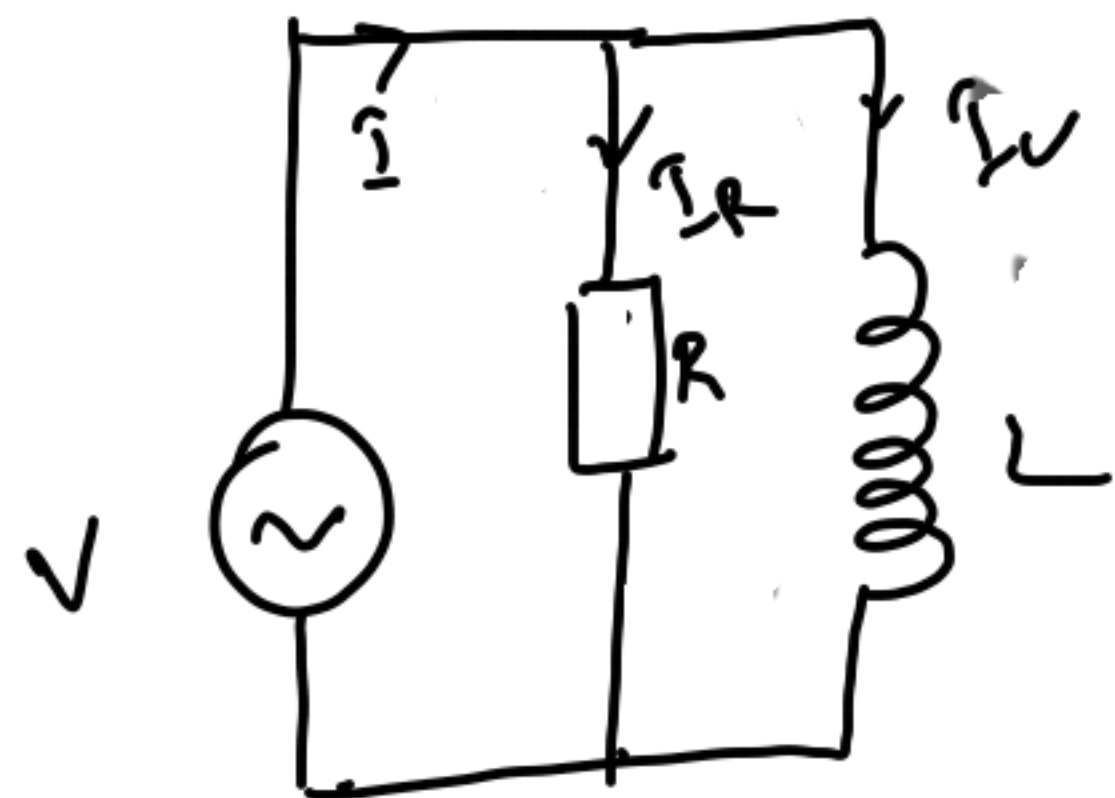


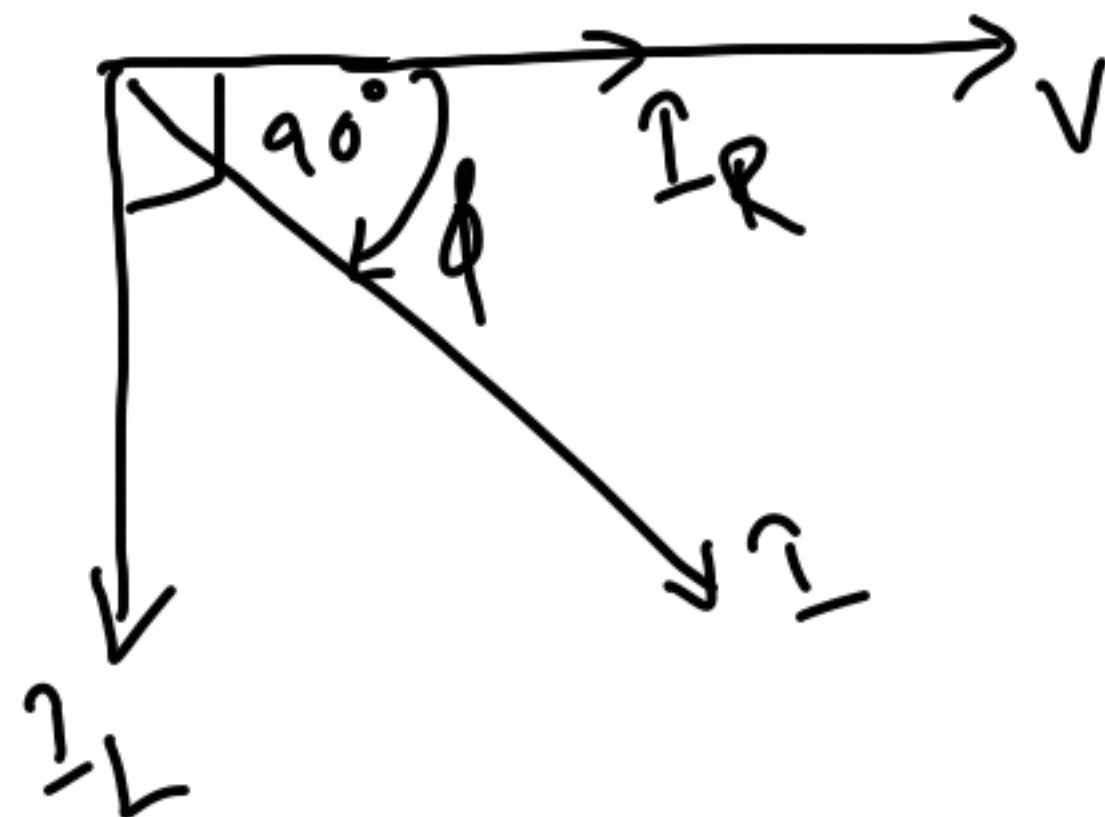
RL Parallel AC Circuits



$$X_L = 2\pi fL$$

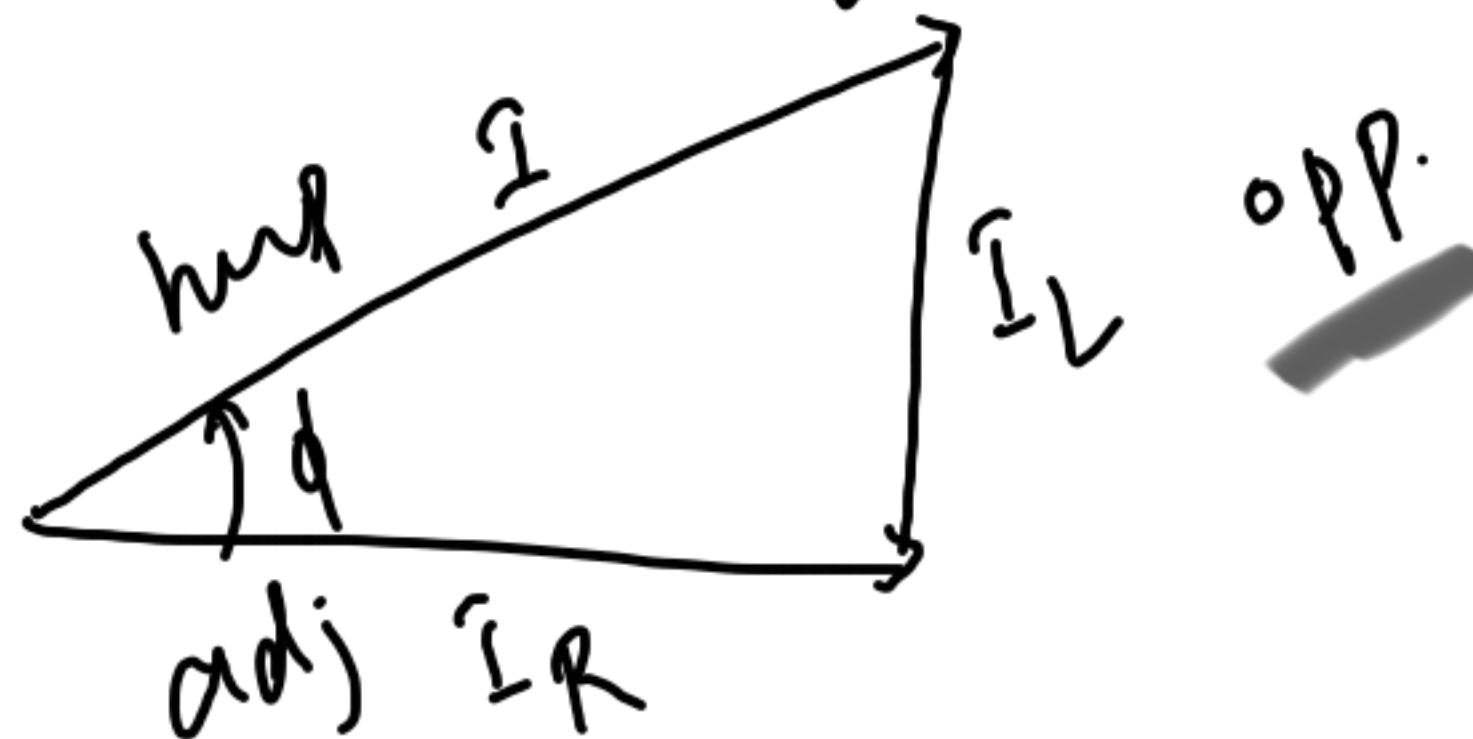
$$I = \sqrt{(I_R)^2 + I_L^2}$$

Phasor diagram



I_R is in phase with V

I_L is in lagging Voltage (V) by 90°



$$\tan \phi = \frac{\text{opp}}{\text{adj}} = \frac{I_L}{I_R}$$

$$\cos \phi = \frac{\text{adj}}{\text{hyp}} = \frac{I_R}{I}$$

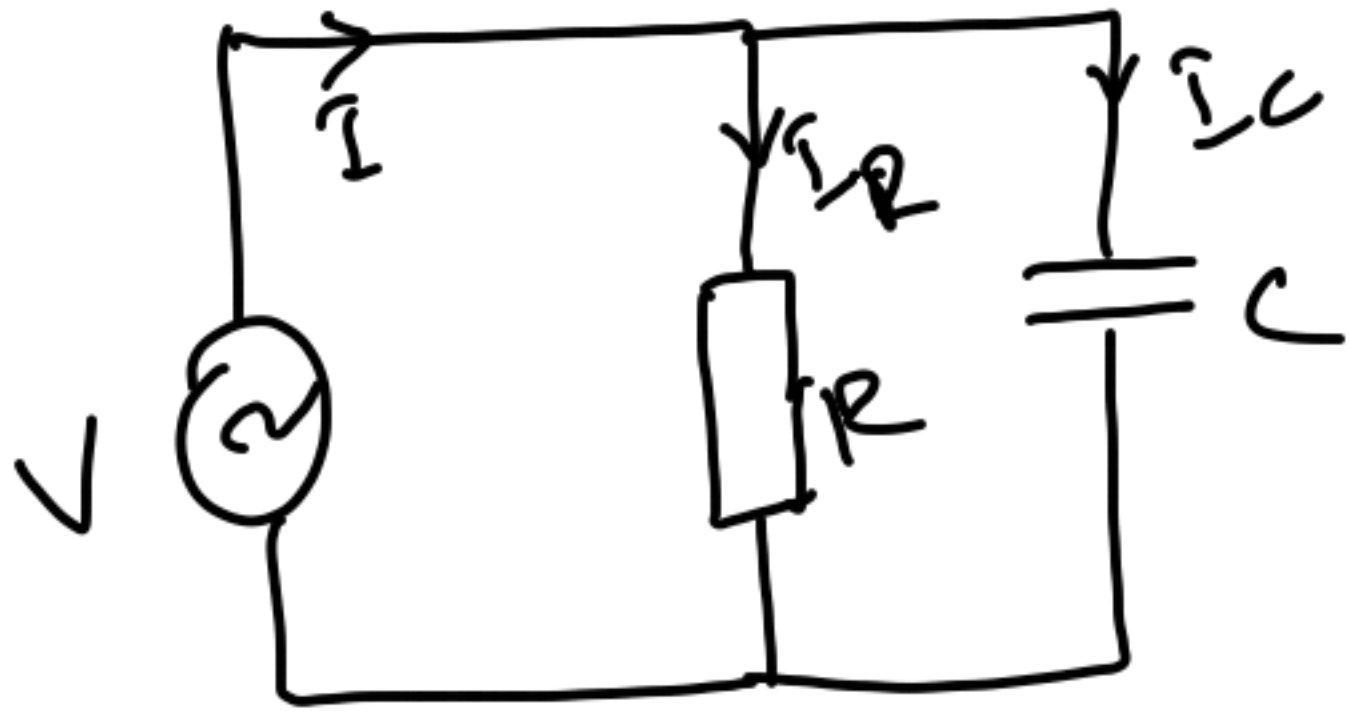
$$\sin \phi = \frac{\text{opp}}{\text{hyp}} = \frac{I_L}{I}$$

$$\text{Impedance} = \frac{V}{I}$$

Inductor current

$$I_L = \frac{V}{X_L}$$

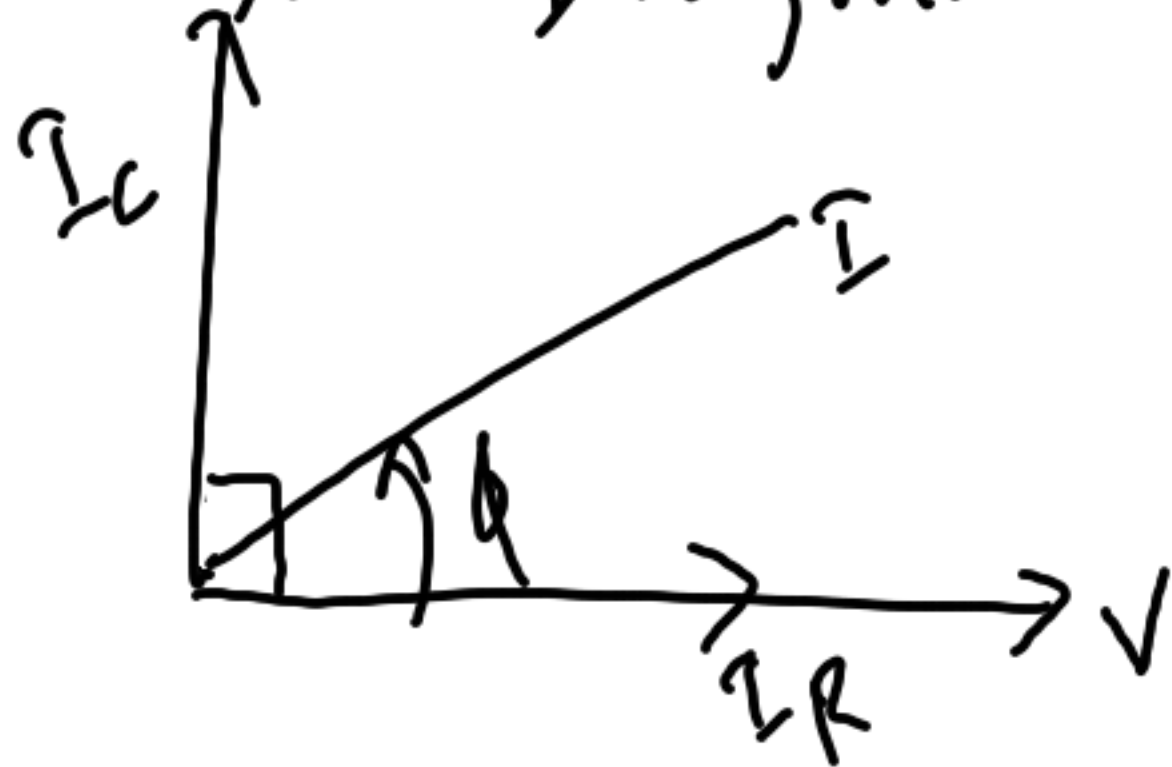
RC Parallel Circuit



$$I = \sqrt{(I_R)^2 + (I_C)^2}$$

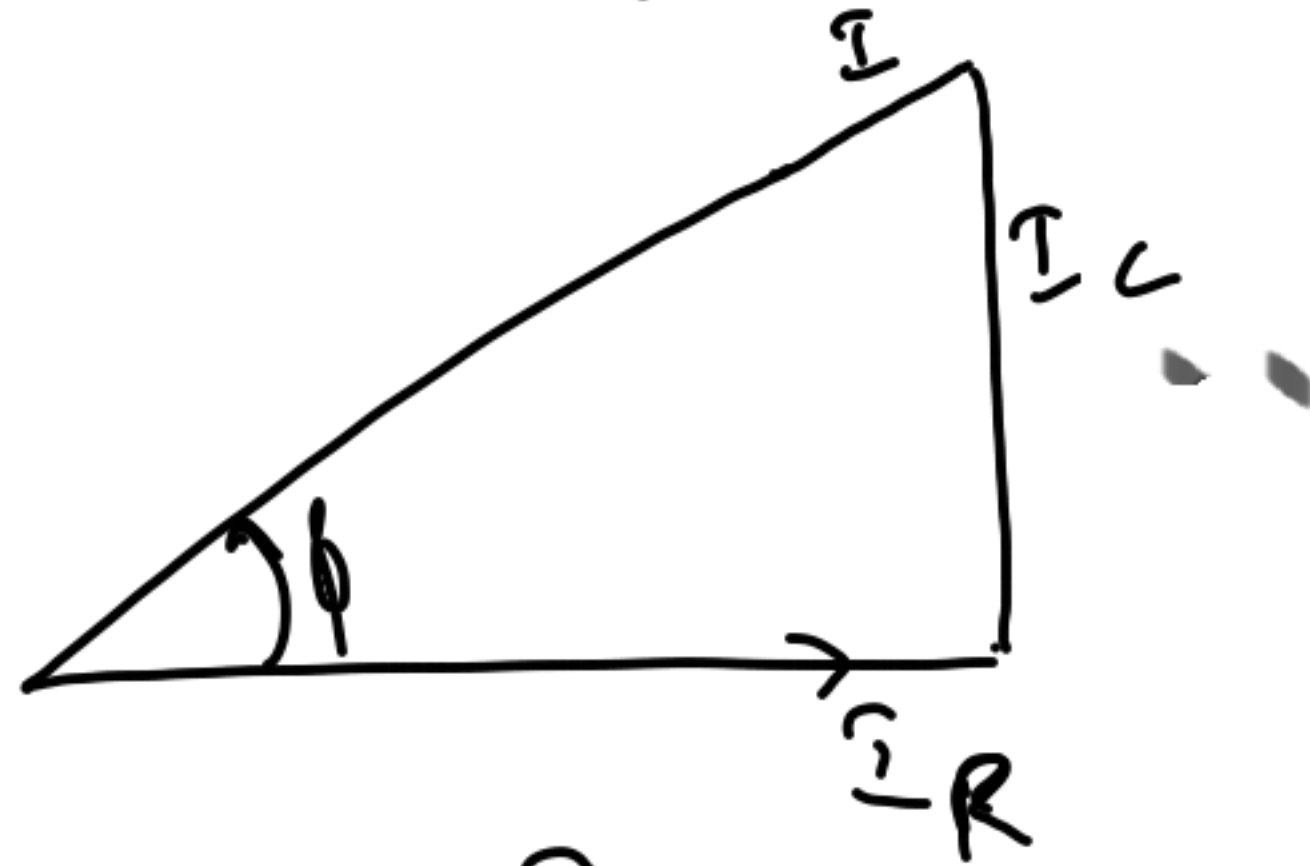
$$I_R = \frac{V}{R}, \quad I_C = \frac{V}{X_C}$$

Phasor Diagram.



I_R is in phase with V

I_C is leading V by 90°



$$\tan \phi = \frac{I_C}{I_R}$$

$$\cos \phi = \frac{I_R}{I}$$

$$\sin \phi = \frac{I_C}{I}$$

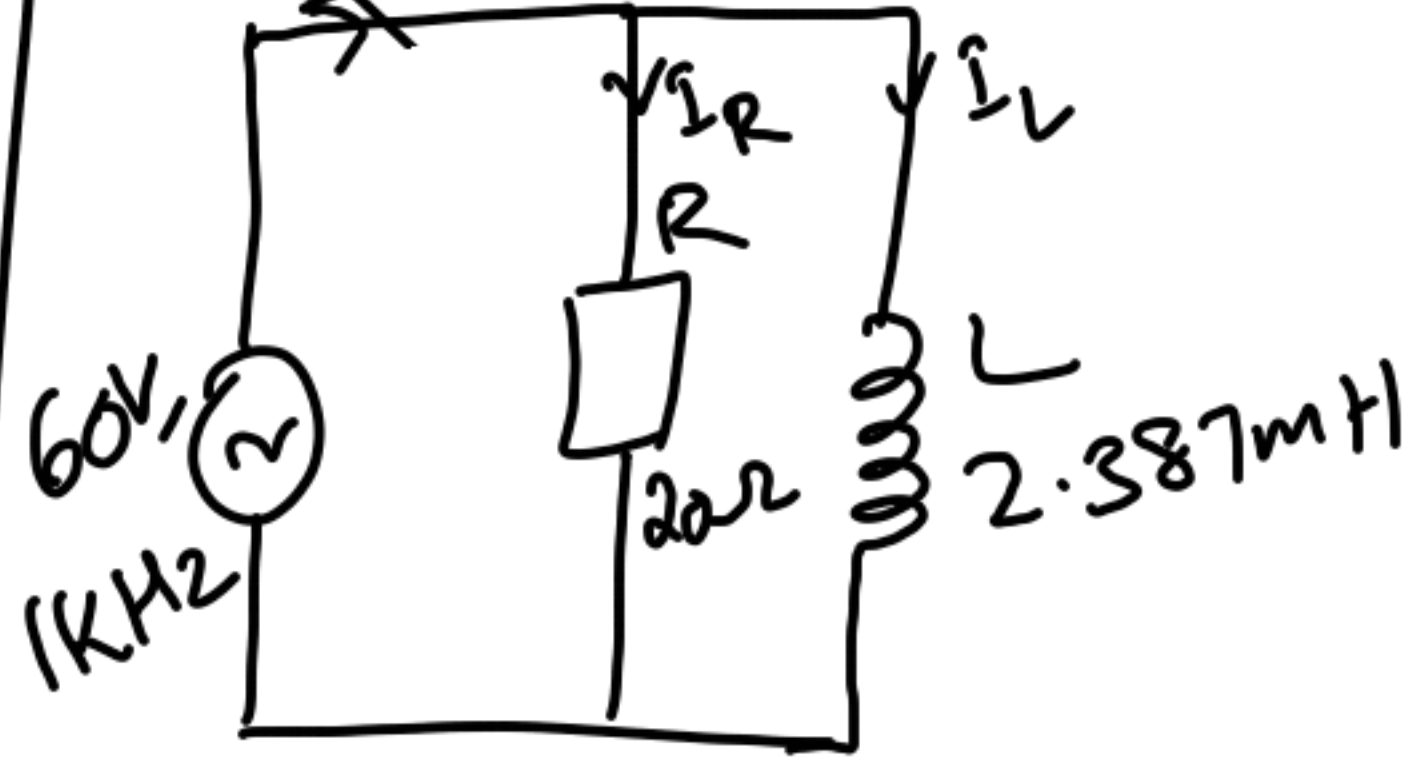
Impedance

$$Z = \frac{V}{I}$$

1) A 20Ω Resistor is connected in parallel with an inductance of 2.387mH across a 60V 1kHz Supply Calculate.

- Current in each branch
- the Supply current
- the Circuit phase angle
- the Circuit impedance
- the power consumed.

Soln:



$$I_R = \frac{V}{R} = \frac{60}{20} = 3\text{A}$$

$$I_L = \frac{V}{X_L} = \frac{60}{2\pi fL}$$

$$= \frac{60}{2\pi \times 1 \times 10^3 \times 2.387 \times 10^{-3}}$$

$$I_L = 4\text{A}$$

$$I = \sqrt{(I_R)^2 + (I_L)^2}$$

$$= \sqrt{3^2 + 4^2}$$

$$I = 5A$$

$$c) \phi = \tan^{-1}\left(\frac{I_L}{I_R}\right)$$

$$= \tan^{-1}\left(\frac{4}{3}\right)$$

$$\phi = 53.13^\circ (\text{lagging})$$

$$d) Z = \frac{V}{I} = \frac{60}{5} = 12 \Omega$$

$$\text{Power consumed} = VI \cos \phi$$

$$= 60 \times 5 \cos(53.13^\circ)$$

$$= 180 \text{ W}$$

2) A 30μF Capacitor is connected in parallel with an 80Ω resistor across a 240V, 50Hz Supply. Calculate

a) the Current in each branch,

b) the Supply Current

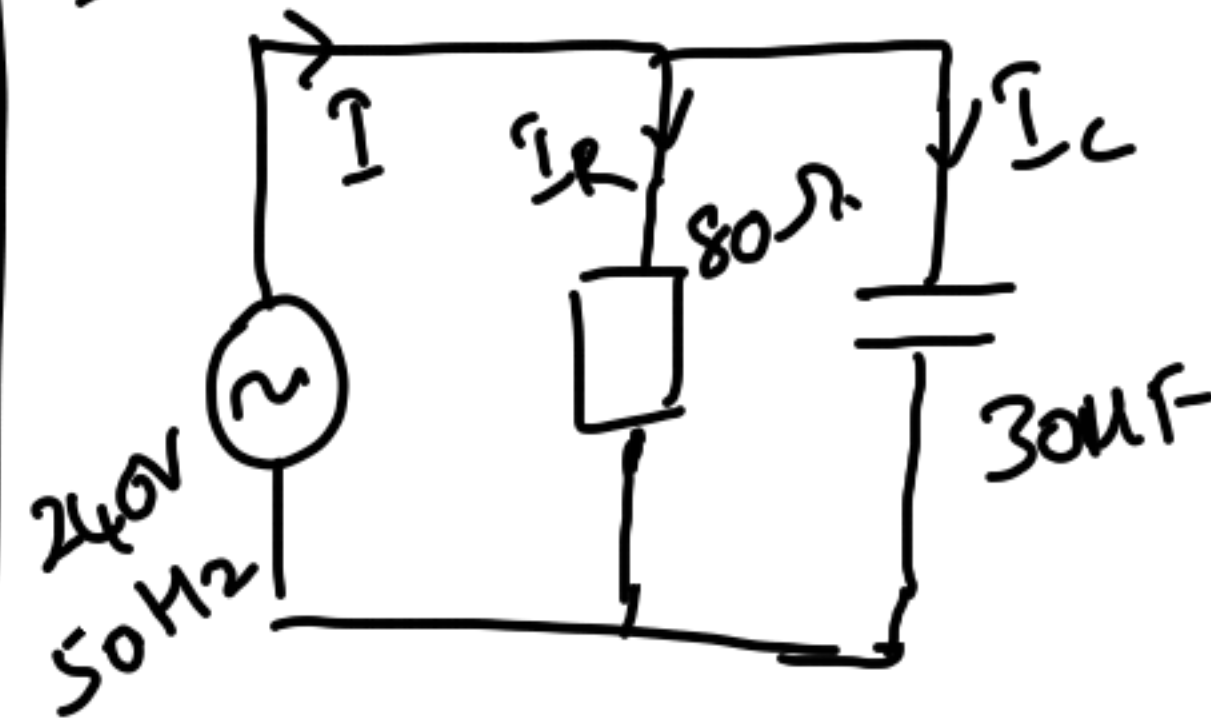
c) the Circuit phase angle

d) the Circuit impedance

e) Power dissipation.

f) apparatus power.

Soln.



a) Resistor Current

$$I_R = \frac{V}{R} = \frac{240}{80} = 3 \text{ A}$$

Capacitor Current

$$I_C = \frac{V}{X_C} = \frac{V}{\frac{1}{2\pi fC}}$$

$$= 240 \times 2\pi \times 50 \times 30 \times 10^{-6}$$
$$= 2.26 \text{ A}$$

b) Supply current.

$$\begin{aligned}\hat{I} &= \sqrt{\hat{I}_R^2 + \hat{I}_C^2} \\ &= \sqrt{3^2 + (2.26)^2}\end{aligned}$$

$$\hat{I} = 3.76 \text{ A}$$

c) Phase angle

$$\phi = \tan^{-1}\left(\frac{\hat{I}_C}{\hat{I}_R}\right)$$

$$\begin{aligned}\phi &= \tan^{-1}\left(\frac{2.26}{3}\right) \\ \phi &= 37.0^\circ \text{ [leading]}\end{aligned}$$

d) Circuit impedance.

$$Z = \frac{V}{\hat{I}} = \frac{240}{3.76}$$

$$Z = 63.8 \Omega.$$

e) Power dissipated (active)

$$P = V \hat{I} \cos \phi$$

$$P = 240 \times 3.76 \cos 37^\circ$$

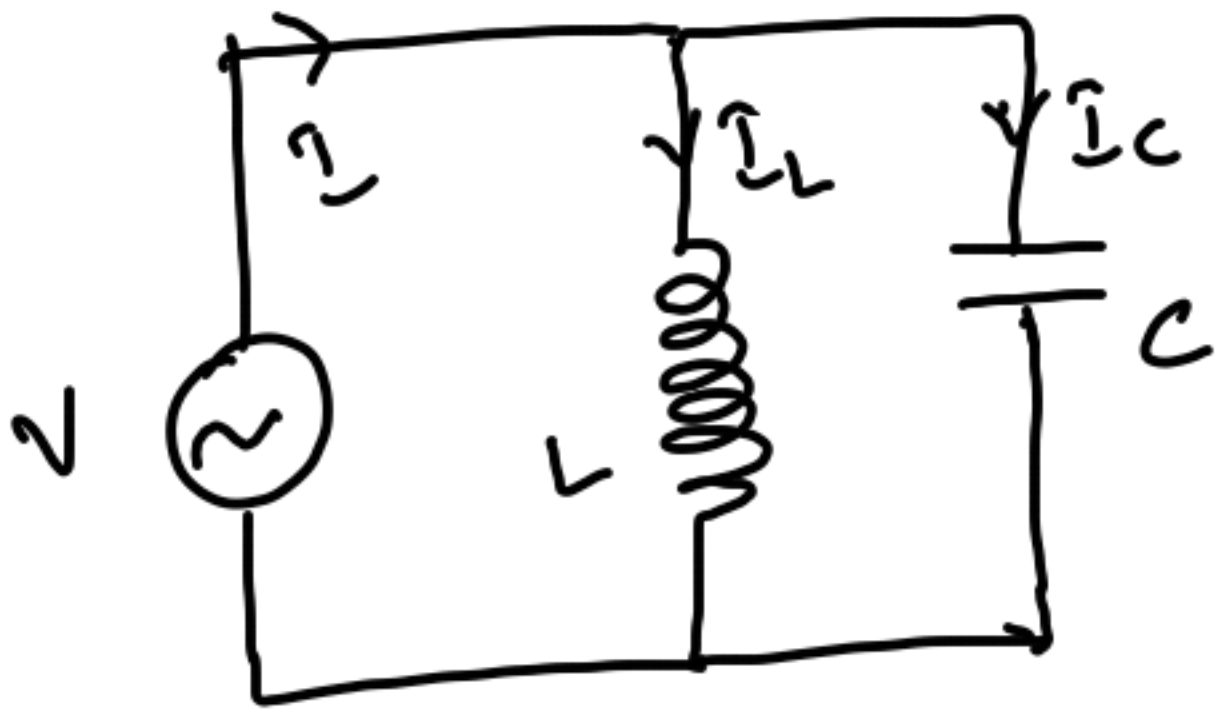
$$P = 720.7 \text{ W}$$

f) apparent power (S)

$$S = V \hat{I} = 240 \times 3.76$$

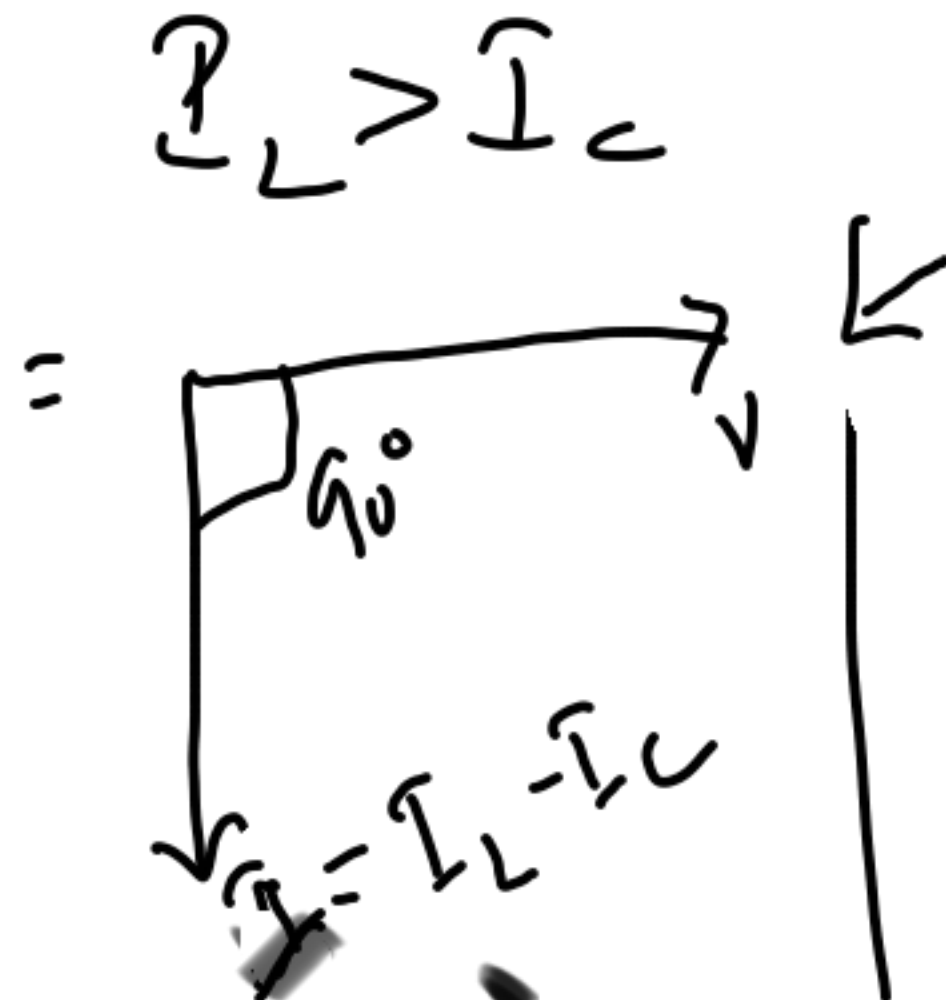
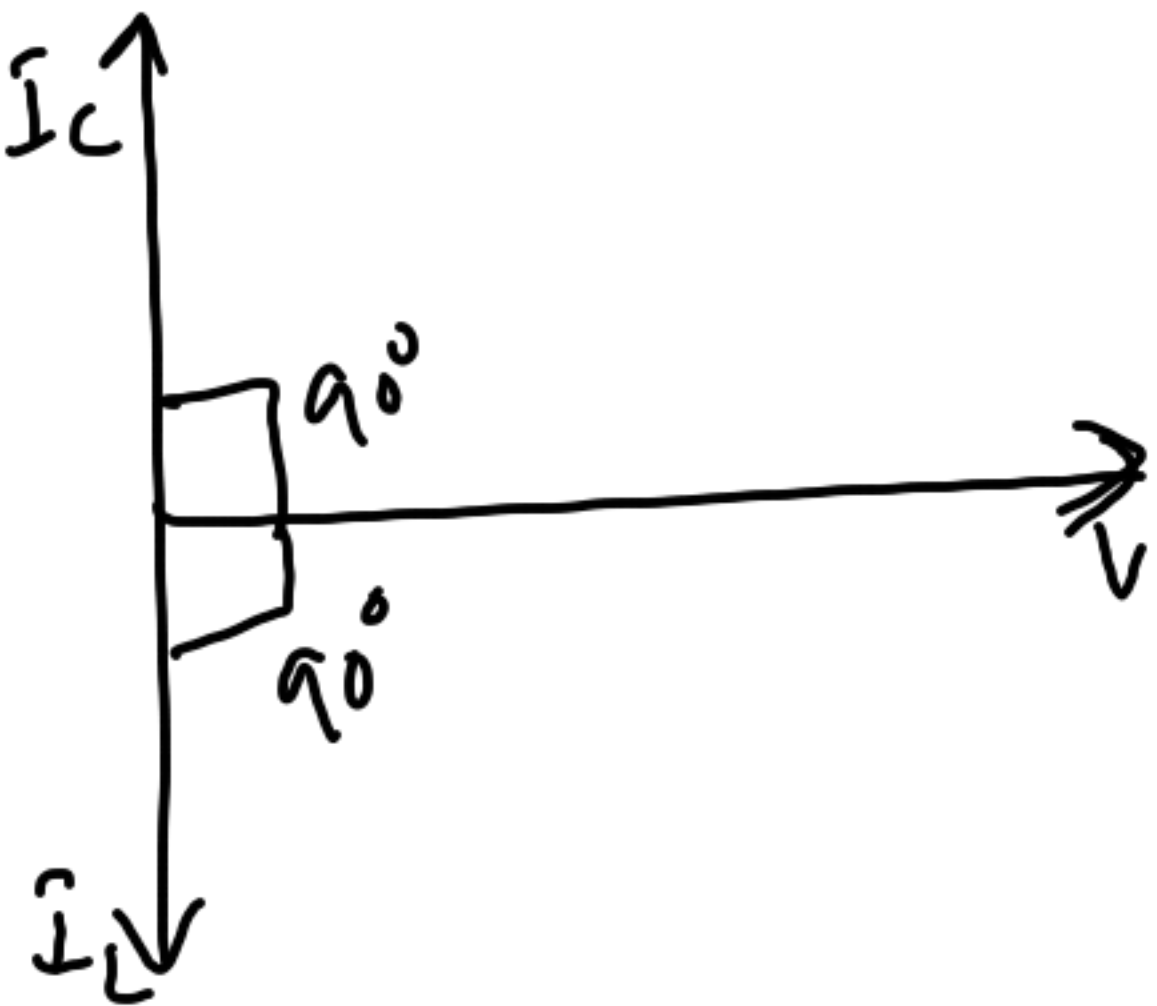
$$S = 901.7 \text{ VA}$$

L-C parallel circuit



I_L with $V \rightarrow I_L$ lags 90°

I_C with $V \rightarrow I_C$ leads 90°

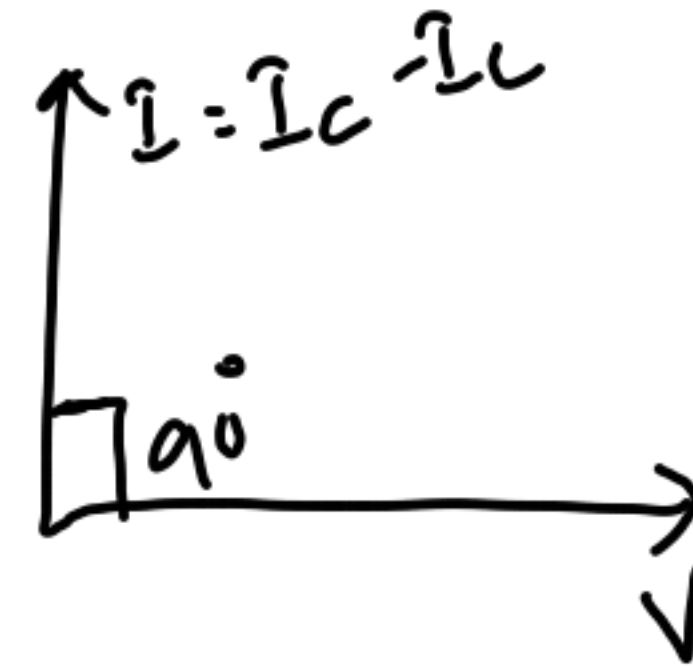


$$1) I_L > I_C \rightarrow I = I_L - I_C \quad \checkmark$$

$$2) I_C > I_L \rightarrow I = I_C - I_L \quad \checkmark$$

$$3) I_L = I_C \rightarrow I = 0 \quad [\text{This is not possible}]$$

$$I_C > I_L$$



$$I_L = \frac{V}{X_L} \quad I_C = \frac{V}{X_C}$$

$$Z = \frac{V}{I}$$

3) A pure inductance of 120mH is connected in parallel with a 25μF capacitor and the network is connected to a 100V, 50Hz supply. Determine (a) the branch current.

b) the supply current and its phase angle.

c) Circuit impedance.

d) the power consumed.

$$a) X_L = 2\pi fL = 2\pi \times 50 \times 120 \times 10^{-3} \\ = 37.7\Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 25 \times 10^{-6}} \\ = 127.3\Omega.$$

$$I_L = \frac{V}{X_L} = \frac{100}{37.7} = 2.653 \text{ A}$$

$$I_C = \frac{V}{X_C} = \frac{100}{127.3} = 0.786 \text{ A}$$

$$b) \underline{I_L > I_C}$$

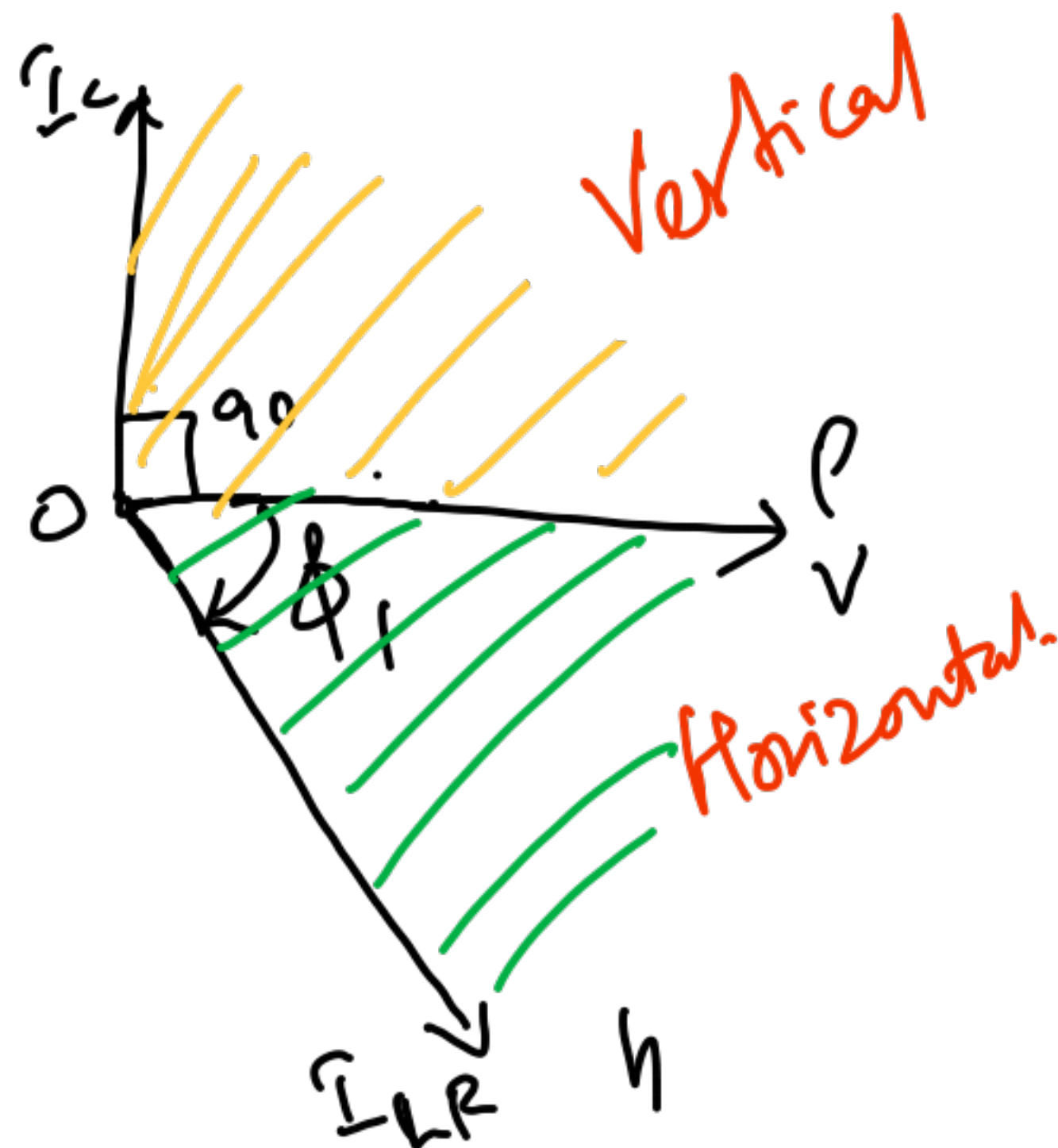
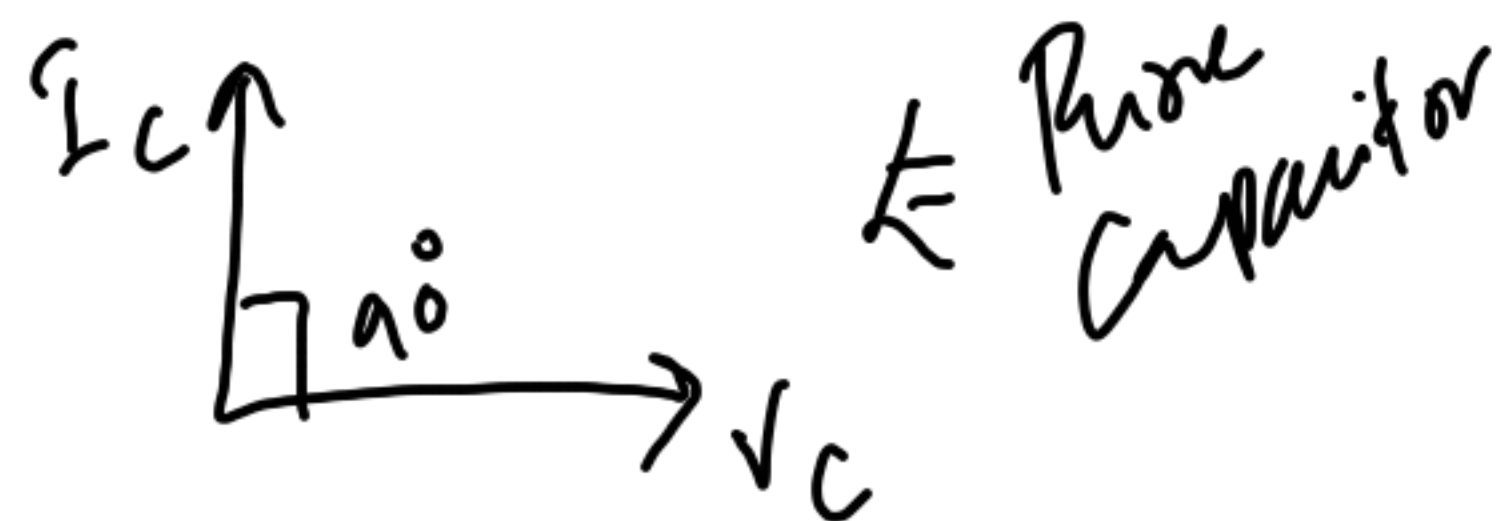
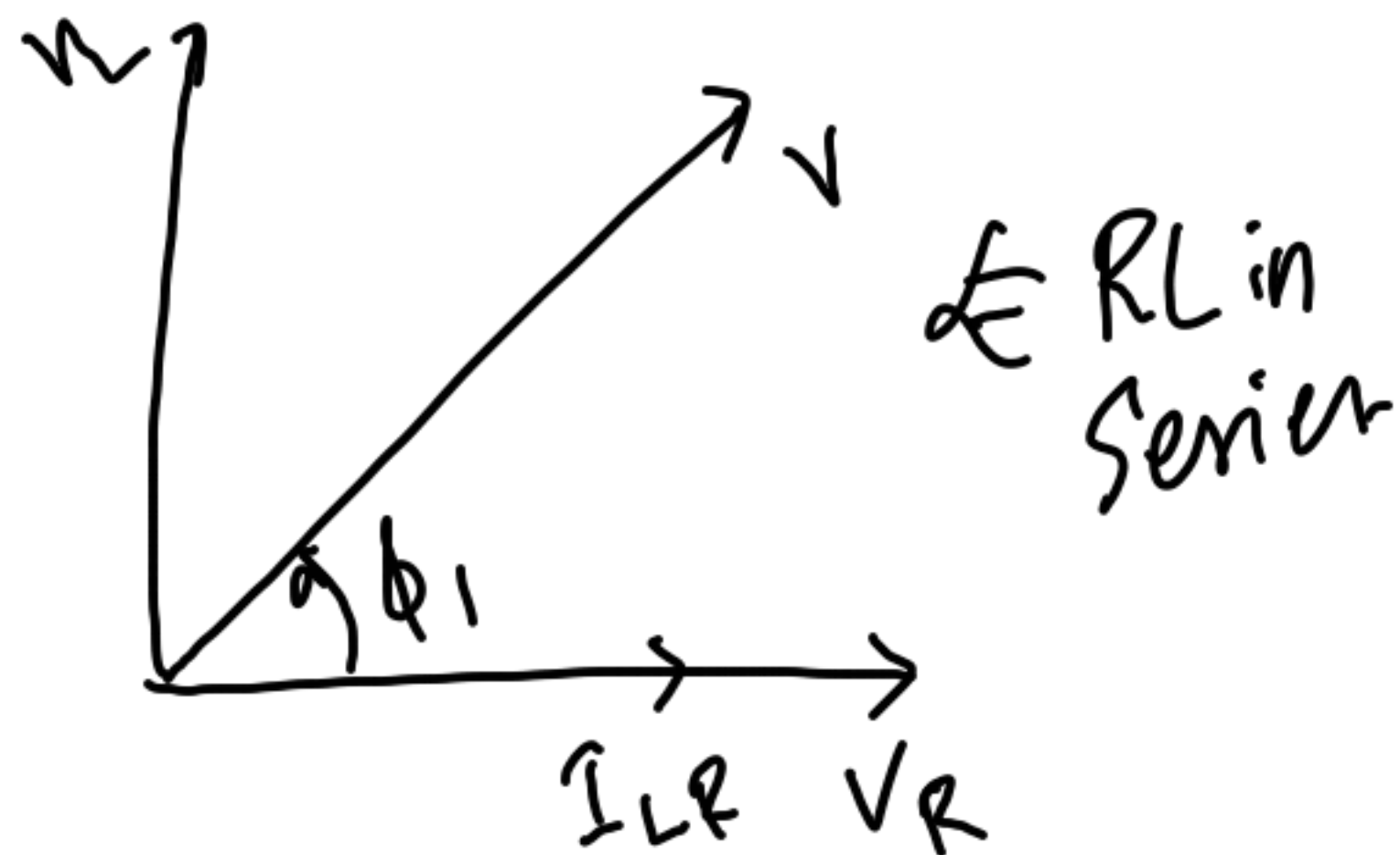
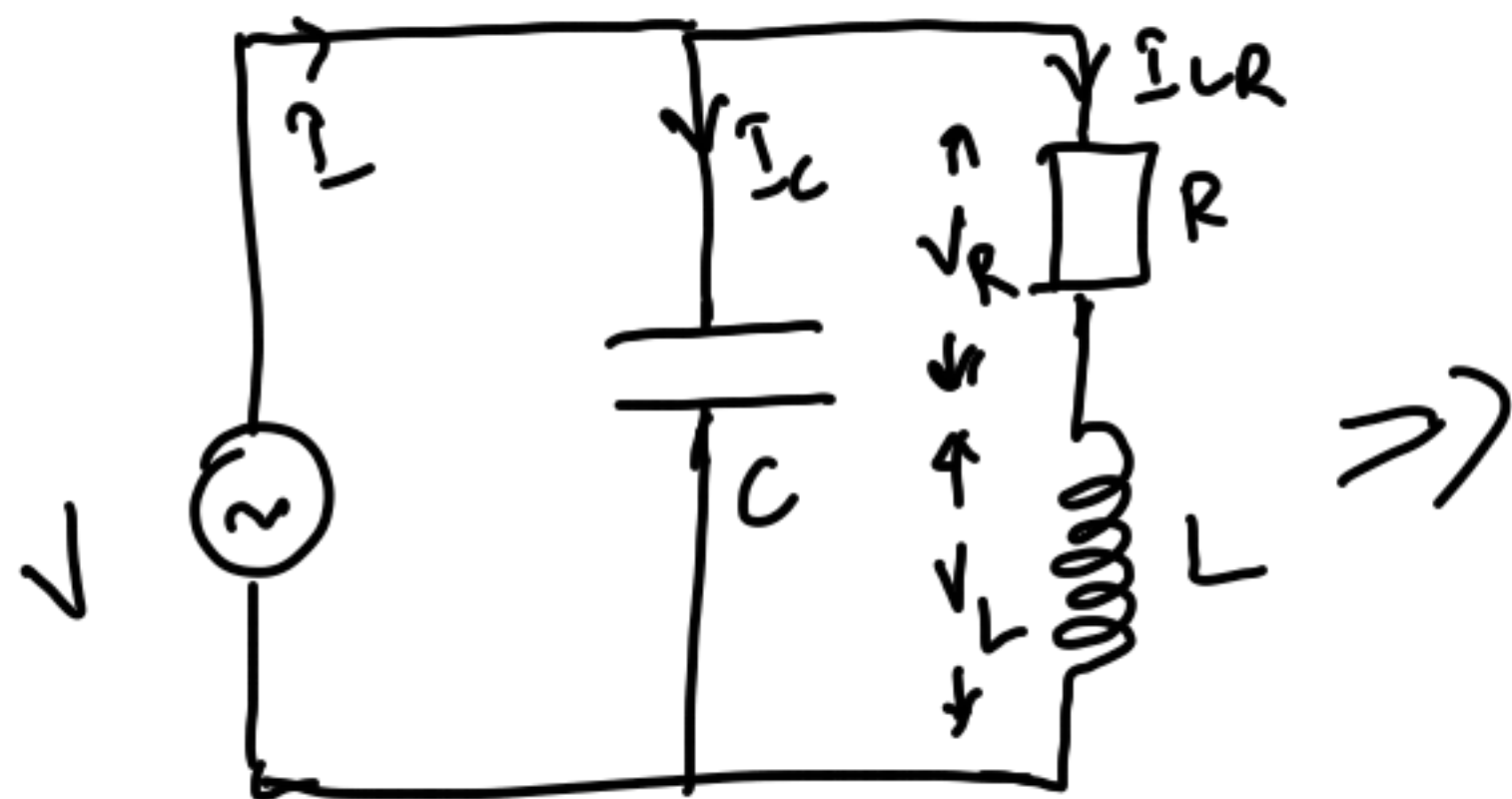
$$I = I_L - I_C = 2.653 - 0.786$$

$$I = 1.867 \text{ A} \quad [I_L \text{ lags } V \text{ by } 90^\circ]$$

$$c) Z = \frac{V}{I} = \frac{100}{1.867} = 53.56\Omega.$$

$$d) \text{ Power consumed } P = VI \cos \phi = 0 \text{ W}$$

LR-C parallel circuit

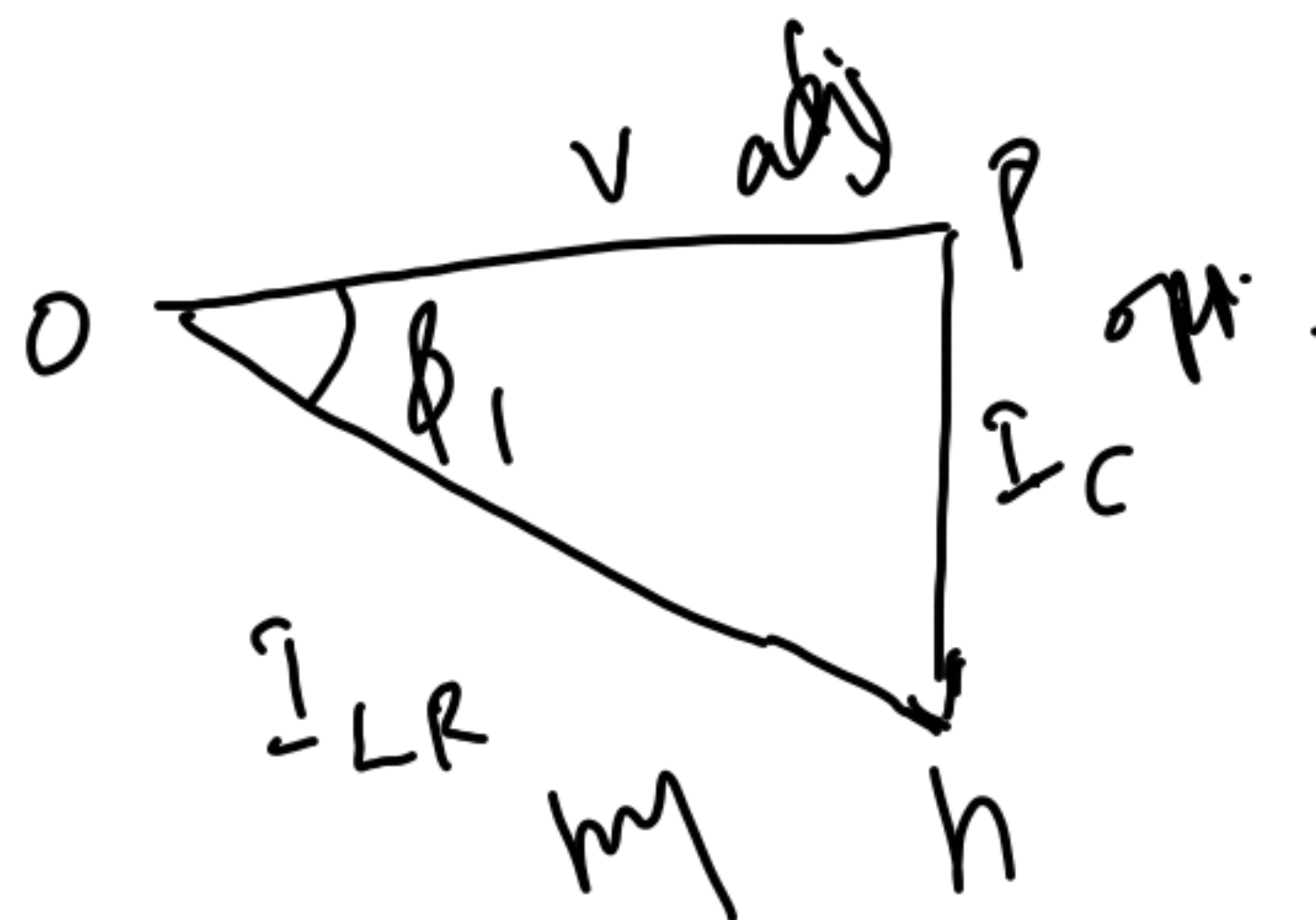


$$\sin \phi_1 = \frac{\text{opp}}{\text{hyp}} = \frac{I_C}{I_{LR}}$$

$$I_{LR} \sin \phi_1 = I_C = \phi h$$

$$\cos \phi_1 = \frac{\text{adj}}{\text{hyp}} = \frac{V}{I_{LR}}$$

$$I_{LR} \cos \phi_1 = V = OP$$

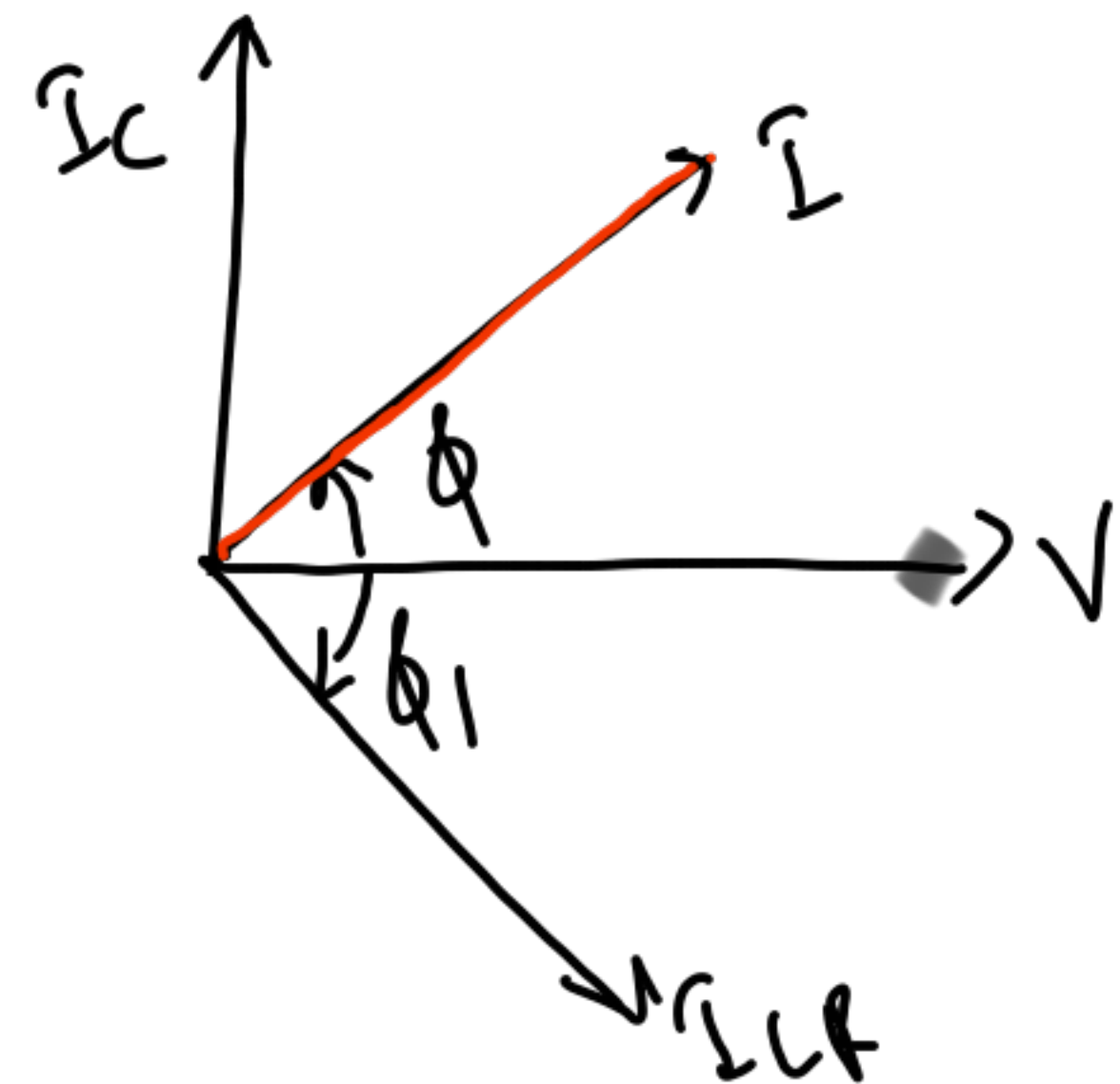


$\hat{I}_C > \hat{I}_{LR} \sin \phi_1 \rightarrow$ giving a supply current $\hat{I}$, leading by angle ϕ

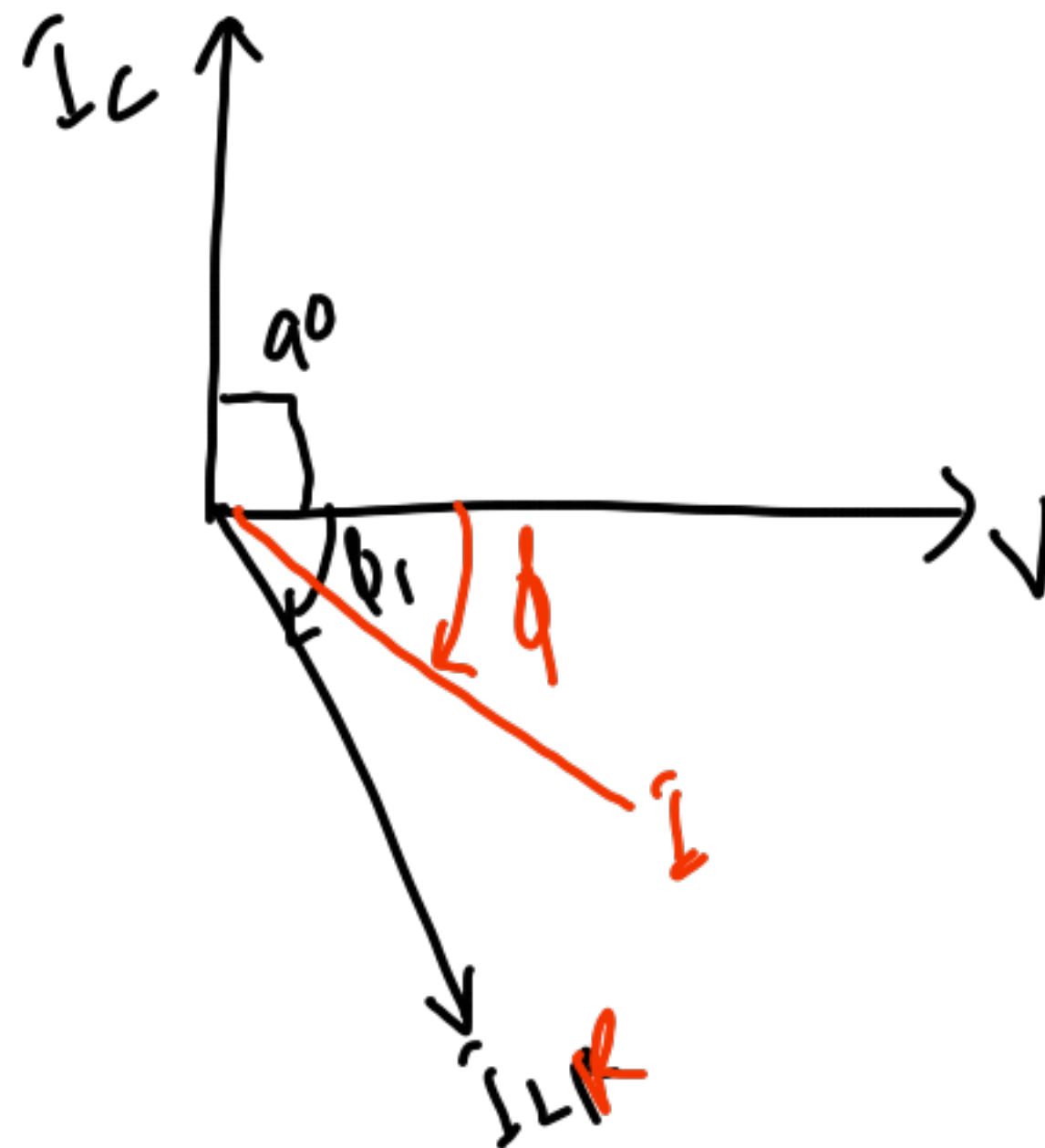
$\hat{I}_C < \hat{I}_{LR} \sin \phi_1 \rightarrow$ giving a supply current $\hat{I}$, lagging by angle ϕ

$\hat{I}_C = \hat{I}_{LR} \sin \phi_1 \rightarrow$ Parallel Resonance.

Phasor for $\hat{I}_C > \hat{I}_{LR} \sin \phi_1$



Phasor for $\hat{I}_C < \hat{I}_{LR} \sin \phi_1$



Impedance of LR branch

$$Z_{LR} = \sqrt{R^2 + X_L^2}$$

Current $I_{LR} = \frac{V}{Z_{LR}}$

$$I_C = \frac{V}{X_C}$$

$$\tan \phi_1 = \frac{V_L}{V_R} = \frac{X_L}{R}$$

$$\sin \phi_1 = \frac{V_L}{Z_{LR}}$$

$$\cos \phi_1 = \frac{R}{Z_{LR}}$$

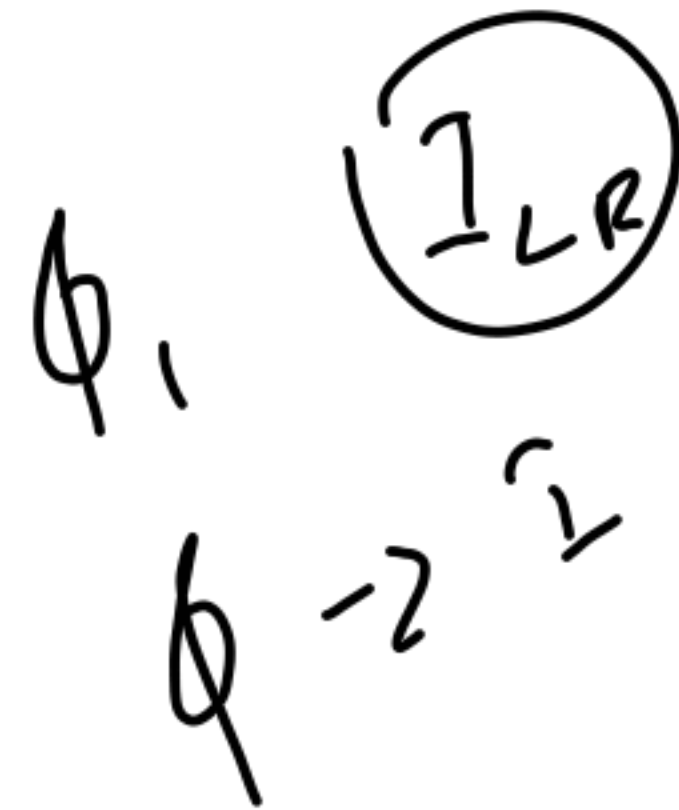
$$\tan \phi = \frac{I_{LR} \sin \phi_1 \sim \hat{I}_C}{I_{LR} \cos \phi_1}$$

$$\cos \phi = \frac{I_{LR} \cos \phi_1}{\hat{I}}$$

Supply Current $\hat{I}$ = phasor sum of I_{LR} and I_C

$$= \sqrt{(I_{LR} \cos \phi_1)^2 + (I_{LR} \sin \phi_1 \sim \hat{I}_C)^2}$$

Circuit Impedance $Z = \frac{V}{\hat{I}}$



Parallel Resonance and Q-Factor

When $I_C = I_{LR} \sin \phi$, Parallel Resonance occurs.

Resonance frequency.

$$f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

If R is too small

$$f_r = \frac{1}{2\pi \sqrt{LC}} \quad [\text{like Series circuit}]$$

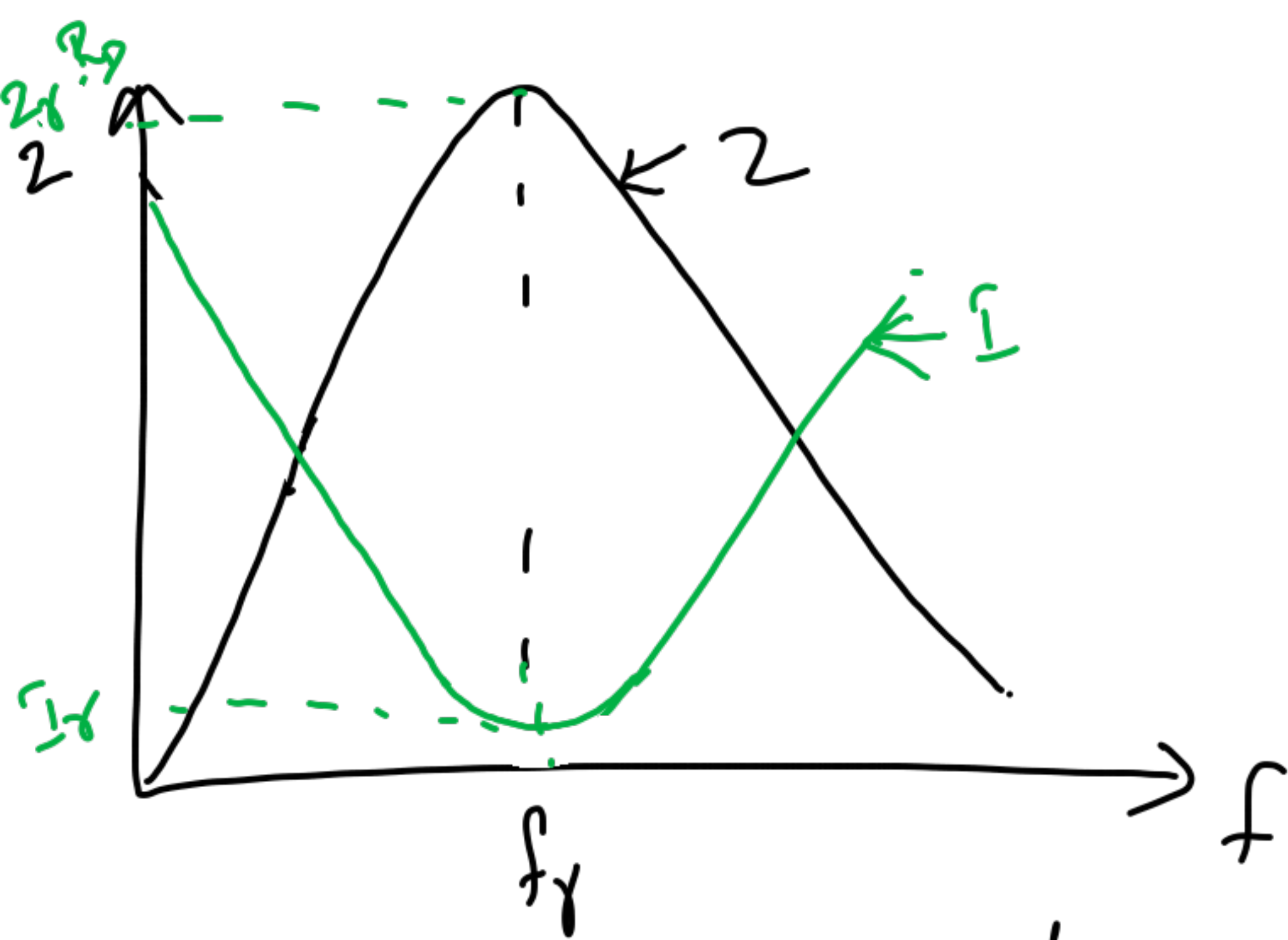
Current at Resonance. Since $\Rightarrow \therefore$

$$I_r = \frac{VR}{Z_{LR}^2} \quad [\because Z_{LR}^2 = \frac{L}{C}]$$

$$I_r = \frac{VRC}{L}$$

Dynamic Resistance

$$R_D = \frac{V}{I_r} = \frac{V}{\frac{VRC}{L}} = \frac{L}{RC}$$



Resonator circuit - 11th Resonance det.

Application:

- provide a stable freq
- filter

Q-factor

Q-factor at Resonance

$$= \frac{\text{Circulating Current}}{\text{Supply Current}}$$

$$Q = \frac{I_c}{I_s}$$

$$Q = \frac{2\pi f_r L}{R}$$

Series $\rightarrow$ Q-factor
Voltage magnification

Parallel $\rightarrow$ Q-factor
Current magnification

Power factor correction

$$\cos(\phi) = \underline{\underline{1}}$$
$$(\phi < 1)$$

