Bernoulli's equation is sometimes written in a different form. Dividing throughout by g gives:

$$\frac{P_1}{\rho \cdot g} + \frac{v_1^2}{2 \cdot g} + h_1 = \frac{P_2}{\rho \cdot g} + \frac{v_2^2}{2 \cdot g} + h_2$$

(no frictional resistance)

The units of each of the above terms are meters.

Proof:

$$\blacklozenge \text{ in term of units } \frac{P}{\rho \cdot g} \equiv \frac{\frac{N}{m^2}}{\frac{kg}{m^3} \cdot \frac{m}{s^2}} = \frac{N \cdot m^3 \cdot s^2}{m^2 \cdot kg \cdot m}$$

$$\text{but: } N = kg \cdot \frac{m}{s^2}$$

$$\text{therefore: } \frac{P}{\rho \cdot g} \equiv \frac{kg \cdot m \cdot m^3 \cdot s^2}{s^2 \cdot m^2 \cdot kg \cdot m} = \frac{m^3}{m^2} = m$$

$$\blacklozenge \text{ in term of units } \frac{v^2}{2 \cdot g} \equiv \frac{\frac{m^2}{s^2}}{\frac{m}{s^2}} = \frac{m^2}{m} = m$$

$$\blacklozenge \text{ in term of units } h \equiv m$$

If frictional resistance to flow is considered:

$$\frac{P_1}{\rho \cdot g} + \frac{v_1^2}{2 \cdot g} + h_1 = \frac{P_2}{\rho \cdot g} + \frac{v_2^2}{2 \cdot g} + h_2 + h_f$$

Where h_f is the frictional resistance "head".

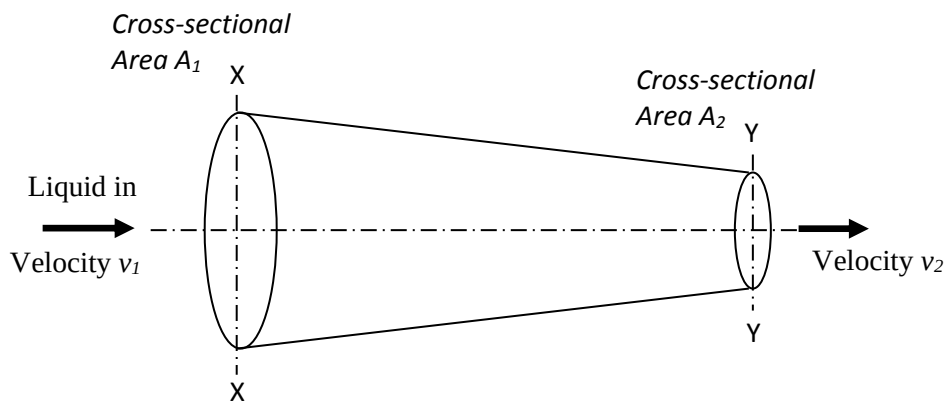
Each quantity in the latter form of the Bernoulli's equation is measured in terms of "head of liquid", i.e. height of liquid above a given datum. The unit for each quantity is the meter.

Therefore,

$$\begin{aligned} &(\text{pressure head} + \text{kinetic head} + \text{potential head}) \text{ all for the section } XX \\ &= \\ &(\text{pressure head} + \text{kinetic head} + \text{potential head} + \text{frictional resistance head}) \text{ all for the section } YY \end{aligned}$$

II. Continuity equation

Consider sections XX and YY in the tapered pipe shown below, which is full of steadily flowing fluid. At section XX, the cross-sectional area is A_1 and the velocity of the fluid is v_1 . At section YY, the area and velocity are A_2 and v_2 respectively.



Since the pipe is full of fluid and the flow is steady, then, applying the principle of conservation of mass (the mass of a closed system remains constant over time),

$$\text{mass of fluid passing section XX per unit time} = \text{mass of fluid passing section YY per unit time}$$

Now,

$$\text{mass flow rate } \dot{m} \text{ (kg/s)} = \text{volume flow rate} \times \text{density } \rho$$

and,

$$\text{volume flow rate } Q \text{ (m}^3\text{/s)} = \text{area} \times \text{velocity}$$

Thus:

$$\text{mass flow rate at section XX} = \text{mass flow rate at section YY}$$

$$\Rightarrow A_1 \cdot v_1 \cdot \rho = A_2 \cdot v_2 \cdot \rho$$

$$\Rightarrow A_1 \cdot v_1 = A_2 \cdot v_2$$

The volume flow rate (area $\times$ velocity) is denoted by Q :

$$Q = A_1 \cdot v_1 = A_2 \cdot v_2$$

This is the **continuity equation** for incompressible flow.

Note: When a real fluid (i.e. one with non-zero viscosity) flows through a pipe, the velocity varies across a section of the pipe. In fact it is zero at the pipe walls. The continuity equation can still be used, however, in the form $Q = A_1 \cdot \bar{v}_1 = A_2 \cdot \bar{v}_2$, where $\bar{v}$ is now the mean flow velocity.

Example: At a section in a tapered pipe, the diameter is 0.7m and the velocity of the liquid is 5m/s. Assuming the flow of the fluid to be steady, determine the velocity when the pipe diameter is 0.5m.

III. Notes

- 1 cubic meter is equal to 1,000 litres:

$$1 \text{ m}^3 = 1000 \text{ l}$$

- For water:

$$1 \text{ kg} = 1 \text{ l} \quad (\text{for water only})$$

- The energy lost in friction, i.e. the loss of power, is given by:

$$\text{J / s or Watt} \quad \text{loss of power} = \text{mass flow rate} \times g \times h_f \quad \text{m}$$

$\text{kg / s} \qquad \text{m / s}^2$