

# THERMOGRAPHY – NEWTON'S COOLING LAW

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## Introduction

Newton's Law of Cooling states (Yakubov, 2007) that the rate of change of temperature of a body is directly proportional to the difference between the bodies temperature and the environment temperature. This translates mathematically to the below equation;

$$\frac{\delta T}{\delta t} = -k (T - T_{amb})$$

*Equation 1: Newton's Law of Cooling, differential form*

Where **k** is the positive proportionality constant, **T<sub>amb</sub>** is the ambient temperature and **T** is the body temperature at time **t**.

Using a thermography camera and the connected software – TIM Connect – this experiment shall monitor the temperature loss of the two containers filled with water, one open, one closed. Using the Newton's Cooling Law, a heat loss investigation will be performed. This raw data will then be used to determine a cooling constant **k**.

## Apparatus

The experimental apparatus includes a thermal imaging camera directly connected to a desktop computer that has the thermal imaging software installed. The thermal camera was situated on the same desk as the computer display. The desk located opposite the camera held the two plastic containers wrapped in PVC black electrical tape, the containers placed on the carbon fibre board. The scale was also placed alongside.

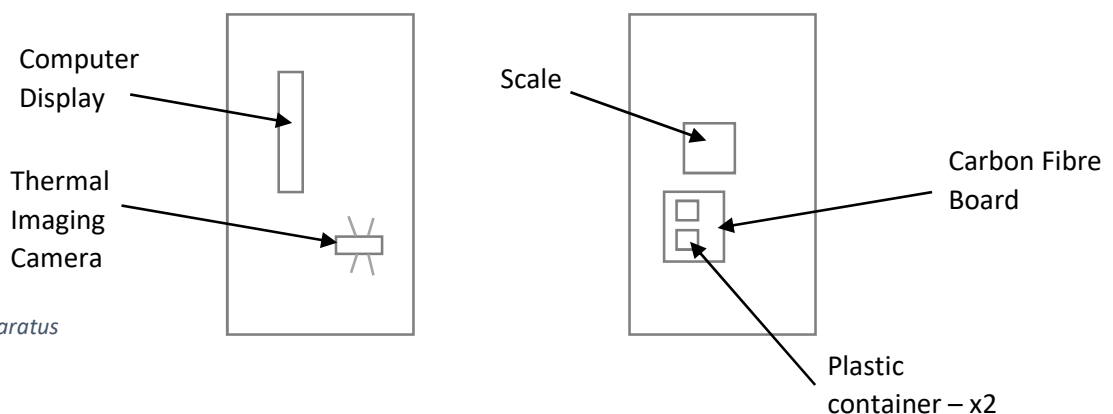


*Figure 1: Apparatus*

*Table 1 : Equipment overview*

| Equipment                                      | Uncertainty                                   |
|------------------------------------------------|-----------------------------------------------|
| Thermal imaging camera – Micro Epsilon TIM 160 | $\pm 2^{\circ}\text{C}$ (Micro-Epsilon, 2018) |
| Digital Balance – AND CVT 610                  | $\pm 0.1\text{g}$                             |
| Carbon fibre board                             | N/A                                           |
| 2 x Plastic container                          | N/A                                           |
| PVC black electrical tape                      | N/A                                           |
| Thermal imaging software – TIM Connect         | N/A                                           |
| Desktop computer and display                   | N/A                                           |
| Kettle                                         | N/A                                           |

## Method

The equipment was setup as figure 1 illustrates. Both containers of water were taped with black PVC prior to the experiment starting. Initially the empty containers were positioned in front of the thermal imaging camera to setup the measurement windows, the windows of measurement were adjusted to fit entirely on the container, allowing for no miscalculation of the mean temperature of the area. The emissivity value of the software was then inputted at 0.95, this is due to the black tape surrounding the containers, this value was obtained from FLIR (2015). Both containers were then filled with above ambient temperature water from the kettle. One container being covered and closed, the other left open. The containers were then weighed and a tare value of the water obtained for each of the samples. The containers were then placed onto the designated areas on the carbon fibre board. Before conducting the temperature measurements, the ambient temperature for the room was taken. The thermal imaging software then allowed a temperature value to be recorded every thirty seconds and plotted for both the closed and open containers. After ten minutes of obtained data the containers were then immediately weighed, and a tare value of water recorded.

## Measurements and Raw Data

The temperature data obtained are shown in appendix 1 those values have been plotted within the graph below;

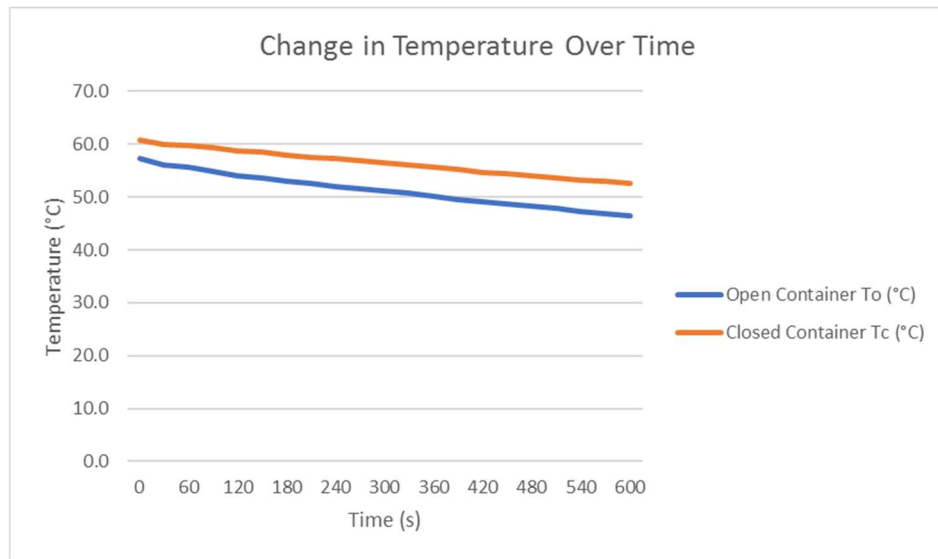


Figure 2: Temperature v time graph

The initial temperature of the open container =  $57.4^{\circ}\text{C}$

The initial temperature of the closed container =  $60.8^{\circ}\text{C}$

Initial mass of open container = 108.1g

Final mass of open container = 107.0g

Initial mass of closed container = 116.3g

Final mass of closed container = 116.2g

Ambient temperature =  $20^{\circ}\text{C}$

## Calculations and Error Estimates

Using Newton's Law;

$$\frac{\delta T}{\delta t} = -k (T - T_{amb})$$

Equation 2: Newton's Cooling Law

the cooling constant  $k$  can be determined. From the values obtained in the experiment the below values of  $k$  were calculated.

|          | Open Container | Closed Container | Open Container | Closed Container |
|----------|----------------|------------------|----------------|------------------|
| Time (s) | $T_o$ (°C)     | $T_c$ (°C)       | $k_o$          | $k_c$            |
| 0        | 57.4           | 60.8             | 0.000000       | 0.000000         |
| 30       | 56.0           | 60.0             | -0.000833      | -0.000444        |
| 60       | 55.6           | 59.7             | -0.000240      | -0.000168        |
| 90       | 54.9           | 59.4             | -0.000425      | -0.000168        |
| 120      | 54.0           | 58.7             | -0.000556      | -0.000398        |
| 150      | 53.7           | 58.5             | -0.000186      | -0.000114        |
| 180      | 53.0           | 58.0             | -0.000440      | -0.000287        |
| 210      | 52.6           | 57.6             | -0.000253      | -0.000231        |
| 240      | 52.0           | 57.3             | -0.000385      | -0.000175        |
| 270      | 51.5           | 56.8             | -0.000324      | -0.000293        |
| 300      | 51.1           | 56.4             | -0.000261      | -0.000236        |
| 330      | 50.7           | 56.1             | -0.000263      | -0.000178        |
| 360      | 50.2           | 55.7             | -0.000332      | -0.000239        |
| 390      | 49.6           | 55.2             | -0.000403      | -0.000302        |
| 420      | 49.1           | 54.7             | -0.000339      | -0.000305        |
| 450      | 48.6           | 54.4             | -0.000343      | -0.000184        |
| 480      | 48.2           | 54.1             | -0.000277      | -0.000185        |
| 510      | 47.8           | 53.7             | -0.000279      | -0.000248        |
| 540      | 47.3           | 53.3             | -0.000352      | -0.000250        |
| 570      | 46.8           | 52.9             | -0.000356      | -0.000252        |
| 600      | 46.5           | 52.5             | -0.000215      | -0.000254        |

Table 2: Cooling Constant Value Data

The differential form of the cooling law can be used again to determine the units of  $k$ .

$$\frac{\delta T}{\delta t} = -k (T - T_{amb})$$

$$\frac{^{\circ}\text{C}}{\text{s}} = k (^{\circ}\text{C})$$

$$\frac{^{\circ}\text{C}}{\frac{\text{s}}{^{\circ}\text{C}}} = k$$

$$\frac{1}{^{\circ}\text{C}} \times \frac{^{\circ}\text{C}}{\text{s}} = k \quad k = \frac{1}{\text{s}} \quad k = \text{s}^{-1}$$

Mean value of  $k$  for the open container =  $-0.58 \times 10^{-3} \text{ s}^{-1}$

Mean value of  $k$  for the closed container =  $-0.381 \times 10^{-3} \text{ s}^{-1}$

Using a standard deviation from our values of  $k$  we can show which values are within one standard deviation of the mean, these are observed below for the open and closed container results.

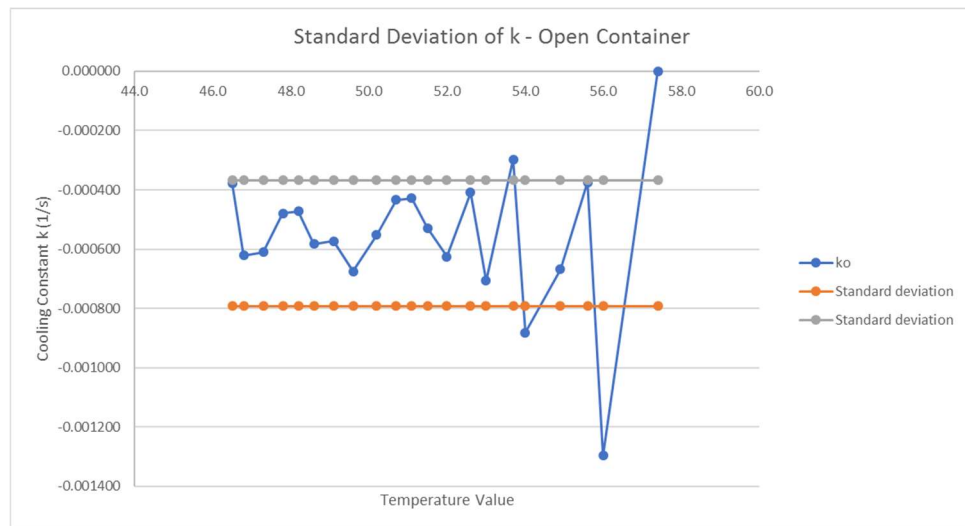


Figure 3: Standard Deviation of  $k$  - Open Container

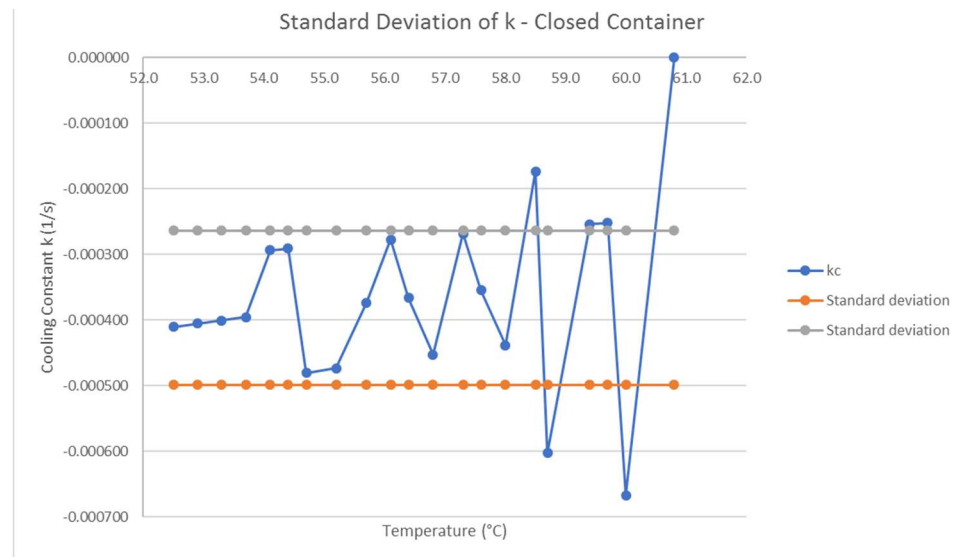


Figure 4: Standard Deviation of  $k$  - Closed Container

Both set of results show  $k$  values beyond one standard deviation, these values also deviate at higher temperatures on both set of results. The graph of both open and closed containers

show a similar pattern, the k values on both results at lower temperatures show less deviation from the mean.

### Heat Loss Calculations

To estimate the accumulative heat loss over the duration of the experiment, the specific heat loss equation can be used;

$$Q = m \cdot C_p \cdot \Delta T$$

*Equation 3: Specific Heat Loss*

The specific heat of water is 4180 J/kg.°C, to estimate the heat loss the mean value of the water mass will be used.

$$\text{Open Container} - Q_o = \left( \frac{0.1081 - 0.107}{2} \right) \cdot 4180 \cdot (57.4 - 46.5) = 4900.2J = 4.9kJ$$

$$\text{Closed Container } Q_c = \left( \frac{0.1163 - 0.1162}{2} \right) \cdot 4180 \cdot (60 - 52.5) = 3644.4J = 3.64kJ$$

Due to the closed container being sealed, a heat energy loss through evaporation would be negligible, the change of mass from the beginning of the experiment to end I likely due to the uncertainty in the scale ( $\pm 0.1g$ ). Using the observed water mass loss from the open container, the heat loss energy through evaporation can be estimated. The specific latent heat value of water will be assumed to be 2260 kJ/kg.

$$\text{Mass loss} = 0.1081 - 0.107 = 0.0011 \text{ kg}$$

*Equation 4: Specific Latent Heat*

$$Q_{evap} = m \cdot L$$

$$Q_{evap} = 0.0011 \times 2260$$

$$Q_{evap} = 2.49 \text{ kJ}$$

Using the estimated accumulated heat loss of the open container, the heat lost from other heat transfer methods can be determined.  $Q_0 - Q_{\text{evap}} = 4.9 - 2.49 = 2.41 \text{ kJ}$

### Radiation Intensity Calculations

Stefan Boltzmann's Law (Johnson, 2012) describes the power emitted from a body that absorbs all radiation that hits its surface in terms of its temperature, this type of surface is described as a black body. The Stefan Boltzmann equation for a black body is written as below;

$$I = \frac{P}{A} = \sigma T^4$$

I – Radiation Intensity

P – Power

A – Area

$\sigma$  – Stefan Boltzmann Constant

T – Temperature

Assuming the container surface behaves like a grey body the equation must consider the emissivity of the container, therefore the equation must then be written as;

$$I = \epsilon \sigma T^4$$

With the container being wrapped in black electrical tape it is assumed that the emissivity value is 0.95. Using Stefan Boltzmann's constant of  $5.67 \times 10^{-8} \text{ W.m}^2.\text{k}^4$  the total radiation intensity for each temperature value can be calculated, these values have been plotted on

figure 5.

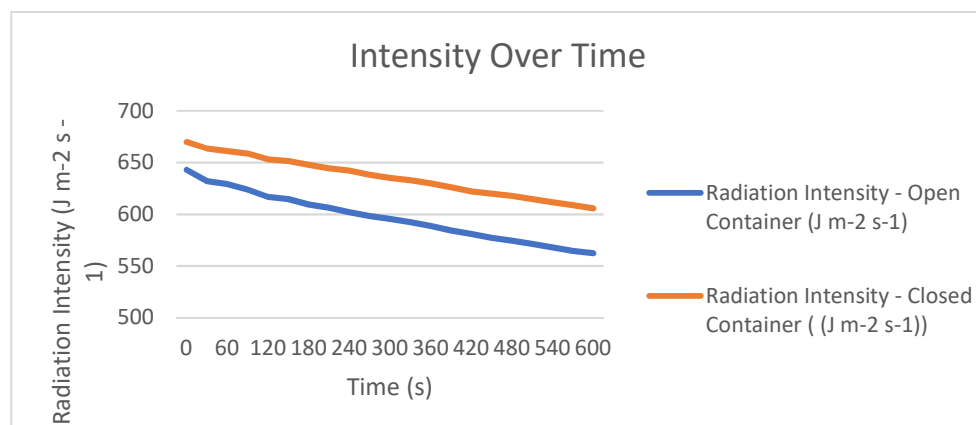


Figure 5: Intensity over time graph

## Discussion

### Emissivity Value

The emissivity value of a body must be calculated and assumed correctly due to the importance this has on the temperature readings, this is agreed by PBSI (2017) who state that slight errors in emissivity values can lead to significant errors in temperature. Due to the lower emissivity value of an object it will radiate less heat as Stefan Boltzmann's Law states. Therefore, when two objects of the same temperature are evaluated the low emissivity value body will be calculated at a lower temperature due to the lower radiation intensity value. This can be mathematically presented using the Stefan Boltzmann calculation;

Temperature value = 20°C = 293.15K

$$I = \varepsilon \sigma T^4$$

$$I = 0.95 \times 5.67 \times 10^{-8} \times 293.15^4$$

$$I = 397.8 \text{ Jm}^{-2}\text{s}^{-1}$$

$$I = 0.98 \times 5.67 \times 10^{-8} \times 293.15^4$$

$$I = 410.4 \text{ Jm}^{-2}\text{s}^{-1}$$

Value of radiation intensity is noticeably lower when changing from 0.95 emissivity to 0.98, giving a significant difference in temperature values.

### Heat Transfer Methods

From the heat energy loss calculations of the open container it can be observed that more than half of the heat loss was estimated to be through evaporation, however other forms of heat transfer methods are occurring. Heat loss through radiation will occur through the outer edge of the container, this will then be lost to the surrounding air. The hotter liquid

within the water will also move to the colder liquid making its way to the internal edge of the container using convection. The heated inner edge will then transfer the heat to the outer edge using conduction transfer, this can then radiate to the surrounding or conducting the carbon fibre board. The closed container will lose heat in a similar way without the heat loss from evaporation.

## Conclusion

The initial graph (figure 2) demonstrates that Newton's Law of Cooling is obeyed throughout the experiment, the graph also reveals that due to the evaporation heat transfer method in the open container the heat is lost at a faster rate. Other heat transfer methods – conduction, radiation, convection - are also present. Using Newton's Cooling Law differential form the report could estimate the cooling constant  $k$  of the open and closed container.

Mean value of  $k$  for the open container =  $-0.58 \times 10^{-3} \text{ s}^{-1}$

Mean value of  $k$  for the closed container =  $-0.381 \times 10^{-3} \text{ s}^{-1}$

The report has illustrated the importance of the emissivity value inputted within the infrared camera, this is shown when applying Stefan Boltzmann's Law.

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## Appendix

### Appendix 1

| Time (s) | Open Container<br>$T_o$ (°C) | Closed Container<br>$T_c$ (°C) |
|----------|------------------------------|--------------------------------|
| 0        | 57.4                         | 60.8                           |
| 30       | 56.0                         | 60.0                           |
| 60       | 55.6                         | 59.7                           |
| 90       | 54.9                         | 59.4                           |
| 120      | 54.0                         | 58.7                           |
| 150      | 53.7                         | 58.5                           |
| 180      | 53.0                         | 58.0                           |
| 210      | 52.6                         | 57.6                           |
| 240      | 52.0                         | 57.3                           |
| 270      | 51.5                         | 56.8                           |
| 300      | 51.1                         | 56.4                           |
| 330      | 50.7                         | 56.1                           |
| 360      | 50.2                         | 55.7                           |
| 390      | 49.6                         | 55.2                           |
| 420      | 49.1                         | 54.7                           |
| 450      | 48.6                         | 54.4                           |
| 480      | 48.2                         | 54.1                           |
| 510      | 47.8                         | 53.7                           |
| 540      | 47.3                         | 53.3                           |
| 570      | 46.8                         | 52.9                           |
| 600      | 46.5                         | 52.5                           |

## Appendix 2

| Time (s) | Open Container |         | Closed Container |         | Open Container |          | Closed Container |          | Open Container | Closed Container |
|----------|----------------|---------|------------------|---------|----------------|----------|------------------|----------|----------------|------------------|
|          | To (°C)        | ΔT (°C) | Tc (°C)          | ΔT (°C) | ΔT/Δt          | T - Tamb | ΔT/Δt            | T - Tamb | ko             | kc               |
| 0        | 57.4           | 0       | 60.8             | 0       | 0.000000       | 37.4     | 0.000000         | #REF!    | 0.000000       | 0.000000         |
| 30       | 56.0           | 1.4     | 60.0             | 0.8     | 0.046667       | 36.0     | 0.026667         | 40.0     | -0.001296      | -0.000667        |
| 60       | 55.6           | 0.4     | 59.7             | 0.3     | 0.013333       | 35.6     | 0.010000         | 39.7     | -0.000375      | -0.000252        |
| 90       | 54.9           | 0.7     | 59.4             | 0.3     | 0.023333       | 34.9     | 0.010000         | 39.4     | -0.000669      | -0.000254        |
| 120      | 54.0           | 0.9     | 58.7             | 0.7     | 0.030000       | 34.0     | 0.023333         | 38.7     | -0.000882      | -0.000603        |
| 150      | 53.7           | 0.3     | 58.5             | 0.2     | 0.010000       | 33.7     | 0.006667         | 38.5     | -0.000297      | -0.000173        |
| 180      | 53.0           | 0.7     | 58.0             | 0.5     | 0.023333       | 33.0     | 0.016667         | 38.0     | -0.000707      | -0.000439        |
| 210      | 52.6           | 0.4     | 57.6             | 0.4     | 0.013333       | 32.6     | 0.013333         | 37.6     | -0.000409      | -0.000355        |
| 240      | 52.0           | 0.6     | 57.3             | 0.3     | 0.020000       | 32.0     | 0.010000         | 37.3     | -0.000625      | -0.000268        |
| 270      | 51.5           | 0.5     | 56.8             | 0.5     | 0.016667       | 31.5     | 0.016667         | 36.8     | -0.000529      | -0.000453        |
| 300      | 51.1           | 0.4     | 56.4             | 0.4     | 0.013333       | 31.1     | 0.013333         | 36.4     | -0.000429      | -0.000366        |
| 330      | 50.7           | 0.4     | 56.1             | 0.3     | 0.013333       | 30.7     | 0.010000         | 36.1     | -0.000434      | -0.000277        |
| 360      | 50.2           | 0.5     | 55.7             | 0.4     | 0.016667       | 30.2     | 0.013333         | 35.7     | -0.000552      | -0.000373        |
| 390      | 49.6           | 0.6     | 55.2             | 0.5     | 0.020000       | 29.6     | 0.016667         | 35.2     | -0.000676      | -0.000473        |
| 420      | 49.1           | 0.5     | 54.7             | 0.5     | 0.016667       | 29.1     | 0.016667         | 34.7     | -0.000573      | -0.000480        |
| 450      | 48.6           | 0.5     | 54.4             | 0.3     | 0.016667       | 28.6     | 0.010000         | 34.4     | -0.000583      | -0.000291        |
| 480      | 48.2           | 0.4     | 54.1             | 0.3     | 0.013333       | 28.2     | 0.010000         | 34.1     | -0.000473      | -0.000293        |
| 510      | 47.8           | 0.4     | 53.7             | 0.4     | 0.013333       | 27.8     | 0.013333         | 33.7     | -0.000480      | -0.000396        |
| 540      | 47.3           | 0.5     | 53.3             | 0.4     | 0.016667       | 27.3     | 0.013333         | 33.3     | -0.000611      | -0.000400        |
| 570      | 46.8           | 0.5     | 52.9             | 0.4     | 0.016667       | 26.8     | 0.013333         | 32.9     | -0.000622      | -0.000405        |
| 600      | 46.5           | 0.3     | 52.5             | 0.4     | 0.010000       | 26.5     | 0.013333         | 32.5     | -0.000377      | -0.000410        |
|          |                |         |                  |         |                |          |                  |          | Mean Value     | Mean Value       |
|          |                |         |                  |         |                |          |                  |          | -0.000580      | -0.000381        |

## Appendix 3

|             |             | Open Container K Standard Deviation |           | Closed Container k Standard Deviation |           |                       |                         |                       |                       |  |  |  |
|-------------|-------------|-------------------------------------|-----------|---------------------------------------|-----------|-----------------------|-------------------------|-----------------------|-----------------------|--|--|--|
|             |             | Standard deviation                  |           | Standard deviation                    |           |                       |                         |                       |                       |  |  |  |
| Open        | Closed      | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | Open Container To (K) | Closed Container Tc (K) | Radiation Intensity - | Radiation Intensity - |  |  |  |
| 5.13301E-07 | 8.13553E-08 | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 330.55                | 333.95                  | 643.0656474           | 669.9346726           |  |  |  |
| 4.21539E-08 | 1.67829E-08 | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 329.15                | 333.15                  | 632.2401936           | 663.5382065           |  |  |  |
| 7.87319E-09 | 1.62897E-08 | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 328.05                | 332.85                  | 629.1724682           | 661.1513809           |  |  |  |
| 9.15105E-08 | 4.9058E-08  | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 327.15                | 331.85                  | 617.0130687           | 658.7710003           |  |  |  |
| 8.01513E-08 | 4.33797E-08 | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 326.85                | 331.65                  | 614.7529499           | 651.6684128           |  |  |  |
| 1.61862E-08 | 3.26708E-09 | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 326.15                | 331.15                  | 609.5034885           | 647.7474347           |  |  |  |
| 2.9189E-08  | 7.19753E-10 | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 325.75                | 330.75                  | 606.5189322           | 644.6234136           |  |  |  |
| 2.03889E-09 | 1.28463E-08 | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 325.15                | 330.45                  | 602.0626663           | 642.2878239           |  |  |  |
| 2.57509E-09 | 5.10659E-09 | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 324.65                | 329.95                  | 598.3679078           | 638.4092853           |  |  |  |
| 2.28377E-08 | 2.29152E-10 | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 324.25                | 329.55                  | 595.4243664           | 635.3191235           |  |  |  |
| 2.11805E-08 | 1.09056E-08 | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 323.85                | 329.25                  | 592.4916985           | 633.0088746           |  |  |  |
| 7.82293E-10 | 6.32886E-11 | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 323.35                | 328.85                  | 588.8411135           | 629.9383506           |  |  |  |
| 9.18335E-09 | 8.4726E-09  | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 322.75                | 328.35                  | 584.4827092           | 626.1159202           |  |  |  |
| 5.05265E-11 | 9.77513E-09 | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 322.25                | 327.85                  | 580.8692257           | 622.3109119           |  |  |  |
| 8.43728E-12 | 8.23383E-09 | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 321.75                | 327.55                  | 577.272523            | 620.0362475           |  |  |  |
| 1.1456E-08  | 7.77624E-09 | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 321.35                | 327.25                  | 574.4072082           | 617.7678246           |  |  |  |
| 1.0046E-08  | 2.01917E-10 | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 320.95                | 326.85                  | 571.5525731           | 614.7529499           |  |  |  |
| 9.39712E-10 | 3.59567E-10 | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 320.45                | 326.45                  | 567.9992573           | 611.7491237           |  |  |  |
| 1.76775E-09 | 5.67886E-10 | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 319.95                | 326.05                  | 564.4625355           | 608.7563191           |  |  |  |
| 4.10011E-08 | 8.30493E-10 | -0.000792                           | -0.000367 | -0.000499                             | -0.000264 | 319.65                | 325.65                  | 562.3484459           | 605.7745089           |  |  |  |
| Mean        | Mean        |                                     |           |                                       |           |                       |                         |                       |                       |  |  |  |
| 4.52116E-08 | 1.38111E-08 |                                     |           |                                       |           |                       |                         |                       |                       |  |  |  |
| 0.00021263  | 0.00011752  |                                     |           |                                       |           |                       |                         |                       |                       |  |  |  |