

$$(ii) \int_1^e \ln x \, dx$$

$$\left[x \ln x - x + C \right]_1^e$$

$$= 1$$

$$3. a. \left(\frac{1+i}{1-i} \right)^3$$

$$\left[\frac{1+i}{1-i} \times \frac{1+i}{1-i} \times \frac{1+i}{1-i} \right]$$

$$\downarrow -1 \times \frac{1+i}{1-i} = -i \quad = \boxed{0-i}$$

$$ii \quad \frac{3}{(1+i)(2+i)} = \frac{3}{2+xi+2i+i^2} = \frac{3}{2+3i+i^2} = \frac{3}{1+3i}$$

$$iii \quad \frac{3}{1+3i} \times \frac{1-3i}{1-3i} = \frac{3-9i}{1-9i^2} = \boxed{\frac{3-9i}{10}}$$

$$iii \quad \frac{1+i}{1-i} \left(\frac{(1+2i)(2+2i)}{2+2i+4i+4i^2} \right) + \frac{3i}{1+i}$$

$$2+6i-4$$

$$\frac{1+i}{1-i} = (6i-2) + \frac{3-i}{1+i} = (1-i) - (6i+2)$$

$$\boxed{= -3-7i}$$

$$\begin{aligned}
 \text{iv. } & 2i(i-1) + (\sqrt{3}-i) + (1+i)(1-i) \\
 & (2i^2 - 2i) + (\sqrt{3}-i) + (1+i)(1-i) \\
 & 2i^2 - 2i + (\sqrt{3}-i) + (1-i^2) \\
 & (2(-1) - 2i) + (\sqrt{3}-i) + (1-(-1)) \\
 & -2 - 2i + \sqrt{3} - i + 2 \\
 & = \sqrt{3} - 3i
 \end{aligned}$$

$$\begin{aligned}
 6. \quad z &= (4+7i)(3-2i) \\
 & 12 - 8i + 21i - 14i^2 \\
 & 12 - 14(-1) + 13i \\
 & 26 + 13i
 \end{aligned}$$

$$\sqrt{26^2 + 13^2} = 13\sqrt{5} = 29.069$$

$$\tan^{-1} \frac{13}{26} = 26.565$$

$$= 29.069 (\cos 26.57 + i \sin 26.57)$$

$$\begin{aligned}
 6. \quad \frac{1+3i}{1+i} \times \frac{1-i}{1-i} &= \frac{1-i+3i-3i^2}{1-i^2} = \frac{1-3(-1)+2i}{1-(-1)} = \frac{4+2i}{2} \\
 &= 2+i
 \end{aligned}$$

$$2. \quad C_i \int_{\ln 2}^{\ln 5} \frac{3e^{2x}}{\sqrt{e^x - 1}} dx$$

$$\int_{\ln 2}^{\ln 5} \frac{e^{2x}}{\sqrt{e^x - 1}}$$

$$u = e^x - 1$$

$$u' = e^x$$

$$dx = \frac{e^{-x}}{e}$$

$$du = e^{2x} = (u+1)$$

$$= \int \frac{u+1}{\sqrt{u}} du$$

$$\int \sqrt{u} \left(+ \frac{1}{\sqrt{u}} \right)$$

$$= \frac{2}{3} u^{3/2} + 2\sqrt{u}$$

$$\left(\frac{2}{3} (e^x - 1)^{3/2} + 2\sqrt{e^x - 1} \right)$$

$$2(e^x - 1)^{3/2} + 6\sqrt{e^x - 1}$$

$$= \left[\sqrt{2e^x - 1} (e^x + 2) + C \right]_{\ln 2}^{\ln 5}$$

$$= 20$$

$$2 \text{ III} \quad 3 \int_0^4 \frac{1}{2x+1} = 3 \left(\ln(2x+1) \right) \Big|_0^4$$

$$4 \int_0^4 \frac{1}{x+4} = 4 \left(\ln(x+4) \right) \Big|_0^4$$

$$\left[\frac{3}{2} \ln(2x+1) + \ln(x+4) \right]_0^4$$

$$\left(\frac{3}{2} \ln 9 + \ln 8 \right) - \left(\frac{3}{2} \ln 1 + \ln 4 \right)$$

$$= 3.989$$

$$b. \int \frac{4}{x(1+4 \ln x)^2} dx$$

$$u = 4 \ln x + 1$$

$$u' = \frac{4}{x} \quad dx = \frac{x}{4}$$

$$\int \frac{1}{u^2} du$$

$$= -\frac{1}{u} = -\frac{1}{(4 \ln x + 1)} + C$$

2. K. i $\int \frac{2x}{(4+3x^2)} dx$ $u = 4+3x^2$
 $u' = 6x$

$$\int \frac{1}{3u} = \frac{1}{3} \int \frac{1}{u}$$

$$\frac{1}{3} \ln(u) = \frac{\ln u}{3}$$

$$\frac{\ln u}{3} = \frac{\ln 4+3x^2}{3} + C$$

ii $\int_0^4 e^{\frac{1}{2}x} dx$

$$\int_0^4 \left[2e^{\frac{1}{2}x} \right]_0^4 \left(2e^{\frac{1}{2}(4)} + C \right) - \left(2e^{\frac{1}{2}(0)} + C \right)$$

$$= 12.778$$

iii $\int_0^4 \frac{5x+13}{(2x+1)(x+4)} dx$

$$\int_0^4 \frac{3}{2x+1} + \frac{1}{x+4} dx$$

$$\int_0^4 \frac{1}{2x+1} + \int_0^4 \frac{1}{x+4}$$

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$$b. y = \frac{1}{x} + \cos 2x$$

$$\frac{d^5 y}{dx^5}$$

$$\frac{dy}{dx} = -\frac{1}{x^2} - 2\sin(2x)$$

$$\frac{d^2 y}{dx^2} = \frac{2}{x^3} - 4\cos(2x)$$

$$\frac{d^3 y}{dx^3} = -\frac{6}{x^4} + 8\sin(2x)$$

$$d^4 y$$

$$\frac{d^4 y}{dx^4} = \frac{24}{x^5} + 16\cos(2x)$$

$$\frac{d^5 y}{dx^5} = -\frac{120}{x^6} - 32\sin(2x)$$

$$\frac{d^5 y}{dx^5} = -\left(32\sin(2x) + \frac{120}{x^6}\right)$$

$$c. f(x) = \ln \operatorname{cosec} x - \cot x$$

$$= \frac{1}{\operatorname{cosec} x - \cot x} \times d(\operatorname{cosec} x - \cot x)$$

$$= \frac{-(\cot x) \operatorname{cosec} x - (-\operatorname{cosec}^2 x)}{\operatorname{cosec} x - \cot x} = \underline{\underline{\operatorname{cosec}(x)}}$$

$$1. iv \quad y = \sec^3 4x$$

$$\frac{dy}{dx} = 3 \sec^2 4x \times \frac{d}{dx} \sec 4x$$

$$\cancel{3 \sec^2(4x)} \cancel{\sec(4x)} \cancel{\tan(4x)} (4x)$$

$$= \cancel{12 \sec^3 4x}$$

$$3 \sec^2(4x) \sec(4x) \tan(4x) (4x)$$

$$\downarrow \quad 3 \sec^3(4x) \times 4 \times x'$$

$$\downarrow \quad 12 \tan(4x) \sec(4x) \times 1$$

$$= 12 \sec^3(4x) \tan(4x)$$

$$V. \quad y = (x + \sin 2x)$$

$$= \cos 2x \frac{d}{dx} 2x + 1$$

$$= \cos 2x \times 2 \times \frac{d}{dx} x + 1$$

$$= 2 \cos(2x) + 1$$

$$vi \quad y = \frac{1-10x}{(2x-1)^2} = \frac{u'v - uv'}{v^2}$$

$$= \frac{-10(2x-1) - (1-10x)(2)}{(2x-1)^3}$$

$$= \frac{-10(2x-1) - 4(1-10x)}{(2x-1)^3} = \frac{20x+6}{(2x-1)^3}$$

Problem Paper 2.

i. a; $y = \frac{x^2}{2x+1}$ $\frac{d}{dx} \frac{1}{g} = \frac{\frac{dy}{dx} \cdot g - 1 \cdot \frac{dg}{dx}}{g^2}$

$$\frac{2x(2x+1) - x^2(2)}{(2x+1)^2} = \frac{2x(2x+1)}{(2x+1)^2}$$

ii $y = \sqrt{x^2+1}$
 $y = (x^2+1)^{\frac{1}{2}}$

$u = x^2+1$
 $u' = 2x$

$y = u^{\frac{1}{2}}$
 $\frac{dy}{du} = \frac{1}{2} u^{-\frac{1}{2}} = \frac{1}{2} (x^2+1)^{-\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2} (x^2+1)^{-\frac{1}{2}} \times 2x$

$\frac{dy}{dx} = \frac{2x}{2\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}$

iii $y = \frac{3}{2}x^{\frac{3}{2}} + \frac{3}{x^2} - 6x$

$\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}} - \frac{6}{x^3} - 6 = \frac{1}{2}x^{\frac{1}{2}} - \frac{6}{x^3} - 6$