

Differentiation of a simple power

To differentiate $s = t^n$. • Bring the existing power down and use it to multiply. • Reduce the old power by one and use this as the new power. Answer $\frac{ds}{dt} = nt^{n-1}$	Example $s = t^3$ $\frac{ds}{dt} = 3t^{3-1}$ $= 3t^2$
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(a) $s = t^9, \quad \frac{ds}{dt}$	(b) $v = t^4, \quad \frac{dv}{dt}$	(c) $a = t^5, \quad \frac{da}{dt}$	(d) $y = x^7, \quad \frac{dy}{dx}$
(e) $y = x^{12}, \quad \frac{dy}{dx}$	(f) $V = r^3, \quad \frac{dV}{dr}$	(g) $P = v^6, \quad \frac{dP}{dv}$	(h) $A = r^2, \quad \frac{dA}{dr}$
(i) $T = \theta^8, \quad \frac{dT}{d\theta}$	(j) $R = t^{19}, \quad \frac{dR}{dt}$	(k) $v = t^{11}, \quad \frac{dv}{dt}$	(l) $p = V^{10}, \quad \frac{dp}{dV}$

Differentiation of a unit power

To differentiate $s = t$. • The phrase 'a unit power' refers to the fact that the power is 1. • Note that t is the same as t^1. We can use the previous rule. • Differentiating the term $t = t^1$ gives $1t^0 = 1$ Answer $\frac{ds}{dt} = 1$

(a) $r = t, \quad \frac{dr}{dt}$	(b) $v = t, \quad \frac{dv}{dt}$	(c) $a = t, \quad \frac{da}{dt}$	(d) $y = x, \quad \frac{dy}{dx}$
(e) $z = y, \quad \frac{dz}{dy}$	(f) $V = r, \quad \frac{dV}{dr}$	(g) $P = v, \quad \frac{dP}{dv}$	(h) $W = z, \quad \frac{dW}{dz}$
(i) $T = \theta, \quad \frac{dT}{d\theta}$	(j) $Q = r, \quad \frac{dQ}{dr}$	(k) $m = s, \quad \frac{dm}{ds}$	(l) $x = y, \quad \frac{dx}{dy}$

Differentiation of a constant

To differentiate $s = a$ where a is a constant. An isolated constant always differentiates to zero. Answer $\frac{ds}{dt} = 0$	Example $s = 4$ $\frac{ds}{dt} = 0$
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(a) $s = 5, \quad \frac{ds}{dt}$	(b) $v = 0.6, \quad \frac{dv}{dt}$	(c) $\theta = \frac{1}{2}, \quad \frac{d\theta}{dt}$	(d) $r = 1, \quad \frac{dr}{d\theta}$
(e) $T = \pi, \quad \frac{dT}{dt}$	(f) $z = 29.6, \quad \frac{dz}{dx}$	(g) $x = 3^{-1}, \quad \frac{dx}{dt}$	(h) $f = 2.7, \quad \frac{df}{dx}$
(i) $p = 4, \quad \frac{dp}{dr}$	(j) $q = \sqrt{2}, \quad \frac{dq}{dt}$	(k) $w = \frac{3}{4}, \quad \frac{dw}{dz}$	(l) $m = 2.7, \quad \frac{dm}{d\theta}$

Differentiation of a simple power multiplied by a constant

To differentiate $s = at^n$ where a is a constant. - Bring the existing power down and use it to multiply. - Reduce the old power by one and use this as the new power. Answer $\frac{ds}{dt} = ant^{n-1}$	Example $s = 3t^4$ $\frac{ds}{dt} = 4 \times 3t^{4-1}$ $= 12t^3$
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(a) $s = 3t^4, \quad \frac{ds}{dt}$	(b) $y = 7x^3, \quad \frac{dy}{dx}$	(c) $r = 0.4\theta^5, \quad \frac{dr}{d\theta}$	(d) $v = 4r^3, \quad \frac{dv}{dr}$
(e) $A = \pi r^2, \quad \frac{dA}{dr}$	(f) $T = 6\theta^4, \quad \frac{dT}{d\theta}$	(g) $v = 5t^9, \quad \frac{dv}{dt}$	(h) $p = 6v^3, \quad \frac{dp}{dv}$
(i) $y = 14x^2, \quad \frac{dy}{dx}$	(j) $r = 5t^7, \quad \frac{dr}{dt}$	(k) $P = 7v^6, \quad \frac{dP}{dv}$	(l) $C = 12N^8, \quad \frac{dC}{dN}$

Differentiation of a unit power multiplied by a constant

To differentiate $s = at$ where a is a constant. The result is always the same as the constant. Answer $\frac{ds}{dt} = a$	Example $s = 3t$ $\frac{ds}{dt} = 3$
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(a) $y = 5x, \quad \frac{dy}{dx}$	(b) $s = 6t, \quad \frac{ds}{dt}$	(c) $p = 4v, \quad \frac{dp}{dv}$
(d) $C = 7n, \quad \frac{dC}{dn}$	(e) $Q = 0.5v, \quad \frac{dQ}{dv}$	(f) $z = \pi w, \quad \frac{dz}{dw}$
(g) $N = 1.7n, \quad \frac{dN}{dn}$	(h) $\theta = \frac{1}{3}t, \quad \frac{d\theta}{dt}$	(i) $r = \frac{1}{4}t, \quad \frac{dr}{dt}$
(j) $s = 3.14\omega, \quad \frac{ds}{d\omega}$	(k) $w = 5z, \quad \frac{dw}{dz}$	(l) $R = 9.6\phi, \quad \frac{dR}{d\phi}$

Differentiation of a general power

To differentiate $s = t^n$ for any value of n. - Bring the existing power down and use it to multiply. - Reduce the old power by one and use this as the new power. Answer $\frac{ds}{dt} = nt^{n-1}$	Example $s = t^{-3.4}$ $\frac{ds}{dt} = -3.4t^{-3.4-1}$ $= -3.4t^{-4.4}$
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(a) $s = t^{2.8}, \quad \frac{ds}{dt}$	(b) $y = x^{2/3}, \quad \frac{dy}{dx}$	(c) $r = 0.4\theta^{1/4}, \quad \frac{dr}{d\theta}$
(d) $v = 4r^{-3}, \quad \frac{dv}{dr}$	(e) $A = r^{1/2}, \quad \frac{dA}{dr}$	(f) $T = \theta^{-6}, \quad \frac{dT}{d\theta}$
(g) $v = t^{-1}, \quad \frac{dv}{dt}$	(h) $p = v^{-3.5}, \quad \frac{dp}{dv}$	(i) $y = x^{3/4}, \quad \frac{dy}{dx}$
(j) $r = t^{-1/3}, \quad \frac{dr}{dt}$	(k) $P = v^{1.8}, \quad \frac{dP}{dv}$	(l) $C = N^{-6}, \quad \frac{dC}{dN}$

Differentiation of a general power multiplied by a constant

To differentiate $s = at^n$ for any value of n where a is a constant. • Bring the existing power down and use it to multiply. • Reduce the old power by one and use this as the new power. Answer $\frac{ds}{dt} = ant^{n-1}$	Example $s = 2.5t^{-4}$ $\frac{ds}{dt} = -4 \times 2.5t^{-4-1}$ $= -10t^{-5}$
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(a) $y = 5x^{1.5}, \quad \frac{dy}{dx}$	(b) $s = 4t^{-3}, \quad \frac{ds}{dt}$	(c) $p = 8v^{1/2}, \quad \frac{dp}{dv}$
(d) $C = 6n^{-1}, \quad \frac{dC}{dn}$	(e) $Q = 2v^{0.5}, \quad \frac{dQ}{dv}$	(f) $z = 6w^{-2.5}, \quad \frac{dz}{dw}$
(g) $N = 5.2n^{10}, \quad \frac{dN}{dn}$	(h) $\theta = \frac{1}{5}t^6, \quad \frac{d\theta}{dt}$	(i) $r = -5t^{-2}, \quad \frac{dr}{dt}$
(j) $s = 2.5p^{2.5}, \quad \frac{ds}{dp}$	(k) $w = 15z^{-0.1}, \quad \frac{dw}{dz}$	(l) $v = -3r^{-4}, \quad \frac{dv}{dr}$

Differentiation of a sum or difference of terms

To differentiate $s = f(t) + g(t)$ or $s = f(t) - g(t)$. • Differentiate $f(t)$ and $g(t)$ using previous rules. • Add or subtract the results. Answer $\frac{ds}{dt} = \frac{df}{dt} + \frac{dg}{dt}$ or $\frac{ds}{dt} = \frac{df}{dt} - \frac{dg}{dt}$	Example $s = 3t^2 \pm 6t^{\frac{1}{3}}$ $\frac{ds}{dt} = 2 \times 3t^{2-1} \pm \frac{1}{3} \times 6t^{\frac{1}{3}-1}$ $= 6t \pm 2t^{-\frac{2}{3}}$
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(a) $s = t^2 + t^3, \quad \frac{ds}{dt}$	(b) $v = 2t + 3t^4, \quad \frac{dv}{dt}$	(c) $a = 3 + 4t, \quad \frac{da}{dt}$
(d) $v = 5t - 10t^2, \quad \frac{dv}{dt}$	(e) $a = 3t^{4.5} - 4t^{-1}, \quad \frac{da}{dt}$	(f) $s = 4t^{1/2} - 2.5t^{-2}, \quad \frac{ds}{dt}$
(g) $y = x^2 + 3x^4, \quad \frac{dy}{dx}$	(h) $y = 2x^{-1} - x^{-2}, \quad \frac{dy}{dx}$	(i) $x = 3y + \frac{1}{4}y^2, \quad \frac{dx}{dy}$

Differentiation of a simple fraction

To differentiate $s = \frac{a}{t^n}$ where a is a constant. • Rewrite as $s = at^{-n}$. • Differentiate using previous rules. Answer $\frac{ds}{dt} = -ant^{-n-1}$	Example $s = \frac{2}{t^3}$ $= 2t^{-3}$ $\frac{ds}{dt} = -3 \times 2t^{-3-1}$ $= -6t^{-4}$
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(a) $s = \frac{1}{t}, \quad \frac{ds}{dt}$	(b) $s = \frac{2}{t^2}, \quad \frac{ds}{dt}$	(c) $y = \frac{5}{x^3}, \quad \frac{dy}{dx}$
(d) $p = \frac{2.4}{r^7}, \quad \frac{dp}{dr}$	(e) $v = \frac{4.5}{t^{1/2}}, \quad \frac{dv}{dt}$	(f) $a = \frac{10}{t^6}, \quad \frac{da}{dt}$
(g) $v = \frac{4.9}{x^4}, \quad \frac{dv}{dx}$	(h) $s = \frac{2.5}{\theta^3}, \quad \frac{ds}{d\theta}$	(i) $x = \frac{\pi}{\theta}, \quad \frac{dx}{d\theta}$
(j) $\theta = \frac{\sqrt{2}}{t^5}, \quad \frac{d\theta}{dt}$	(k) $z = \frac{7}{2x}, \quad \frac{dz}{dx}$	(l) $q = \frac{3}{r^{2.5}}, \quad \frac{dq}{dr}$

Differentiation of fractions reducible to simpler fractions

To differentiate $s = \frac{a + bt^n}{ct^m}$. • Rewrite as $s = \frac{a}{ct^m} + \frac{bt^n}{ct^m}$. • Use power laws to simplify. • Differentiate using previous rules. Answer $\frac{ds}{dt} = -\frac{amt^{-m-1}}{c} + \frac{b}{c}(n-m)t^{n-m-1}$	Example $s = \frac{4 + 3t^2}{5t^3}$ $= \frac{4}{5t^3} + \frac{3t^2}{5t^3}$ $= \frac{4t^{-3}}{5} + \frac{3t^{-1}}{5}$ $\frac{ds}{dt} = -\frac{3 \times 4t^{-3-1}}{5} + (-1) \times \frac{3t^{-1-1}}{5}$ $= -\frac{12t^{-4}}{5} - \frac{3t^{-2}}{5}$
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(a) $s = \frac{4t+1}{t}, \quad \frac{ds}{dt}$	(b) $s = \frac{5t^3+3t}{t}, \quad \frac{ds}{dt}$	(c) $v = \frac{5t^2+3t+1}{t^3}, \quad \frac{dv}{dt}$
(d) $y = \frac{7+6x+5x^2}{x^2}, \quad \frac{dy}{dx}$	(e) $y = \frac{3+5x}{4x}, \quad \frac{dy}{dx}$	(f) $z = \frac{7+2t}{3t}, \quad \frac{dz}{dt}$
(g) $\theta = \frac{2+5t+2t^3}{t^2}, \quad \frac{d\theta}{dt}$	(h) $w = \frac{8+6z^2}{3z^4}, \quad \frac{dw}{dz}$	(i) $x = \frac{3y+2y^2}{4y}, \quad \frac{dx}{dy}$

Differentiation of a simple bracket

To differentiate $s = (at + b)^n$. • Bring the existing power down and use it to multiply. • Reduce the old power by 1 and use this as the new power. • Differentiate the expression inside the bracket. • Multiply your results together. Answer $\frac{ds}{dt} = an(at + b)^{n-1}$	Example $s = (3t + 6)^5$ $\frac{ds}{dt} = 5 \times (3t + 6)^{5-1} \times 3$ $= 15(3t + 6)^4$
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(a) $s = (2t + 3)^6, \quad \frac{ds}{dt}$	(b) $v = (5 + 2t)^7, \quad \frac{dv}{dt}$	(c) $\theta = (0.4t + 1)^{2.5}, \quad \frac{d\theta}{dt}$
(d) $z = (\pi x + 2.4)^{10}, \quad \frac{dz}{dx}$	(e) $a = (5t - 3)^{-2}, \quad \frac{da}{dt}$	(f) $s = (3 - 6t)^{-3}, \quad \frac{ds}{dt}$
(g) $y = (4x - 1)^{1/2}, \quad \frac{dy}{dx}$	(h) $y = (2 + 0.5t)^3, \quad \frac{dy}{dt}$	(i) $z = (4x - 6)^{-0.5}, \quad \frac{dz}{dx}$

Differentiation of a general bracket

To differentiate $s = [f(t)]^n$. • Bring the existing power down and use it to multiply. • Reduce the old power by 1 and use this as the new power. • Differentiate $f(t)$ to give $\frac{df}{dt}$. • Multiply your results together. • Simplify. Answer $\frac{ds}{dt} = n[f(t)]^{n-1} \cdot \frac{df}{dt}$	Example $s = (5t^3 - 3t^2 + 1)^7$ $\begin{aligned}\frac{ds}{dt} &= 7 \times (5t^3 - 3t^2 + 1)^{7-1} \times (3 \times 5t^{3-1} - 2 \times 3t^{2-1} + 0) \\ &= 7(5t^3 - 3t^2 + 1)^6(15t^2 - 6t) \\ &= 21t(5t^3 - 3t^2 + 1)^6(5t - 2)\end{aligned}$
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(a) $s = (3t^2 + 2t + 4)^5$	$\frac{ds}{dt}$	(b) $r = (4\theta^2 - 2\theta + 1)^{-2}$	$\frac{dr}{d\theta}$	(c) $v = (8x^2 + 5x)^2$	$\frac{dv}{dx}$
(d) $\theta = (3t + 1)^{-1}$	$\frac{d\theta}{dt}$	(e) $p = (5v^2 + 2)^{-2}$	$\frac{dp}{dv}$	(f) $y = (t^3 + t + 1)^2$	$\frac{dy}{dt}$
(g) $z = (5x^3 + 1)^{-4}$	$\frac{dz}{dx}$	(h) $s = (2t + 1)^4$	$\frac{ds}{dt}$	(i) $x = (3y^2 - 4y)^3$	$\frac{dx}{dy}$

Practice

1. In each of the following determine $\frac{ds}{dt}$.

(a) $s = \frac{2}{t}$

(b) $s = t^{1/2}$

(c) $s = 4t^{-2}$

(d) $s = \frac{1}{3}t$

(e) $s = \frac{2t+1}{t}$

(f) $s = \frac{t^3 - 3t + 4}{t^2}$

(g) $s = (5t+1)^4$

(h) $s = (3t+1)^{-1}$

(i) $s = \frac{2}{3t+1}$

(j) $s = 5t^2 + 3t + 6$

(k) $s = 2t^{-3} + t^4$

(l) $s = (2t + 3t^{-1})^{-5}$

2. In each of the following determine $\frac{dy}{dx}$.

(a) $y = x^{1/2}$

(b) $y = (x+1)^{1/2}$

(c) $y = \frac{3}{x} + 1$

(d) $y = 5x + \frac{1}{x}$

(e) $y = 5x^2$

(f) $y = 7(x^2 + 1)^5$

(g) $y = (2x+1)^{1/3}$

(h) $y = \frac{7+2x}{x}$

(i) $y = \frac{5}{3x^2}$

(j) $y = 4x^{-2} + 3x^{-1}$

(k) $y = 24 - 2x^{-3/4}$

(l) $y = (2x^3 + 3x^2 - 5x)^2$

Answers

Section 1

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|-----------------|----------------|----------------|-------------|
| (a) $9t^8$ | (b) $4t^3$ | (c) $5t^4$ | (d) $7x^6$ |
| (e) $12x^{11}$ | (f) $3r^2$ | (g) $6v^5$ | (h) $2r$ |
| (i) $8\theta^7$ | (j) $19t^{18}$ | (k) $11t^{10}$ | (l) $10V^9$ |

Section 2

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|-------|-------|-------|-------|
| (a) 1 | (b) 1 | (c) 1 | (d) 1 |
| (e) 1 | (f) 1 | (g) 1 | (h) 1 |
| (i) 1 | (j) 1 | (k) 1 | (l) 1 |

Section 3

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|-------|-------|-------|-------|
| (a) 0 | (b) 0 | (c) 0 | (d) 0 |
| (e) 0 | (f) 0 | (g) 0 | (h) 0 |
| (i) 0 | (j) 0 | (k) 0 | (l) 0 |

Section 4

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|--------------|------------------|-----------------|-------------|
| (a) $12t^3$ | (b) $21x^2$ | (c) $2\theta^4$ | (d) $12r^2$ |
| (e) $2\pi r$ | (f) $24\theta^3$ | (g) $45t^8$ | (h) $18v^2$ |
| (i) $28x$ | (j) $35t^6$ | (k) $42v^5$ | (l) $96N^7$ |

Section 5

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|-----------|-----------|---------|-----------|
| (a) 5 | (b) 6 | (c) 4 | (d) 7 |
| (e) 0.5 | (f) π | (g) 1.7 | (h) $1/3$ |
| (i) $1/4$ | (j) 3.14 | (k) 5 | (l) 9.6 |

Section 6

(a) $2.8t^{1.8}$	(b) $\frac{2}{3}x^{-1/3}$	(c) $0.1\theta^{-3/4}$	(d) $-12r^{-4}$
(e) $\frac{1}{2}r^{-1/2}$	(f) $-6\theta^{-7}$	(g) $-t^{-2}$	(h) $-3.5v^{-4.5}$
(i) $\frac{3}{4}x^{-1/4}$	(j) $-\frac{1}{3}t^{-4/3}$	(k) $1.8v^{0.8}$	(l) $-6N^{-7}$

Section 7

(a) $7.5x^{0.5}$	(b) $-12t^{-4}$	(c) $4v^{-1/2}$	(d) $-6n^{-2}$
(e) $v^{-0.5}$	(f) $-15w^{-3.5}$	(g) $52n^9$	(h) $\frac{6}{5}t^5$
(i) $10t^{-3}$	(j) $6.25p^{1.5}$	(k) $-1.5z^{-1.1}$	(l) $12r^{-5}$

Section 8

(a) $2t + 3t^2$	(b) $2 + 12t^3$	(c) 4	(d) $5 - 20t$
(e) $13.5t^{3.5} + 4t^{-2}$	(f) $2t^{-1/2} + 5t^{-3}$	(g) $2x + 12x^3$	(h) $-2x^{-2} + 2x^{-3}$
(i) $3 + \frac{y}{2}$			

Section 9

(a) $-t^{-2}$	(b) $-4t^{-3}$	(c) $-15x^{-4}$	(d) $-16.8r^{-8}$
(e) $-2.25t^{-3/2}$	(f) $-60t^{-7}$	(g) $-19.6x^{-5}$	(h) $-7.5\theta^{-4}$
(i) $-\pi\theta^{-2}$	(j) $-5\sqrt{2}t^{-6}$	(k) $-\frac{7}{2}x^{-2}$	(l) $-7.5r^{-3.5}$

Section 10

(a) $-t^{-2}$	(b) $10t$	(c) $-5t^{-2} - 6t^{-3} - 3t^{-4}$	(d) $-14x^{-3} - 6x^{-2}$
(e) $-\frac{3}{4}x^{-2}$	(f) $-\frac{7}{3}t^{-2}$	(g) $-4t^{-3} - 5t^{-2} + 2$	(h) $-\frac{32}{3}z^{-5} - 4z^{-3}$
(i) $\frac{1}{2}$			

Section 11

- (a) $12(2t+3)^5$ (b) $14(5+2t)^6$ (c) $(0.4t+1)^{1.5}$ (d) $10\pi(\pi x+2.4)^9$
(e) $-10(5t-3)^{-3}$ (f) $18(3-6t)^{-4}$ (g) $2(4x-1)^{-1/2}$ (h) $1.5(2+0.5t)^2$
(i) $-2(4x-6)^{-1.5}$

Section 12

- (a) $10(3t^2+2t+4)^4(3t+1)$ (b) $-4(4\theta^2-2\theta+1)^{-3}(4\theta-1)$ (c) $2(8x^2+5x)(16x+5)$
(d) $-3(3t+1)^{-2}$ (e) $-20v(5v^2+2)^{-3}$ (f) $2(t^3+t+1)(3t^2+1)$
(g) $-60x^2(5x^3+1)^{-5}$ (h) $8(2t+1)^3$ (i) $6(3y^2-4y)^2(3y-2)$

Section 13

Exercise 1

- (a) $-2t^{-2}$ (b) $\frac{1}{2}t^{-1/2}$ (c) $-8t^{-3}$
(d) $1/3$ (e) $-t^{-2}$ (f) $1+3t^{-2}-8t^{-3}$
(g) $20(5t+1)^3$ (h) $-3(3t+1)^{-2}$ (i) $-6(3t+1)^{-2}$
(j) $10t+3$ (k) $-6t^{-4}+4t^3$ (l) $-5(2t+3t^{-1})^{-6}(2-3t^{-2})$

Exercise 2

- (a) $\frac{1}{2}x^{-1/2}$ (b) $\frac{1}{2}(x+1)^{-1/2}$ (c) $-3x^{-2}$
(d) $5-x^{-2}$ (e) $10x$ (f) $70x(x^2+1)^4$
(g) $\frac{2}{3}(2x+1)^{-2/3}$ (h) $-7x^{-2}$ (i) $-\frac{10}{3}x^{-3}$
(j) $-8x^{-3}-3x^{-2}$ (k) $\frac{3}{2}x^{-7/4}$ (l) $2(2x^3+3x^2-5x)(6x^2+6x-5)$
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Exercise 3

$(a) \quad 3$

$(b) \quad -2.1t^{-4} - 18t^2$

$(c) \quad -2.5(5t - 6)^{-1.5}$

$(d) \quad -4t^{-2} + 3$

$(e) \quad 0$

$(f) \quad -4t^{-3} + 3t^{-2} + 5$

$(g) \quad -3(6t - 2)^{-1.5}$

$(h) \quad -6t^{-3}$

$(i) \quad 3 + 0.1t^{-1.1}$

$(j) \quad 32t^3 - 1.5t^{-1/2}$

$(k) \quad 6(5t^{-2} + 3t^2)^2(-5t^{-3} + 3t)$

$(l) \quad -\pi$