

UWTSD MBA (Nov 2018 intake)

(5-1)

Application of Game Theory in Businesses

Che-Cheong POON

10 February 2019; 10:00 am to 1:30 pm

Lecture Agenda :

- Introduction to game Theory
- The Prisoners' Dilemma
- Applications of the Prisoners' Dilemma
- Two Examples for One Nash Equilibrium Games:
 1. Ice Cream Sellers Game
 2. A Threat Game
- An Example no Nash Equilibria Games

津津樂道

《道教格言》

仁義有過禍者矣，而行業不可惰。《抱朴子外篇·廣譬》

躬行仁義的人有時也會遭遇禍患，但是節操修持絕不可以鬆懈。聖人指出，任何人不都有不如意的時候，但人生遇到低谷也必須繼續進修及裝備自己，並堅守道德底線，不可放棄理想及原則。即使遭遇禍患，也絕不可忘記慈仁、禮義之心。《南華經·雜篇》載：「君子通於道之謂通，窮於道之謂窮。今丘抱仁義之道以遭亂世之患，其何窮之為！故內省而不疚於道，臨難而不失其德！」明乎天道法則叫「通」，不明所以叫「窮」。堅持正道而遭逢難關，這不算是窮困。只要反省自己有沒有愧疚於良心道德，面對屈辱又不失做人底線及原則，這才是「通」，才是真正的行道者！

《道學淺談》

為甚麼幫人即是幫自己？

我們每天都會面對不同的人 and 事，遇上困難時未必每次都可以自己一人去解決，有時候總需要嘗試找人幫忙。同樣，我們遇到別人有困難時，也需主動幫忙。主動幫人便是種下善緣，大家互相幫助，最終其實在幫自己。修行上，我們也需要與同門互相鼓勵，王重陽祖師便提出了「合道伴」，即修行要與道伴互相扶持才更見進步。以下故事或可啟發：

京城有一個賣包子的，一個賣棉被的，某夜因躲避突如其來的風雪，同時住進了一間破廟內。兩人互不理會，一個吃飽了餘下的包子便睡在廟裡東南角，卻乾冷著；一個蓋上被子睡在西北角，卻乾餓著！兩個人心想：要是對方主動找我，我肯定和他合作。結果第二天早上，附近的人發現了他們，一個凍死了，一個餓死了！

很多人因為自尊心強，不想求人幫忙，所以有困難、有不開心的事只會收起來，令自己躲起來痛苦，獨自面對。不過仔細想想，其實尋求幫助真的這麼難於啟齒嗎？我們見到別人有需要的時候，也應主動伸出援手。這個世界就是這樣互相協助、扶持，廣結善緣，才會有更多正能量，不斷改善和進步。



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Introduction to Game Theory

<https://www.youtube.com/watch?v=xxjQ2jDc6O8>

The concept of a game

Game theory studies the optimal strategies for players involved in multi-person interactions, and each game has three essential elements: :

1. The rules of the game
2. Strategies
3. Payoff

Game theory is a bag of analytical tools designed to help us understand the phenomena that we observe when decision-makers interact. The basic assumptions that underlie the theory are that decision-makers pursue well-defined exogenous objectives (they are rational) and take into account their knowledge or expectations of other decision-makers' behavior. (i.e., what one does affects and is affected by what others do).

Martin J. Osborne & Ariel Rubinstein (1994). *A Course in Game Theory*. The MIT Press. Cambridge, Massachusetts. London, England

Game Theory

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Interactive Decision Theory

- Game theory is a tool for telling us where incentives will lead.
- Game theory studies selfishness, it is ethically neutral.
- The decision makers are rational

The development of game theory

- 1928: John von Neumann (1903-1957) is usually credited with founding the modern discipline of game theory. (Von Neumann, J., 1928. Theorie der Gesellschaftsspiele. *Mathematische Annalen*. 100, 295-320.) This paper is focused on winner-takes-all games (**zero-sum games**— such that one person's gains exactly equal net losses of the other participants.) games with pure competition and to prove the concept of **the minimax theorem**.
- 1944: (Von Neumann, J. and Morgenstern, O., 1944. Theory of Games and Economic Behavior. *Princeton: Princeton University Press*.)
- 1951: (Nash, John F., 1951. Non-cooperative games. *Annals of Mathematics*. 54, 289-92.); (Tucker, A.W., 1951. A Two-Person Dilemma. Stanford University mimeo, reprinted in Straffin (1980)

- 1960: (Schelling, Thomas, 1960. *The Strategy of Conflict*. Cambridge, MA: Harvard University Press.)
- 1994: The award of the Nobel Prize in Economics to three game theorists, John Forbes Nash, Reinhard Selten and John Harsanyi, in 1994 sealed the intimate relationship between economics and game theory.
- 2005: Robert J. Aumann and Thomas C. Schelling won the Nobel Memorial Prize for their work in game theory, which explains the choices that competitors make in situations that require strategic thinking.

Nature of game theory

- Game theory is concerned with the actions of individuals who are conscious that their actions affect each other.
- Players' planned decisions are called strategies, and **payoffs** to players are the profits or losses resulting from **strategies**.
- Managers must make decisions in the context of existing rivals and take into account the possible actions of potential rivals.
- The market structure that focuses on this interdependence is called **oligopoly**.

- The elements of a game are players, actions, information, strategies, payoffs, outcomes, and equilibria.
- A description of the players, actions, and outcomes are collectively referred to as the rules of the game, and the modeler's objective is to use the rules of the game to determine the equilibrium.
- The rules of the game may be defined by laws or government definitions, by societal convention, or by the actors themselves.

Example: OPEC members choosing their annual output

Players:

Islamic Republic of Iran, Iraq, Kuwait, Saudi Arabia, and Venezuela.

Information:

All payers know the value of demand, but they choose their actions simultaneously.

Actions and events:

1. The players simultaneously choose supply schedules consisting of the quantity each will supply at each possible market price in a given year.
2. Expected world demand for oil is either weak or strong, with equal probability.

Strategies:

The strategies are the same as actions, since no information is revealed that might affect the action a player chooses.

Payoffs:

For each country the payoff is +US\$100 billion. (If its oil revenue is above a country-specific amount necessary to avoid a military coup.)

Outcomes:

The outcome includes the quantities supplied, the state of demand, the resulting revenue and market price, and whether or not each country has a coup.

Game Environments

- Order of play
 - **Simultaneous-move game**: each player chooses his action without knowledge of the actions chosen by other players.
 - **Sequential-move game**: players make moves at different times or in turn. This means that each player's strategy makes the actions that he or she chooses conditional on the additional information received during the game.
- Frequency of rival interaction
 - **One-shot game**: game is played once.
 - **Repeated game**: game is played more than once; either a finite or infinite number of interactions.

The Prisoners' Dilemma

Tucker, Albert (unpub) “A Two-Person Dilemma” Stanford University mimeo. May 1950. Reprinted in Straffin (1980). 13, 38n.

Straffin, Phillip (1980) “The prisoner’s Dilemma” UMAP (Undergraduate Mathematics and Its Applications) Journal. 1:101-3. 38n.

21-9-2017

考小學爭崩頭之囚犯兩難

女人心 光明女樂
黃明樂 隔兩日見報
wong_minglok@yahoo.com.hk

近日，所有家長朋友，話題獨沽一味：考小學。

小學，跟幼稚園，兩碼子事。哪怕是相信「快樂童年」的爸媽，都醒覺茲事體大，嚟真，馬虎不得。

壓力無限，心力有限。熱門選擇，拿起手十間八間，每間平均面試兩至三次，合共二三十次走唔甩，還要同期發生。舟車勞頓，密集操練，人都癲。

少報考一點可以嗎？但搶手貨坐底幾千人爭，要打定輸數。爲了增加機會，唯有多報。然而當所有人都大包圍，人龍就愈長，競爭也愈大，取錄機會自然更低。惡性循環。

學校也不見得好過。數千人報名，全校老師總動員，每人平均面見上百個考生，每個十分鐘，第一輪已花上三四天。在繁重的教學和行政工作以外，年年咁玩法，命都短幾年。

家長們不諱言，報十間，是買保險，真正心儀的，一不離二。但有鳥在手，總好

過攪珠大抽獎。意思是什麼？即是花了這許多氣力，不爲得到最好，只爲逃避最差。這種不合比例的集體虛耗，每年一度，世世代代困擾天下父母心。

有無出路？即管……即管大膽假設，如果每位考生只報考三間小學，由十變三，全港報讀人次驟跌七成，又會是怎樣的光景？

家長奔波少了，孩子輕鬆了，競爭細了，入讀心儀學校的機會也高了。學校也有更大信心，報考者都是鎖定目標而非但求水泡，篩選自也不需要那麼緊張嚴謹。

問題是，如何建立這種集體信任，去達至「團體最佳選擇」，同時也是「個人最佳選擇」？典型的囚犯兩難（prisoner's dilemma），贏要一齊贏，輸必一齊輸。眼前死路一條，生路也只有一條。要逃出生天，唯有靠默契，但默契何來？

囚犯兩難，要在「欠缺溝通和不完整資訊」下，才有效果。如今，家長群體早有太多溝通，又有機會群起撥亂反正嗎？

Interpretation

- The prisoners' dilemma is a particular “game” between two captured prisoners that illustrates why cooperation is difficult to maintain even when it is mutually beneficial.
- Often firms fail to cooperate with one another (so they compete with each other) even when cooperation would make them better off.

The situation

- Two suspected criminals were arrested by the police.
- The police think that they were trying to rob a bank, but the police only have enough evidence to prove that they were trespassing. Thus, the police need one of the suspected criminals to rat out the other to charge them for the bigger crime.
- How can they do it?

Promise to the suspected criminals

- If no one confesses to the robbery, the police can only charge the prisoners for trespassing. The punishment will be 1 year in jail each.
- If one confesses and the other doesn't, the police will be lenient on the rat and severely punish the quiet one. The punishment will be 10 years in jail for the one who remains silence, and the rat will go free.
- If both confess, the police punish both of them equally. The punishment will be 4 years in jail each.



Can you predict what these two prisoners will do?

Prison A's Decision

Confess

Remain Silent

		Confess	Remain Silent
Prison B's Decision	Confess	Prisoner A gets 4 years Prisoner B gets 4 years	Prisoner A gets 10 years Prisoner B goes free
	Remain Silent	Prisoner A goes free Prisoner B gets 10 years	Prisoner A gets 1 year Prisoner B gets 1 year

Confess

**Prison B's
Decision**

**Remain
Silent**

The possible alternative decisions

- If prisoner A confesses, what should prisoner B do?
Confess is the dominant strategy
- If prisoner A doesn't confess, what should prisoner B do?
Confess is the dominant strategy
- If prisoner B confesses, what should prisoner A do?
Confess is the dominant strategy
- If prisoner B doesn't confess, what should prisoner A do?
Confess is the dominant strategy

A dominant strategy equilibrium is a strategy combination consisting of each player's dominant strategy. In this case, it is [Confess, Confess].

Both prisons A and B will choose Confess → The Nash Equilibrium

Prison A's Decision		Confess	Remain Silent
Prison B's Decision	Confess	Prisoner A gets 4 years Prisoner B gets 4 years	Prisoner A gets 10 years Prisoner B goes free
	Remain Silent	Prisoner A goes free Prisoner B gets 10 years	Prisoner A gets 1 year Prisoner B gets 1 year

Notes

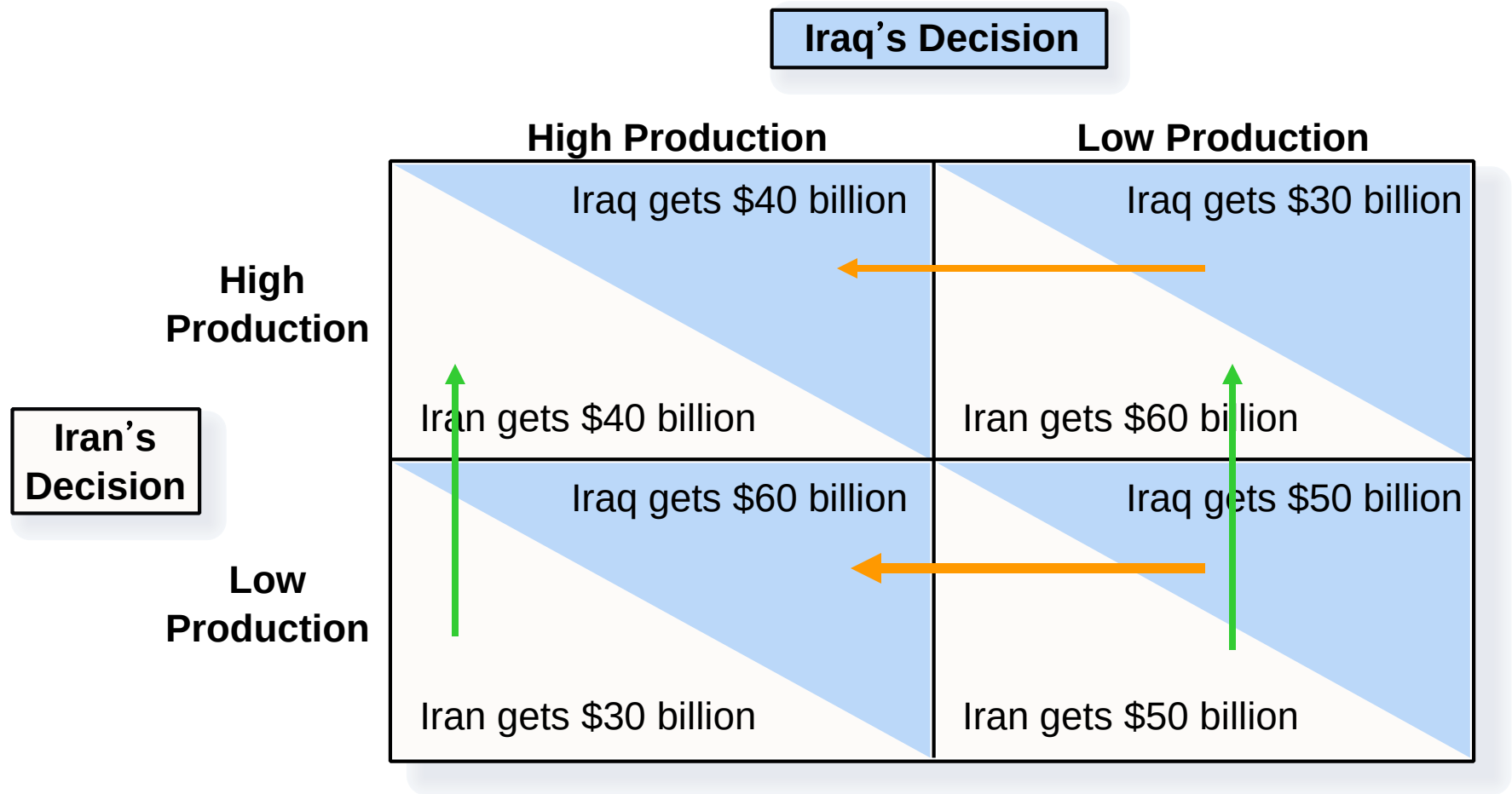
- In the Prisoners' Dilemma game, each player's dominant strategy is to confess, and yet, they would both be better off if they remained silent (the pursuit of **self-interest** leads to misery for all.)
- Cooperation is difficult to maintain, because cooperation is not in the best interest of the individual player.
- There is no guarantee that players will always play an equilibrium strategy, since it is certainly possible to guess incorrectly regarding what the other player is going to do.
- Another reason for the failure to achieve an equilibrium is that the players in the game were learning about the system and cheating by cooperating.

Applications of the Prisoners' Dilemma

Prisoner's Dilemma: An oil production game

		Iraq's Decision	
		High Production	Low Production
Iran's Decision	High Production	Iraq gets \$40 billion Iran gets \$40 billion	Iraq gets \$30 billion Iran gets \$60 billion
	Low Production	Iraq gets \$60 billion Iran gets \$30 billion	Iraq gets \$50 billion Iran gets \$50 billion

Both Iran and Iraq will choose high production → The Nash Equilibrium



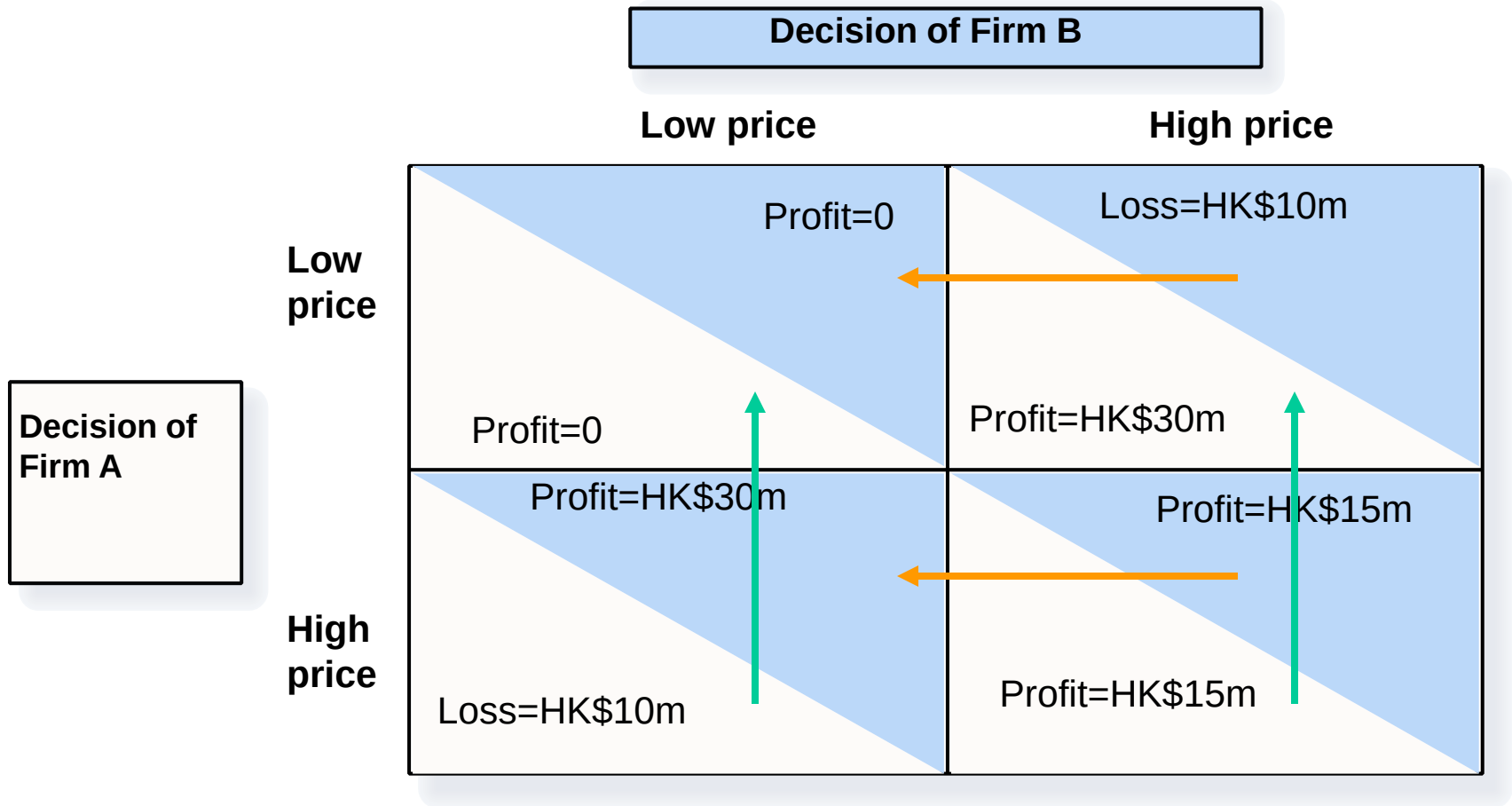
Prisoner's Dilemma:

The options available to each country are high production or low production. In this case, both countries choose high production even though both would be better off choosing low production.

Prisoner's Dilemma: a price war

		Decision of Firm B	
		Low price	High price
Decision of Firm A	Low price	Profit=0 Profit=0	Loss=HK\$10m Profit=HK\$30m
	High price	Profit=HK\$30m Loss=HK\$10m	Profit=HK\$15m Profit=HK\$15m

Both Firms A and will choose low price → The Nash Equilibrium



Prisoner's Dilemma:

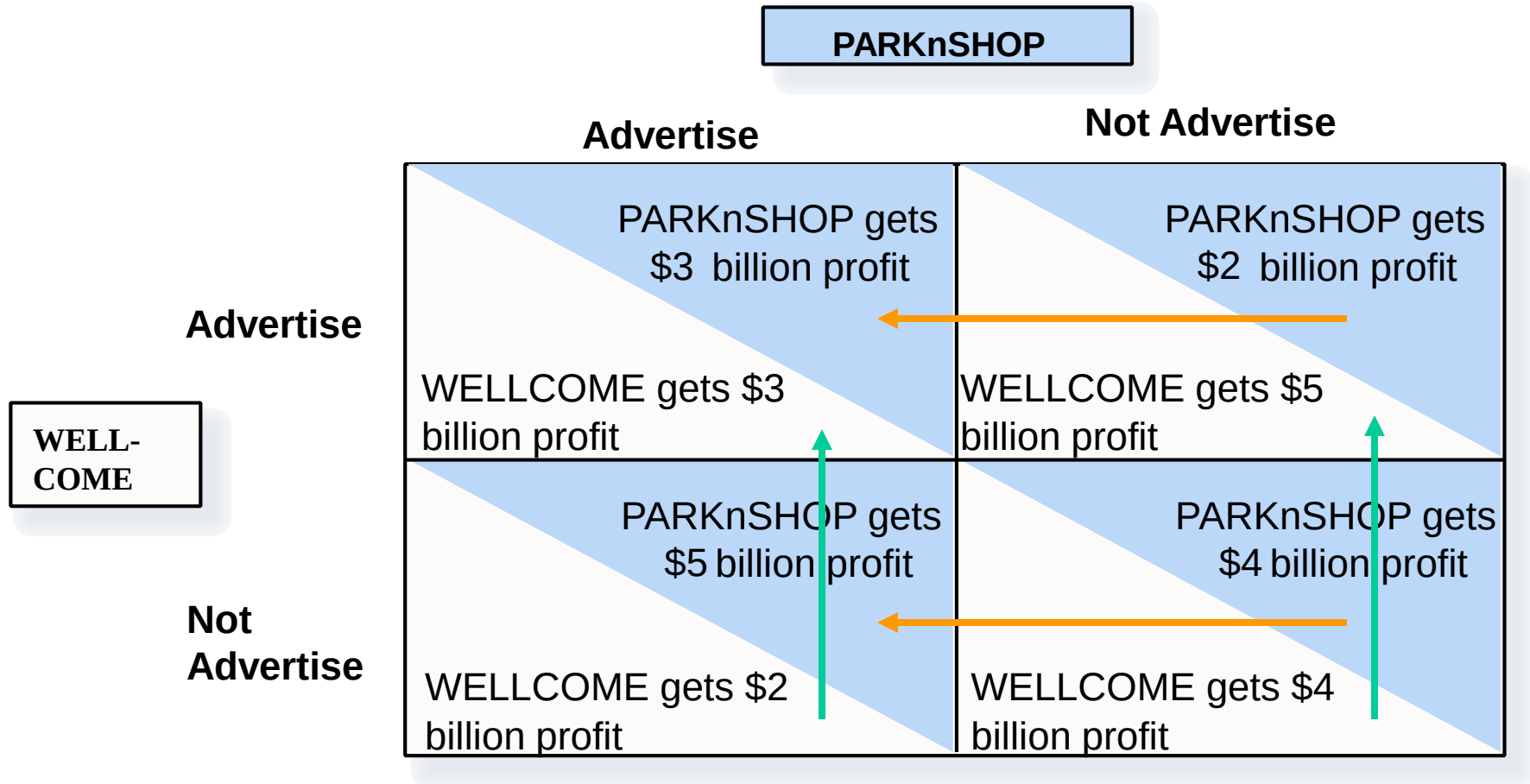
Each firm is better off charging a low price if the other chooses a high price. The dilemma is that they each select the low price which is the worst outcome.

Prisoner's Dilemma: An advertising game

		PARKnSHOP	
		Advertise	Not Advertise
WELL-COME	Advertise	PARKnSHOP gets \$3 billion profit WELLCOME gets \$3 billion profit	PARKnSHOP gets \$2 billion profit WELLCOME gets \$5 billion profit
	Not Advertise	PARKnSHOP gets \$5 billion profit WELLCOME gets \$2 billion profit	PARKnSHOP gets \$4 billion profit WELLCOME gets \$4 billion profit

Both PARKnSHOP and WELLCOME will choose to advertise

→ The Nash Equilibrium



Prisoner's Dilemma:

The options available to each firm are to advertise or not advertise. In this case, both firms advertise even though both would be better off not advertising.

Other Examples for one Nash Equilibrium Games

https://www.youtube.com/watch?v=jILgxeNBK_8

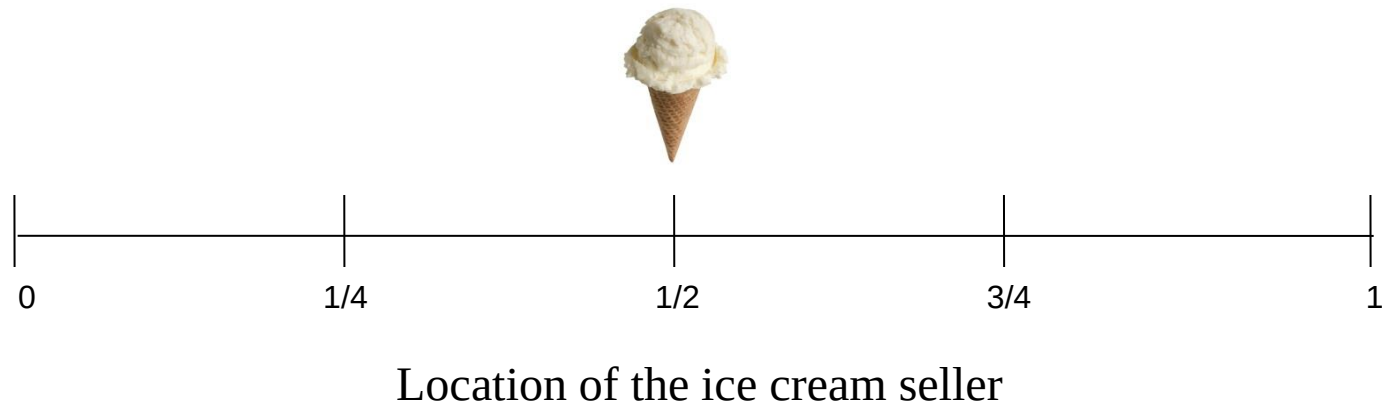
(1) Ice Cream Sellers Game

- There are “ n ” ice cream sellers going to locate their stands on a crowded beach.
- Assumptions :
 - Sellers are selling identical ice cream for the same price.
 - Beach of length l .
 - Beach is populated by consumers that is equally distributed along the beach.
 - Every consumer chooses the closest stand and buy just one ice cream
 - Sellers wants to maximize their revenues.
 - If sellers location is the same place, customers select stands randomly with equal probabilities.

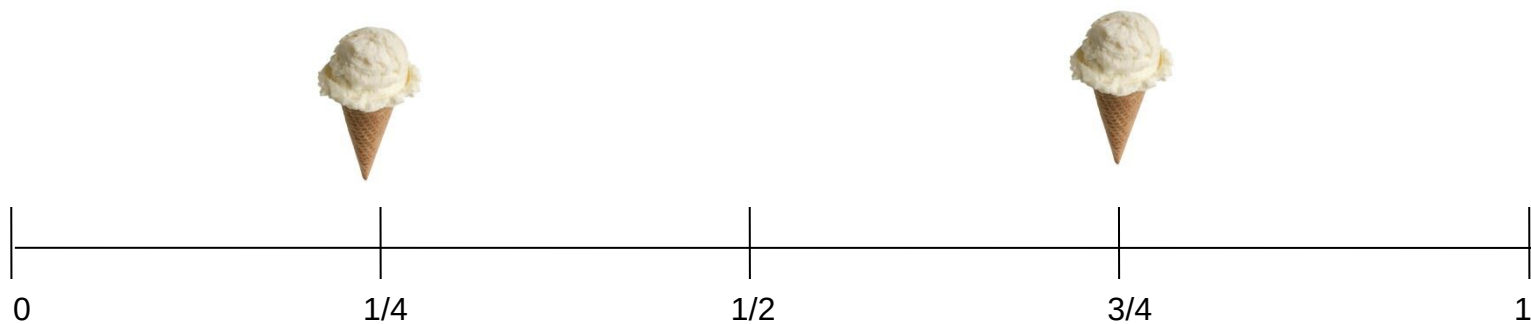


- How the stands should be located such that none of the sellers is willing to deviate?
- Is any seller locate his stand at the ends of the beach?
- What would happen if all the sellers are located at the same side of the beach?
- What would happen if half of the sellers locate their stand on the right side of the center customer and the other half on the left side?

Nash equilibrium: There is only one ice cream seller on the beach

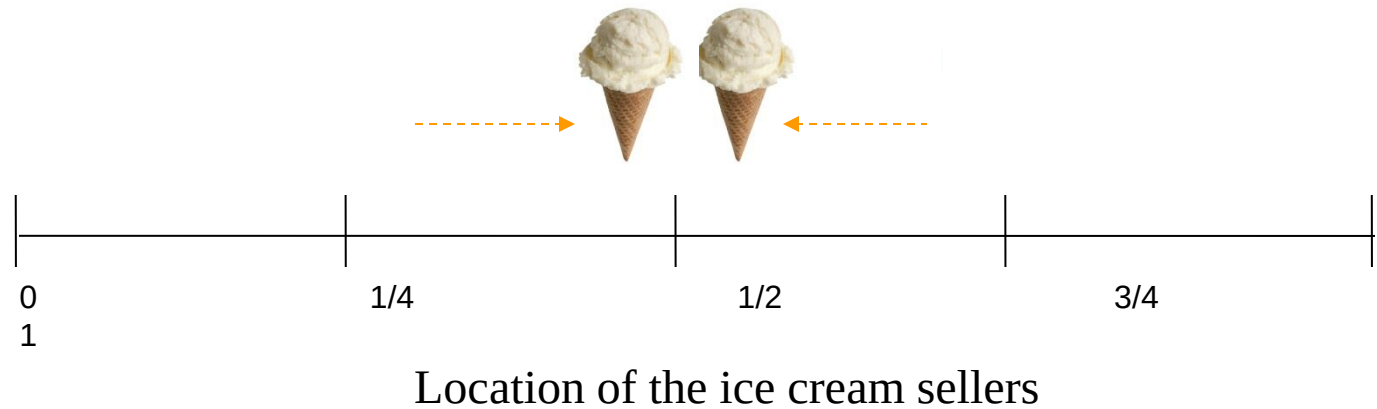


The second ice cream seller enters the market



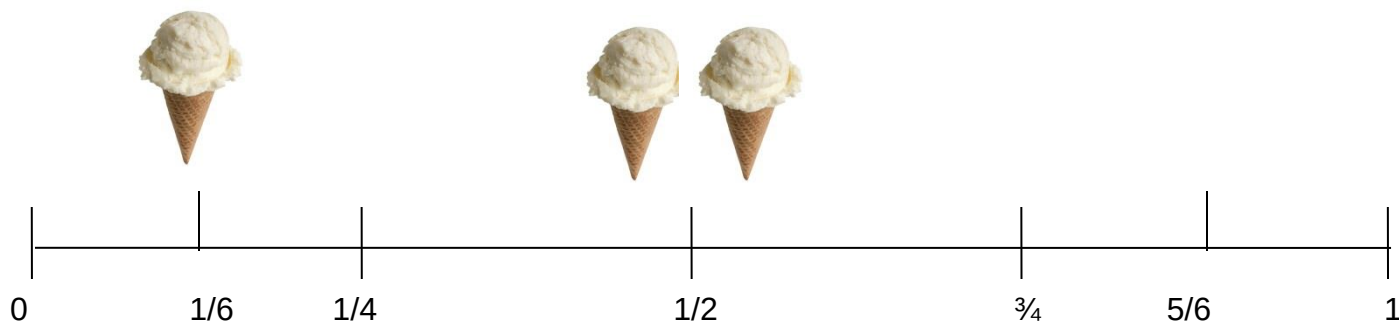
Location of the ice cream sellers

Nash equilibrium for two ice cream sellers on the beach

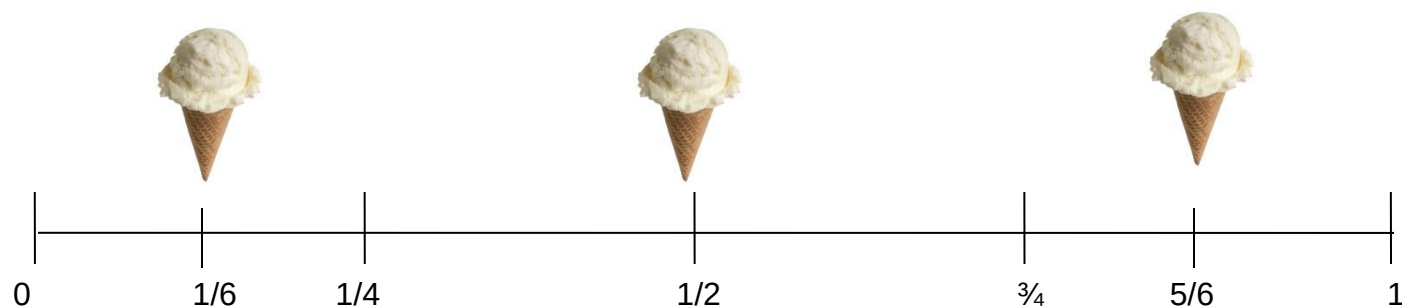


Nash equilibrium: Three ice cream sellers on the beach

Initial

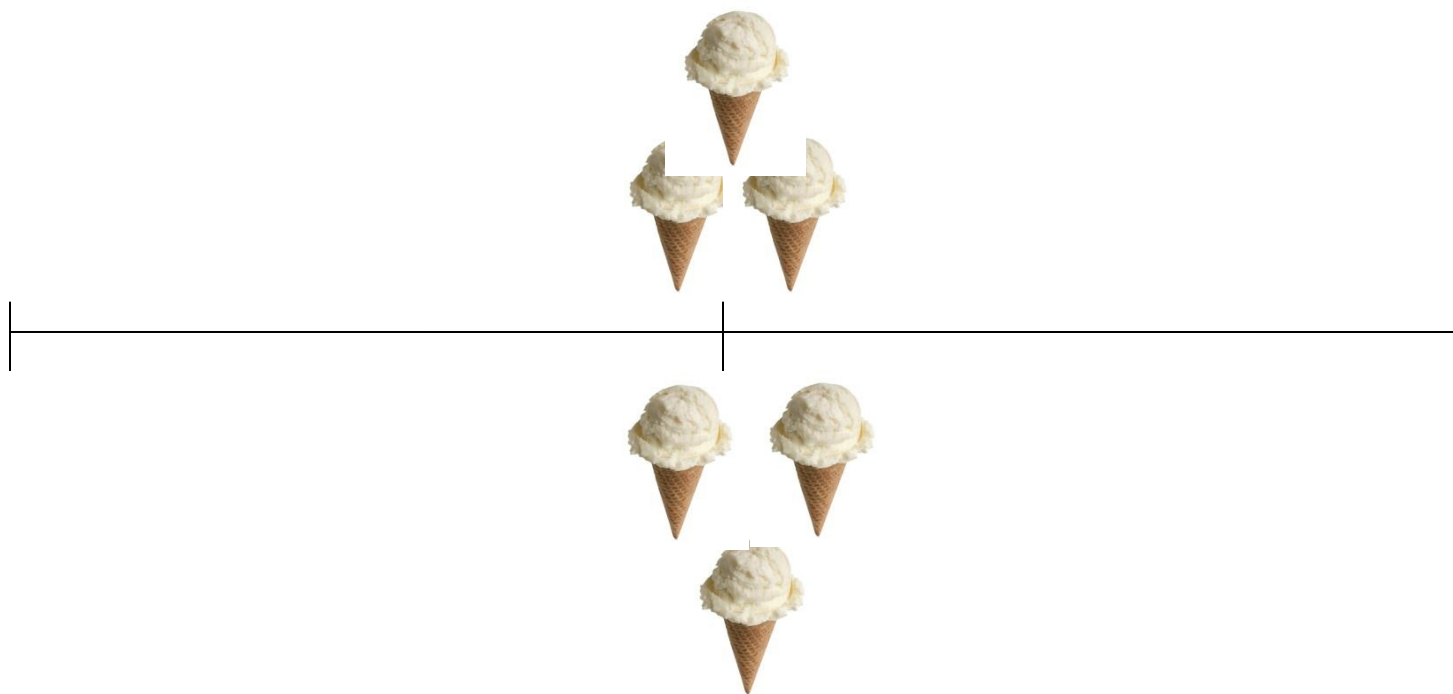


Final



Location of the ice cream sellers

Nash equilibrium: Many ice cream sellers on the beach



The Hotelling's Law

It shows the following points are rational:

- For sellers of same product to locate their shops next to each other (Agglomeration effect) .
- For two political parties to have similar agenda.
- For two producers to produce very similar products.

https://www.youtube.com/watch?v=jILgxeNBK_8

A Threat Game: The existing of a Nash equilibrium but it does not work

You are an economist who has been out of town arriving back on Christmas Eve. Your wife and children are upset that the house is not decorated, so you run out to buy a tree. You come to a tree lot where the owner is tossing trees into a big bonfire. You figure you can get a great tree at a very low price. You find a 12-foot spruce with a price tag of \$1000 on it.



Suppose you have 2 five dollar notes, 3 ten dollar notes, and 5 twenty dollar notes, 3 one hundred dollar notes, and 2 one thousand dollar notes in your wallet, and that you need to make a rational economic decision.

You say to the proprietor, “I want this 12-feet spruce and I will give you ?? for it.



Does the proprietor accept or reject the offer?

The proprietor responds: “Give me \$500 and you can have the tree. If not, I will burn it. **What should you do?**”

If you **believe** the proprietor, then the lower amount will be rejected and you will get nothing. Given the proprietor's **strategy**, your **best choice** is to give him \$500, since that action gives you a tree. And given that you offer the proprietor \$500, his **best choice** is to accept the offer.

Thus, one **Nash equilibrium** outcome of this sequential bargaining process yields a tree for you and \$500 for the proprietor.

But since you are an economist, you might figure \$5 is better than nothing, so you stick with your guns and offer just \$5. The proprietor picks up the tree and tosses it on the bonfire.

What was wrong with your strategy?

- You thought the proprietor's **threat** to be an empty or a non-credible one. The problem was that this threat was credible.
- The only way you might have known this threat was credible was to remember that people tend to look at **bargaining situations** as requiring a “**fair**” solution, and fair in most cases means something where each party gets about the same amount.
- The **ultimatum game** indicates that in bargaining situations people have in mind a division that is **fair** to them.
- If the share they are offered is not **fair**, they reject the offer even though they end up with zero.



Would it be possible to apply the concept of “**economic rationality**” to a real-life decision ?

An example
for
no Nash Equilibria

Rock, scissors, paper Game

- The zero payoffs along the diagonal show that if players adopt the same strategy, no payments are made.
- In other cases, the payoffs indicate a \$1 payment from the loser to winner under the usual hierarchy (Rock breaks Scissors, Scissors cut Paper, Paper covers Rock).

The Payoff Table

B's strategies

A's strategies

	paper	Scissors	Rock
paper	0, 0	1, -1	-1, 1
Scissors	-1, 1	0, 0	1, -1
Rock	1, -1	-1, 1	0, 0

It is a zero-sum game

- This game has no equilibrium.
- Any strategy pair is unstable since it offers at least one of the players an incentive to adopt another strategy.
 - If A forms Scissor and B forms Scissors provides an incentive for either A or B to choose Rock in the next round.
 - Also, if A forms Paper and B forms Rock encourages B to choose Scissors in the next round.

Thank you
for
your attention