



**Complete, fold and seal
(see note 2 below)**

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12 Pages

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

[illegible]

92%
Very good

$$1) i) 3x + 4y - 2z + 7x - 9y + 3z + 8x - 6y + 7z$$

$$= 10x - 11y + 11z$$

(2)

$$ii) x(x+5) - (x-4)$$

$$= x^2 + 5x - x + 4$$

$$= x^2 + 4x + 4$$

(2)

$$iii) 3qr \times 4pq^3 \times 5qr^2$$

$$= 60pq^5r^3$$

(2)

$$iv) \frac{12p^2q^3r}{6p^3q^2r^3}$$

$$= 2p^{-1}r^4$$

if $q \neq 0$ yes

(2)

$$v) \frac{2F_3}{16h} \times \frac{8gh^2}{4Fh} \times \frac{32Fh^3}{64F^3}$$

$$= \frac{512F^2g^2h^5}{4096F^4h^2}$$

$$= \frac{1}{8}F^{-2}g^2h^3$$

(2)

$$2) i) x^2 + 10x = -25$$

$$x^2 + 10x + 25 = 0$$

$$(x+5)(x+5) = 0$$

$$x+5 = 0 \text{ OR } x+5 = 0$$

$$x = -5$$

$$x = -5$$

(5)

$$ii) \frac{(x-8)}{4} = x + \frac{(2x-4)}{8}$$

$$8 \frac{(x-8)}{4} = 8x + (2x-4)$$

$$2(x-8) = 8x + (2x-4)$$

$$2x - 16 = 8x + 2x - 4$$

$$2x = -2$$

$$x = \frac{-2}{2}$$

$$= -1$$

(5)

$$3) a) 4^{2x-1} = 3^{x+1}$$

$$\log_{10} 4^{2x-1} = \log_{10} 3^{x+1}$$

$$(2x-1)\log_{10} 4 = (x+1)\log_{10} 3$$

$$4(2x-1) = 3(x+1)$$

$$8x - 4 = 3x + 3$$

$$5x = 7$$

$$x = \frac{7}{5}$$

$$= 1.40 \text{ (2dp)}$$

$$ii) \log(x^2+4) - \log(x+8) = \log x$$

$$\log\left(\frac{x^2+4}{x+8}\right) = \log x$$

$$\frac{x^2+4}{x+8} = x$$

$$(x+8)$$

$$x^2 + 4 = x(x+8)$$

$$x^2 + 4 = x^2 + 8x$$

$$8x = 4$$

$$x = \frac{4}{8}$$

$$= 0.5$$

$$b) i) \log(x^2yz)$$

$$= \log x^2 + \log y + \log z$$

$$= 2\log x + \log y + \log z$$

$$ii) \log\left(\frac{x^4y^2}{z}\right)$$

$$= \log x^4 + \log y^2 - \log z$$

$$= 4\log x + 2\log y - \log z$$

$$4) \underline{a} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$$

$$\underline{b} = \mathbf{i} + 3\mathbf{j} - \mathbf{k}$$

$$i) \underline{a} \cdot \underline{b} = (2 \times 1) + (-3 \times 3) + (4 \times -1)$$

$$= 2 - 9 - 4$$

$$= -11$$

(3)

$$ii) \underline{a} \cdot \underline{b} = |\underline{a}| |\underline{b}| \cos \theta$$

$$|\underline{a}| = \sqrt{(2)^2 + (-3)^2 + (4)^2}$$

$$= \sqrt{4 + 9 + 16}$$

$$= \sqrt{29}$$

$$|\underline{b}| = \sqrt{(1)^2 + (3)^2 + (-1)^2}$$

$$= \sqrt{1 + 9 + 1}$$

$$= \sqrt{11}$$

$$-11 = \sqrt{29} \sqrt{11} \cos \theta$$

$$\cos \theta = \frac{-11}{\sqrt{29} \sqrt{11}}$$

$$\theta = \cos^{-1} \left(\frac{-11}{\sqrt{29} \sqrt{11}} \right)$$

$$= 128.0160 \dots$$

$$= 128.02^\circ (2dp)$$

(2)

$$iii) \underline{c} = -3\mathbf{i} + \mathbf{j} + 0\mathbf{k}$$

$$|\underline{c}| = \sqrt{(-3)^2 + (1)^2 + (0)^2}$$

$$= \sqrt{9 + 1 + 0}$$

$$= \sqrt{10}$$

$$\underline{c} \cdot \underline{b} = \cancel{(-3 \times 1)} + (1 \times 3) + (0 \times -1)$$

$$= -3 + 3 + 0$$

$$= 0$$

(2)

$$\underline{c} \cdot \underline{b} = |\underline{c}| |\underline{b}| \cos \theta$$

$$0 = \sqrt{10} \sqrt{11} \cos \theta$$

$$\cos \theta = \frac{0}{\sqrt{10} \sqrt{11}}$$

$$\theta = \cos^{-1} \left(\frac{0}{\sqrt{10} \sqrt{11}} \right)$$

$$= 90^\circ$$

$\therefore$ Perpendicular

4) i) $A = 3 \times 3$

~~2x2 matrix~~ $A = \begin{bmatrix} i & j & k \\ 2 & -1 & 4 \\ 1 & 3 & -1 \end{bmatrix} \begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$

$$A = i\{(-3)(-1) - (1)(4)\} - j\{(2)(-1) - (1)(4)\} + k\{(2)(3) - (1)(-1)\}$$

$$= i(3 - 4) - j(-2 - 4) + k(6 + 1)$$

$$= i(-1) - j(-6) + k(7)$$

$$= -i + 6j + 7k$$

$$\underline{\hat{A}} = \frac{A}{|A|}$$

$$= \frac{-i + 6j + 7k}{\sqrt{(-1)^2 + (6)^2 + (7)^2}}$$

$$= \frac{-i + 6j + 7k}{\sqrt{1 + 36 + 49}}$$

$$= \frac{-i + 6j + 7k}{\sqrt{86}}$$

$$= \frac{-i + 6j + 7k}{\sqrt{86}}$$

3

5) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 4 & -2 \end{bmatrix} \begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$

$$\text{Det}(A) = |A| = 1\{(-1)(-2) - (4)(1)\} - 2\{(2)(-2) - (3)(1)\} + 3\{(2)(4) - (3)(-1)\}$$

$$= 1(2 - 4) - 2(-4 - 3) + 3(8 + 3)$$

$$= 1(-2) - 2(-7) + 3(11)$$

$$= -2 + 14 + 33$$

$$= 45$$

$$C = \begin{bmatrix} + \begin{vmatrix} 1 & 1 \\ 4 & -2 \end{vmatrix} - \begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} \\ - \begin{vmatrix} 2 & 3 \\ 4 & -2 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 3 & -2 \end{vmatrix} - \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \\ + \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & 7 & 11 \\ 16 & -17 & 2 \\ 5 & 5 & -5 \end{bmatrix}$$

$$5) C^T = \begin{bmatrix} -2 & 16 & 5 \\ 7 & -11 & 5 \\ 11 & 2 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \cdot C^T$$

$$= \frac{1}{45} \begin{bmatrix} -2 & 16 & 5 \\ 7 & -11 & 5 \\ 11 & 2 & -5 \end{bmatrix}$$

gust

15

$$6) \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 4 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}$$

$$A \cdot X = b$$

$$\det(A) = 45$$

$$C^T = \begin{bmatrix} -2 & 16 & 5 \\ 7 & -11 & 5 \\ 11 & 2 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{45} \begin{bmatrix} -2 & 16 & 5 \\ 7 & -11 & 5 \\ 11 & 2 & -5 \end{bmatrix}$$

$$A^{-1}X = A^{-1}b$$

$$IX = A^{-1}b$$

$$X = A^{-1}b$$

$$X = \frac{1}{45} \begin{bmatrix} -2 & 16 & 5 \\ 7 & -11 & 5 \\ 11 & 2 & -5 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}$$

$$= \frac{1}{45} \begin{bmatrix} (-2 \times 5) + (16 \times 0) + (5 \times 2) \\ (7 \times 5) + (-11 \times 0) + (5 \times 2) \\ (11 \times 5) + (2 \times 0) + (-5 \times 2) \end{bmatrix}$$

$$= \frac{1}{45} \begin{bmatrix} 0 \\ 45 \\ 45 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$AX = b$$

$$A^{-1}AX = A^{-1}b$$

$$IX = A^{-1}b$$

$$X = A^{-1}b$$

8

$$7) i) y = 5x - 3$$

$$\frac{dy}{dx} = 5 - 0$$

$$= 5$$

2

$$ii) y = 4x + 20x^{\frac{1}{2}} + \frac{5}{6}$$

$$\frac{dy}{dx} = 4 + 5x^{-\frac{1}{2}} + 0$$

$$= 4 + 5x^{-\frac{1}{2}}$$

2

$$iii) y = \frac{2}{x^5} + 9x^3$$

$$= 2x^{-5} + 9x^3$$

$$\frac{dy}{dx} = 2(-5x^{-6}) + 9(3x^2)$$

$$= -10x^{-6} + 27x^2$$

2

$$iv) y = \cos 20x$$

$$\frac{dy}{dx} = -20 \sin(20x)$$

2

$$v) y = (8x+5)^5$$

$$u = 8x+5 \quad v = u^5$$

$$\frac{du}{dx} = 8 \quad \frac{dv}{du} = 5u^4$$

$$\frac{dy}{dx} = 8(5u^4)$$

$$= 40u^4$$

$$= 40(8x+5)^4$$

2

$$vii) y = 2e^x + \ln(x)$$

$$\frac{dy}{dx} = 2e^x + \frac{1}{x}$$

2

7) b) $y = \frac{(x+3)}{(1+x^2)}$

$u = x+3 \quad v = 1+x^2$

by

$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = 2x$

$\frac{dy}{dx} = \frac{1+x^2(1) - (x+3)(2x)}{(1+x^2)^2}$

$= \frac{1+x^2 - 2x^2 - 6x}{(1+x^2)^2}$

$= \frac{-x^2 - 6x + 1}{(1+x^2)^2}$

when $x = 2 \dots$

$\frac{dy}{dx} = \frac{-(2)^2 - 6(2) + 1}{(1+(2)^2)^2}$

$= -0.6$

8) i) $\int (4x + x^3) dx$

$= \frac{4x^2}{2} + \frac{x^4}{4} + c$

$= 2x^2 + \frac{x^4}{4} + c$

ii) $\int (2x + 3e^x) dx$

$= \frac{2x^2}{2} + 3e^x + c$

$= x^2 + 3e^x + c$

iii) $\int (\frac{1}{x} + 7) dx$

$= \ln x + 7x + c$

$$8) \text{ii) } \int (x^2 - 1)(3x + 4) dx$$

$$\int (3x^3 + 4x^2 - 3x - 4) dx$$

$$= \frac{3x^4}{4} + \frac{4x^3}{3} - \frac{3x^2}{2} - 4x + C$$

$$\text{vi) } \int_3^6 (3x^2 + 2x) dx$$

$$= \left[\frac{3x^3}{3} + \frac{2x^2}{2} \right]_3^6$$

$$= [x^3 + x^2]_3^6$$

$$= (6^3 + 6^2) - (3^3 + 3^2)$$

$$= 276$$

This is a very good effort Rose,
Very well Done!