

Week 1

Longitudinal Acceleration Braking

Bad braking

1 [How to make a stoppie with a van.mp4](#)

2 [Car Stoppie 2.mp4](#)

3 [Car Stoppie Fail.mp4](#)

Grip between tyre and road

Several mechanisms for generating
'frictional' forces

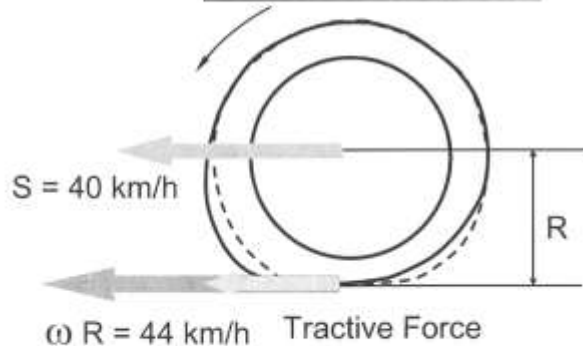
- Hysteresis – interlocking of tyre and road
- Adhesion – chemical bonding
- Conventional friction- smooth surfaces rubbing

Tyre characteristics

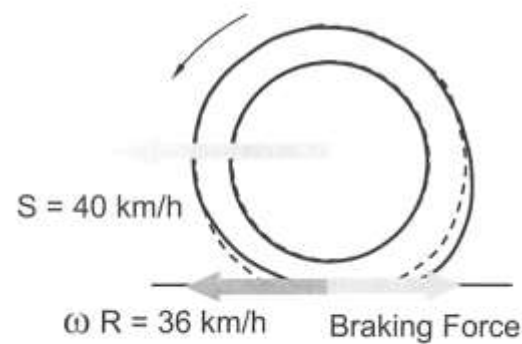
Longitudinal Slip Definition



Sr is 'Slip Ratio' :



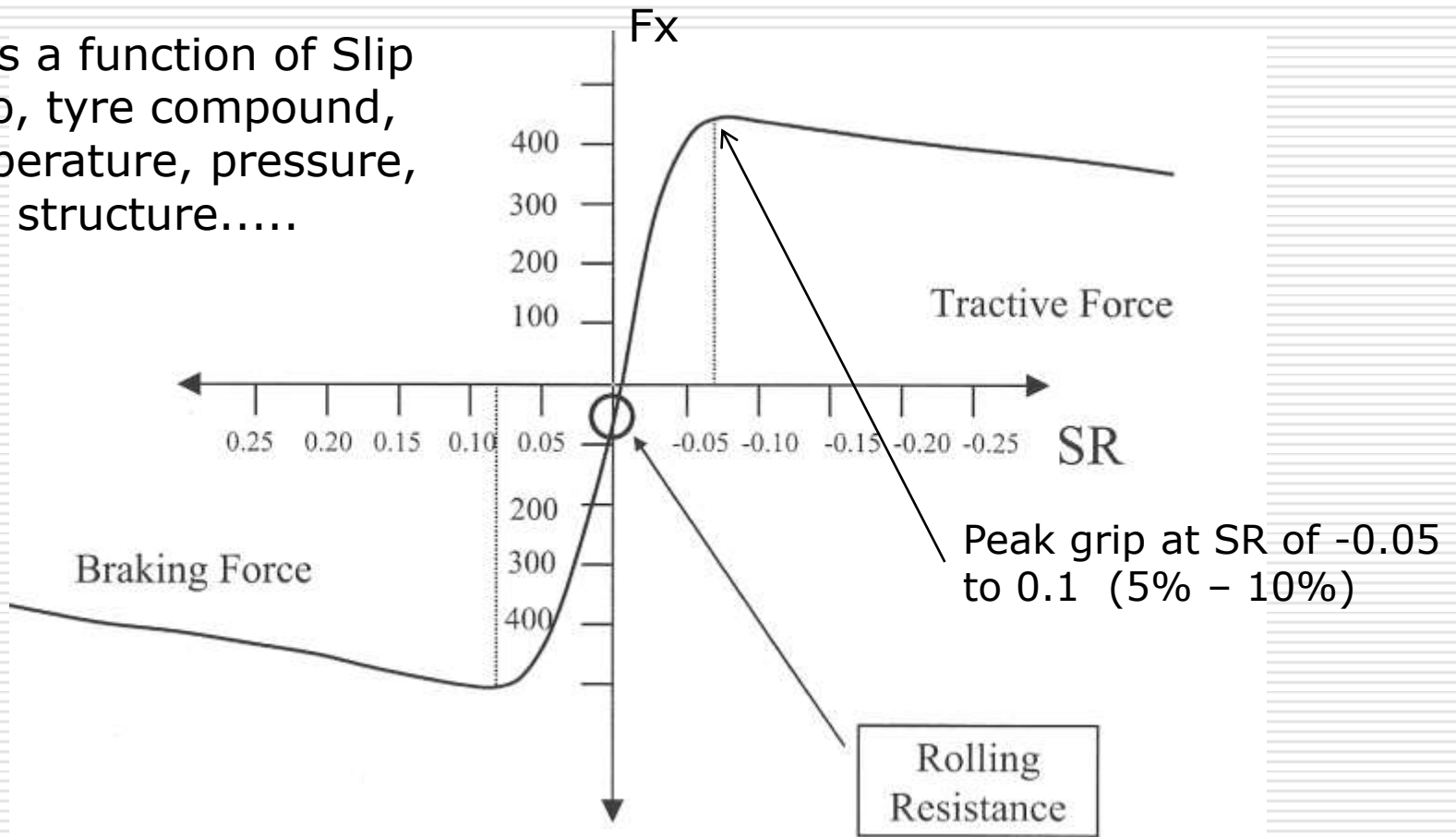
$$Sr = (44 - 40) / 40 = 10 \%$$



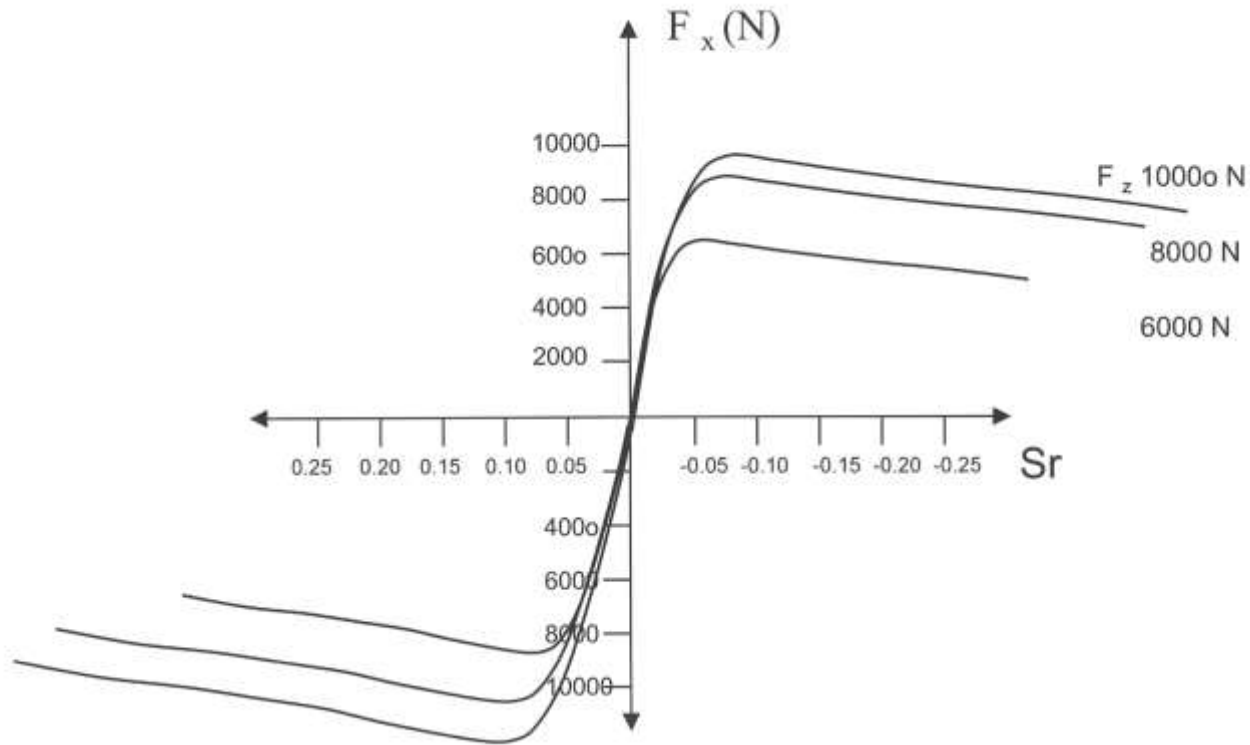
$$Sr = (40 - 36) / 40 = -10 \%$$

Longitudinal Force , F_x

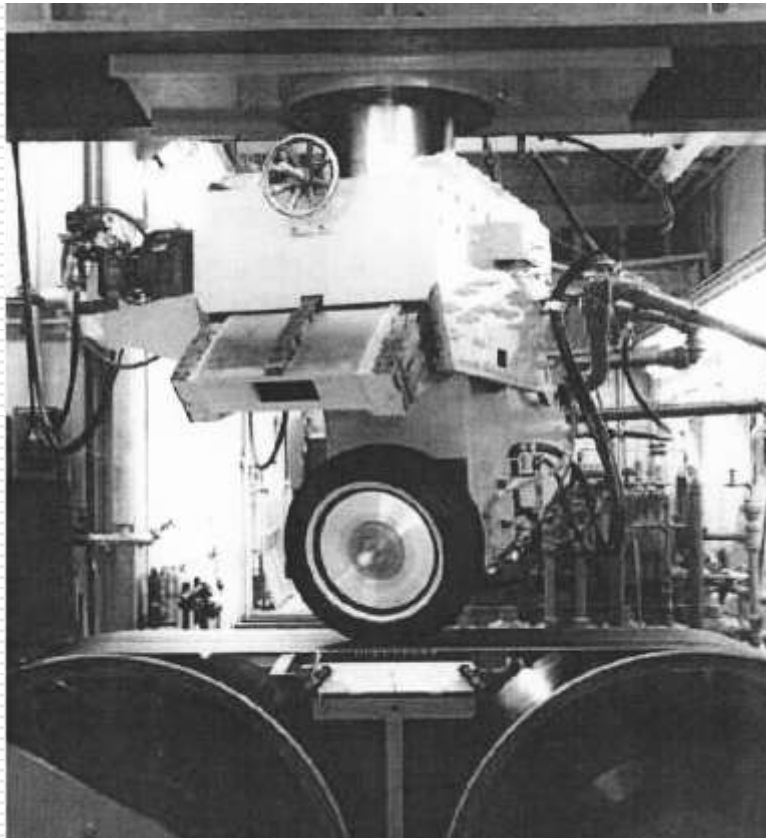
F_x , is a function of Slip Ratio, tyre compound, temperature, pressure, road structure.....



Influence of vertical load



Measuring the Longitudinal Friction Coefficient



Using single tyre forces in a laboratory

[Tire Force Test.mp4](#)

Track Test

How could we obtain a value for the longitudinal friction coefficient of the tyres on one axle with a track test ?

Finding μ from a track test

$$\mu = \frac{\text{Friction Force (F)}}{\text{Reaction Force (R)}}$$

In this example:

$$F = F_x \quad \text{and} \quad R = Wb$$

So

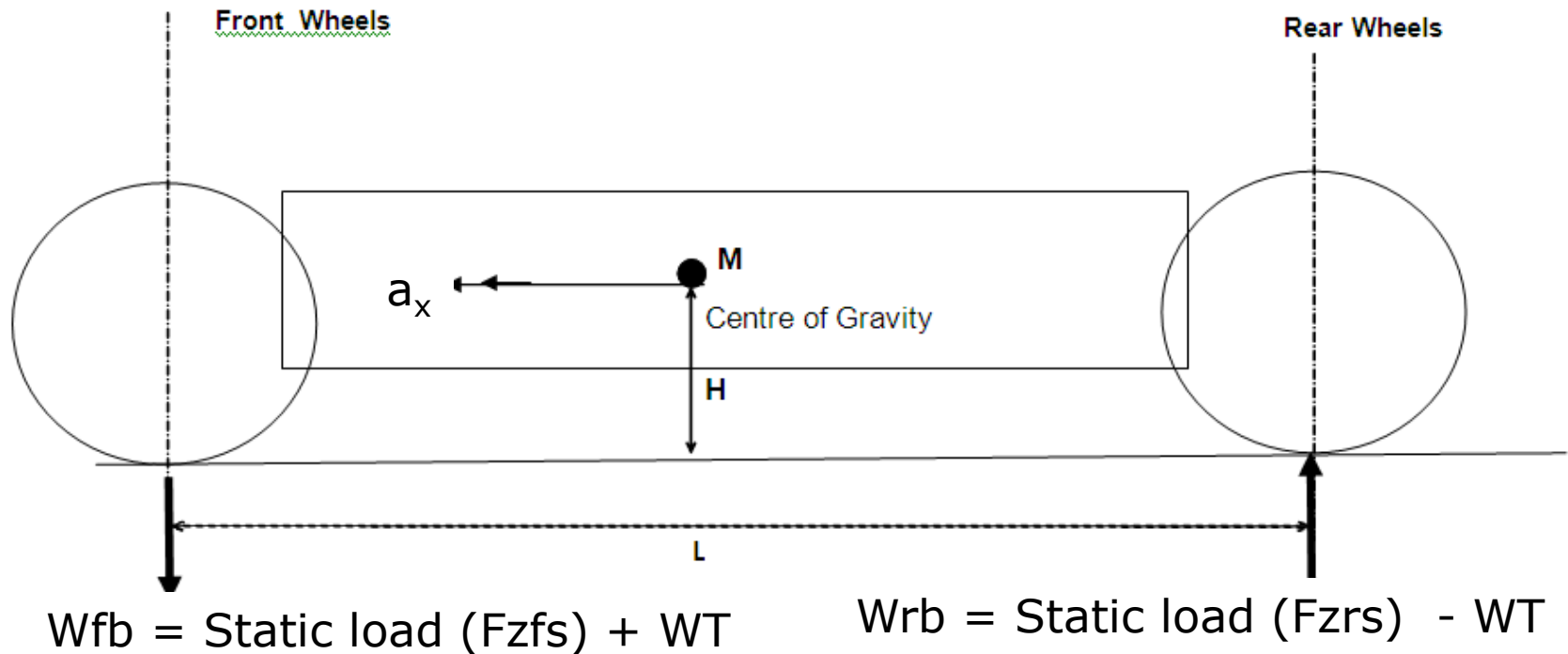
$$\mu = \frac{F_x}{Wb}$$

Finding μ from a track test

To find μ we need to:

- Calculate tyre loads from static load +/- weight transfer = W_{fb} and W_{rb}
- Measure rate of deceleration and hence calculate longitudinal force F_x

Tyre loads when braking with simple rigid model



Finding WT

$$\text{Over turning moment} = Ma_x H$$

$$\text{Reaction moment} = WT \times L$$

For equilibrium:-

$$\text{Reaction} = \text{Overturning moment}$$

$$WT \times L = Ma_x H$$

Giving:

$$WT = \frac{Ma_x H}{L}$$

Example 1

Data:

- $L = 2.5\text{m}$
- $M_{\text{tot}} = 800\text{ kg}$
- $H = 0.355\text{m}$
- $a_x = 1.4g$
- $F_{\text{zfs}} = 3628.9\text{ N} \quad (370\text{ kg})$
- $F_{\text{zrs}} = 4219.08\text{ N} \quad (430\text{ kg})$

Example 1 Weight transfer

$$WT = \frac{Ma_x H}{L}$$

So

$$\begin{aligned} WT &= \frac{800 \times 9.81 \times 1.4 \times 0.355}{2.5} \\ &= 1560 \text{ N} \end{aligned}$$

Example 1 Total tyre loads

$$W_{fb} = F_{zfs} + W_T$$

$$\begin{aligned} W_{fb} &= 3628.9 + 1560 \\ &= 5189.1 \text{ N} \end{aligned}$$

$$W_{rb} = F_{zrs} - W_T$$

$$\begin{aligned} W_{rb} &= 4219.08 - 1560 \\ &= 2659.08 \text{ N} \end{aligned}$$

Continuing Example 1

Longitudinal force F_x

The car is decelerating at 1.4 g
ie 13.734 m/s²

The friction force required to achieve this rate of deceleration is found from:

$$F_x = m a_x$$

So

$$\begin{aligned} F_x &= 800 \times 13.734 \text{ N} \\ &= 10987 \text{ N} \end{aligned}$$

Example 1 Finding μ

If all four wheels are braking then the average value of μ for the four tyres of car will be:

$$\frac{F_x}{W_{fb} + W_{rb}}$$

$$\begin{aligned}\text{So } \mu &= \frac{10987}{5189.1 + 2659.08} \\ &= 1.4\end{aligned}$$

Finding μ

This can be simplified by saying that the combined value of μ for the tyres of a car is the maximum rate of deceleration that can be achieved divided by g .

Or to put it another way , the maximum rate of deceleration that can be achieved is μg

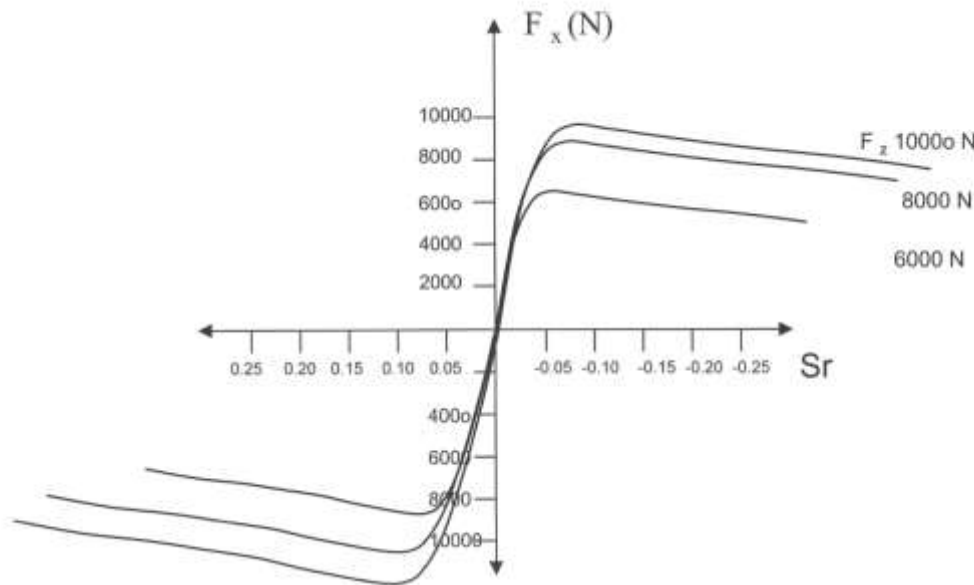
Refining the value of μ

We can determine the value of μ for a pair of front or rear tyres rather than all four tyres by only braking the wheels of one axle at a time

We will then be able to determine the value of μ for the tyres on that axle alone.

Finding μ for front or rear tyres

It has been demonstrated already that μ varies with vertical load.



Two wheels braking

$$\mu = \frac{F_x}{W_{fb}}$$

If the car had still been able to decelerate at 1.4 g using just the front brakes

$$\begin{aligned}\mu &= 10987/5189.1 \\ &= 2.117 !\end{aligned}$$

Two wheel braking

More realistically the value of μ will be slightly lower at the front than the rear due to the increased vertical load.

Assume that the rate of deceleration with just two wheels braking is now just **0.8g**

giving a decelerating force of **6278.4 N** from the two front tyres.

Two wheel braking

$$W_{fb} = F_{zfs} + WT$$

$$F_{zfs} = 3628.9 \text{ N}$$

$$WT = \frac{800 \times 9.81 \times 0.8 \times 0.355}{2.5}$$

$$WT = 891.53 \text{ N}$$

$$\begin{aligned} W_{fb} &= 3628.9 + 891.53 \text{ N} \\ &= 4520.43 \text{ N} \end{aligned}$$

Two wheel braking

So for the two front tyres:

$$\mu = \frac{F_x}{W_{fb}}$$

$$= \frac{6278.4}{4520.43}$$

Giving $\mu = 1.389$

Summary

- The value of μ averaged for all tyres of a car can be found by dividing the maximum rate of deceleration [in m/s^2] by g .
- The maximum rate of deceleration achievable by a vehicle with four wheels braking is μg (*where μ is the averaged value described above*)
- The individual value of μ for front or rear tyres can be found by only braking the wheels on that axle and calculating vertical load on the tyres and the braking force created by those tyres

Determining wheel loads considering sprung and unsprung masses

$$W_{fb} = M_{usf} \times g + M_{sf} \times g + \left(\frac{M_s \times a_x \times H}{L} \right) + \left(\frac{M_{us} \times a_x \times r}{L} \right)$$

Front sprung mass both front wheels	kg	M_{sf}
Total sprung mass	kg	M_s
Unsprung Mass both front wheels	kg	M_{usf}
Total Unsprung mass	kg	M_{us}
Height of CG of sprung mass	kg	H
Longitudinal acceleration	m/s ²	a_x
Rolling radius of tyre	m	r
Wheelbase	m	L

What is the equivalent calculation for the rear wheels?

Finding wheel loads when considering sprung and unsprung masses: Example 2

Data:

Wheelbase $L = 2.5\text{m}$

Individual wheel front scale reading = 300 kg

Individual wheel rear scale reading = 300 kg

Unsprung Mass each front wheel 40 kg

Unsprung mass each rear wheel 40 kg

Unsprung Mass both front wheels $M_{usr} = 80\text{ kg}$

Unsprung mass both rear wheels $M_{usf} = 80\text{ kg}$

Front sprung mass $M_{sf} = 520\text{ kg}$

Rear sprung mass $M_{sr} = 520\text{ kg}$

Total mass $M_{tot} = 1200\text{ kg}$

Height of CG of sprung mass $H = 0.35\text{m}$

Longitudinal acceleration $a_x = 1.2g$

Rolling radius of tyre $r = 0.28\text{m}$

Solution for Example 2

$$W_{fb} = M_{usf} \times g + \frac{M_{sf} \times g}{L} + \frac{(M_s \times a_x \times H)}{L} + \frac{(M_{us} \times a_x \times r)}{L}$$

$$W_{fb} = 80 \times g + 520 \times g + \frac{(1040 \times 1.2g \times 0.35)}{2.5} + \frac{(160 \times 1.2g \times 0.28)}{2.5}$$

$$= 784.8 + 5101.2 + 1714 + 210.95$$

Giving

$$W_{fb} = 7810.95 \text{ N}$$

(Similarly $W_{rb} = 3961 \text{ N}$)

Example 3

Finding wheel loads from typical track test data

CG height for **sprung** mass above ground. 0.299m

Wheelbase 2.351m

Left front scale reading [kg] 110kg

Right front scale reading [kg] 120kg

Left rear scale reading [kg] 185kg

Right rear scale reading [kg] 195kg

Left front unsprung mass [kg] 38kg

Right front unsprung mass [kg] 38kg

Left rear unsprung mass [kg] 44kg

Right rear unsprung mass [kg] 44kg

Rolling radius [m] 0.275 m

Longitudinal acceleration [m/s^2] 0.68g

Example 3 Solution

$$M_{usf} = 76 \text{ kg} \quad M_{usr} = 88 \text{ kg}$$

$$M_{us} = 164 \text{ kg}$$

$$M_{sf} = 154 \text{ kg}$$

$$M_s = 446 \text{ kg}$$

$$W_{fb} = M_{usf} \times g + M_{sf} \times g + \frac{(M_s \times a_x \times H)}{L} + \frac{(M_{us} \times a_x \times r)}{L}$$

$$W_{fb} = 76 \times g + 154 \times g + \frac{(446 \times 0.68g \times 0.299)}{2.351} + \frac{(164 \times 0.68g \times 0.275)}{2.351}$$

$$\text{Giving } W_{fb} = 745.6 + 1510.74 + 378.38 + 127.97$$

$$\text{Similarly} \quad W_{fb} = 2762.7 \text{ N}$$

$$W_{rb} = 3961 \text{ N}$$