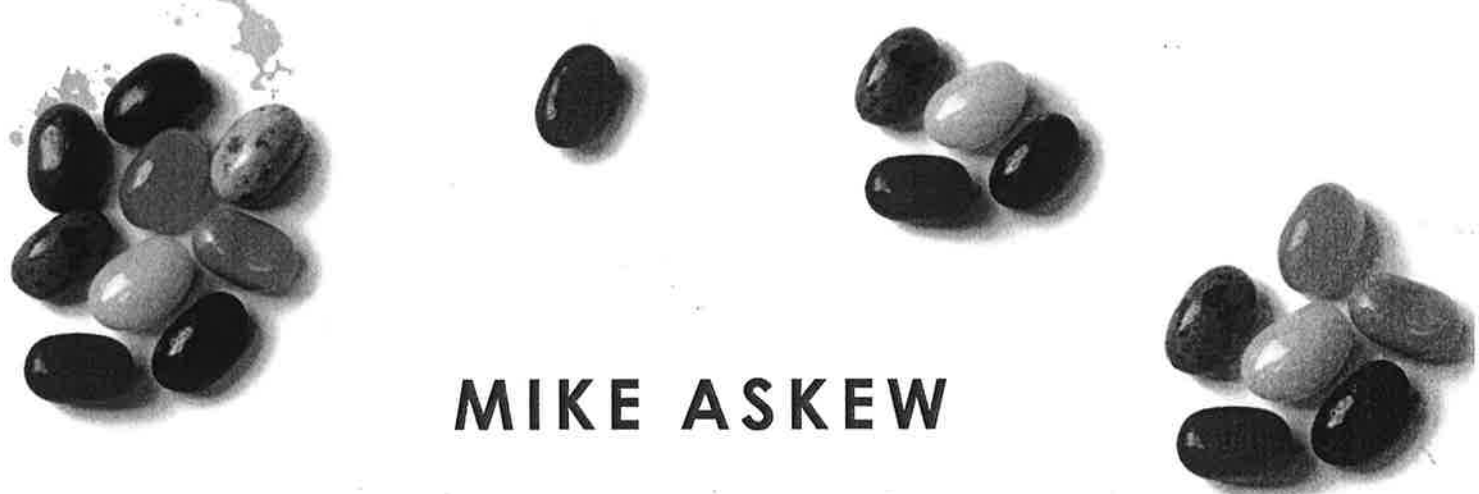




MIKE ASKEW

A PRACTICAL GUIDE TO TRANSFORMING **PRIMARY** MATHEMATICS

Activities and tasks
that really work

Geometry

Teaching geometry largely falls into two aspects of learning and inquiry:

- the properties of two-dimensional and three-dimensional shapes;
- position and movement.

While these two aspects can be treated separately they are connected. For example, pupils need to understand that a shape remains being, say, a square, irrespective of how it is positioned or moved in space. A square does not suddenly become a diamond simply because it is rotated through 45° ! So although these two aspects are dealt with separately here, keep in mind the possible links.

When teaching geometry we also need to be aware of another, subtle, distinction; the distinction between the physical, real world of three- and two-dimensional objects and the theoretical world of mathematical ideas. When we work with images of, say, a circle, there are the physical representations (on paper, screen or cut out of card) of circles and the mathematical concept of a circle (as the points that are all a fixed distance from a given point). However accurately drawn, a physical circle can never be as perfect as the mathematical concept. A circle cut out of paper or card is, strictly speaking, a three-dimensional object - a cylinder, albeit a very short one. As teachers, we need to be clear about this distinction between the imperfect real and the perfect imagined, even if it is not always made explicit to pupils.

Big ideas

- The Big Idea of invariance lies at the heart of geometry - what can be changed and what needs to stay the same for shapes to be well-defined. For example, having four right angles is an invariant property of rectangles, whereas their size can vary.
- Shapes can be considered to be alike in two ways. Shapes that are alike in that they match each other in every way mathematically, both in terms of properties of shape and in terms of size, are described as congruent (non-mathematical properties like colour do not count). Shapes which are alike because they share the same geometrical properties and are not the same size but are scaled versions of each other are known as similar shapes. For example, all equilateral triangles have three equal sides and three equal angles, irrespective of how large or small they are, so all equilateral triangles are similar. But only two equilateral triangles of exactly the same size are congruent. And while all rectangles have four right angles, they are not all similar to each other as the proportions of the sides can vary.
- Geometrical properties are independent of measures. A quadrilateral is a square if it has four equal sides and four equal angles. The fact that the angles are measured to be right angles is not the determining property of 'squareness', it is the fact that the angles all have to be equal. And the actual lengths of the sides have no impact on a quadrilateral being a square, just that the lengths all need to be equal.

Making connections

- Pupils need to make connections between imagining and visualising, shapes in their minds, and the examples of shapes that they experience in the real world.

- The classification of shapes tells us more than just the name of a shape. For example, knowing that a shape is an isosceles triangle not only tells us that it has two equal sides, but also that it must have two equal angles and only one line of symmetry.

Progression in understanding properties of shapes

Often shape is treated as though it were less important than number (it certainly gets less teaching time). In the previous chapters on number, I have suggested that there is a strong spatial element to learning about number. Position and movement on number lines, or comparing arrays and how much space they cover, are just two aspects of foundational ideas in number that are linked to space, and our bodies and movement in space. So while there are aspects of the geometry curriculum that are distinct from the number curriculum, number and geometry are two connected, complementary ways of thinking.

Pupils start school with many experiences of working with three-dimensional objects – balls, bricks, Lego and Duplo, wheels, coins, counters and so forth. Building on these early experiences means that concepts of shape can be grounded in familiar experiences. The development of mathematical language associated with shapes is a key aspect to work on and initially the use of ordinary English words like box instead of cuboid is fine, as long as pupils eventually come to know that 'square boxes' or cubes are also cuboids.

Teaching geometry often focuses mainly on declarative ideas – identifying and naming shapes – but it can do much more than that; geometry can engage pupils in reasoning and problem solving and in considering how to engage pupils in thinking about geometry Duval's (1998) distinction of three types of thinking processes is helpful:

- visualisation processes
- construction processes
- reasoning processes.

Visualisation processes

Although we talk about teaching pupils to visualise, this is a far from straightforward task. Visualising is something that only happens inside the head and we can never know what anyone else is 'internally' seeing. All teachers can do is provide pupils with experiences that get them talking about shapes and describing what they can see (or feel) and hope that this leads to the development of some internal visualising processes. Having shapes on view may help develop the language of description, although working with hidden shapes may bring the processes of visualising into play and in Key Stage 1, working with 'feely bags' introduces hidden shapes. Having shapes in a bag that the learners can feel but cannot see not only gets them to describe the shapes but also encourages them to conjure up a mental image of the shape. Hide a 3D shape in a bag. Pass it round – everyone feels the shape and says something about it – but they cannot say what they think the shape is. For example, 'I can feel a square face' is ok, but, 'I can feel a cube' is not. Stress that it is okay to repeat what someone else has said. Help the pupils refine their language of description – for example, if they talk about feeling a 'side', use another shape that everyone can see to find out if they are using 'side' to refer to face ('a side of a cube') or an edge ('a side of a square') and to refine their description appropriately. When everyone has said something about the shape, get pupils to speculate what it is. Can the pupils identify the shape from the description given? Take it from the bag to check and continue to talk about its properties. (Depending on the age and level of attainment of the pupils you might have a set of all the shapes on the table labelled with their correct names for them to refer to as they hear the description.)

An alternative to having shapes in feely bags is to set things up in such a way that one pupil can see a shape but the rest of the group cannot: *Behind the wall* provides an extended description of such a task. One definition of spatial or geometrical reasoning is that it is 'the set of cognitive processes by which mental representations for spatial objects, relationships, and transformations are constructed and manipulated' (Clements & Battista, 1992, p. 420). Tasks like these where objects are present, but out of sight, prompt the mental activities of such reasoning.

Asked what the object in Figure 7.1 is, most people will answer 'cube'.

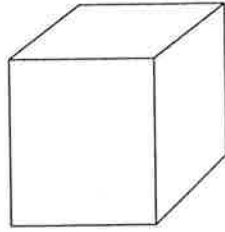


Figure 7.1

However, this is not a cube, but a stylised mathematical representation of a cube: no cube actually looks like that. Just as pupils have to learn to read meaning into the symbols of arithmetic, so too they need to learn to read meaning into the representations of shapes. This is best done through problems that challenge pupils either to make their own representations or to make sense of other people's representations. *Shape treasure hunt* provides both these challenges.

One representation of a right angle is created by a double fold on a piece of paper (see *Right angle everywhere*) - with their own right angle 'tester' pupils can visualise right angles in the environment and then see if they were reasonably accurate in identifying them.

Construction processes

Involving pupils in construction helps to deepen their understanding of geometry. Problems to encourage this can also build on visualising processes. For example, 'feely bags' tasks can be further developed by providing pupils with an object in a bag made up of, say, five interlocking cubes. The bag could be passed around a group, with each person feeling the object and describing it to everyone. Once they have all described the object, each person in the group takes five cubes and builds what they think is a duplicate of the object in the bag, before removing it to check. Or pupils might work with this in pairs: one puts her hand in the bag and describes the object to her partner who has to build a replica from this description.

Other construction activities can engage pupils in moving from two-dimensional representations of shapes to three-dimensional models. For example, the four images in Figure 7.2 show what a building made up of cubes looks like from each of four different directions. Working together, can the pupils use cubes to make a model of the building? Having made a model, is it possible to create it with fewer cubes? (It can be made with as few as six cubes!)

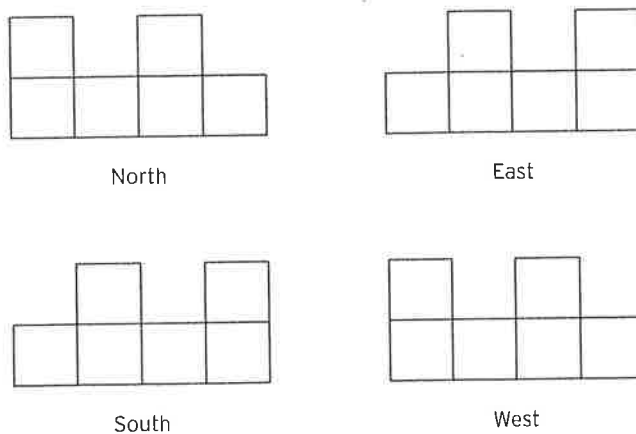


Figure 7.2

Something else to focus on is not just what can be constructed but what are the constraints on constructions, what conditions prevent something from being possible? *Can you make?* . . . provides opportunities for pupils to make two-dimensional shapes from a loop of string and to explore what is then possible or not. For example, an equilateral triangle is very stable in the sense that it cannot be transformed in the plane to create another shape, whereas a square can be transformed to create a (non-square) rhombus.

Reasoning processes

Visualising and constructing come together when pupils reason about results and ask the question 'why?' For example, pupils may observe that the opposite sides of a parallelogram are always the same length and simply take that as a given. Reasoning means asking why that might be so. The writer David Fielker (1983) suggests that properties like this can be explored through pupils drawing a pair of parallel lines on a sheet of paper and creating another pair of parallel lines (a different distance apart as the first pair) on an overhead transparency or sheet of tracing paper. Putting this second pair of lines over the first pair creates a parallelogram. Can the pupils position the paper in such a way that the parallelogram created does not have opposite sides equal? It cannot be done, and although this does not constitute a mathematical proof of the impossibility of a parallelogram not having two pairs of equal sides, it provides pupils with an informal sense of what is happening.

This example demonstrates the importance of exploring properties in a dynamic way. Simply looking at many representations of parallelograms may draw a pupil's attention to the fact that opposite sides are always equal in length, but it will not help her develop a sense of why that is so in the way that playing around with pairs of parallel lines can. Working dynamically is also important in helping pupils understand which properties uniquely define a shape and which properties are a consequence of that definition. A parallelogram, for example, is a quadrilateral with two pairs of parallel sides. We do not need to define it as 'a quadrilateral with two pairs of parallel sides, and opposite sides equal' - the equality of the sides is not an additional requirement, it is a consequence of the sides being parallel. Similarly, a rectangle is not 'a quadrilateral with two pairs of equal sides and four right angles', it is simply a quadrilateral with four right angles (try making a quadrilateral with four right angles that does not have two pairs of equal sides). *All kinds of quadrilaterals* is an extension challenge that leads to reasoning about the links between various properties of quadrilaterals. For example, trying to construct a quadrilateral with exactly one pair of parallel sides and two pairs of equal sides turns out to be an impossible task. Pupils can then explore, informally, what it is about the interaction of these two properties that makes this impossible.

Tree sorts is another construction challenge that engages pupils in reasoning. The naming of different types of quadrilaterals has a certain logic to it (although it is not entirely unambiguous) - this challenge gets pupils reasoning about the names of quadrilaterals and sorting them into various categories.



BEHIND THE WALL

Foundational inquiry.

Level

Key Stage 1.

Direct object of learning

Recognise and name common 2D and 3D shapes.

Indirect object of learning

Reasoning: understand properties of shapes and how rectangles, triangles, cuboids and pyramids are not always similar to each other.

Tools

A collection of two-dimensional shapes cut out from thin card. Include a variety of triangles and rectangles, some of which are similar to each other, and some of which are not. A screen of some sort to hide the shapes behind, for example a flip chart stand. A collection of three-dimensional shapes including cuboids and pyramids that are not similar to each other (have different proportions of lengths). A sturdy opaque bag to put the shapes in.

Task

Public talk

Hold one of the 2D shapes behind the screen. Reveal part of the shape - what do the pupils think the shape might be? Hide the shape again and reveal a different part. Continue until the pupils are confident that they know what the shape is. Remove it from behind the wall - were they correct?

As the shapes are revealed, leave them on view. When the pupils think a shape of the same type has been hidden, talk about whether or not they think it 'matches' any of the shapes on display. When the shape is revealed, hold it up alongside a shape of the same type. Talk about what is the same and what is different about the shapes. An informal sense of the idea of shapes being similar can be introduced by getting two pupils to stand up and each hold one of the shapes. A third pupil stands where they can see both shapes lined up in their vision. If the shapes are similar, then the two pupils holding them can be positioned so that it looks like one shape exactly fits onto the other - can this be done? Invite pupils up one at a time to check if they think the shapes can be made to fit in this way. Introduce the language of shapes being similar.

Private conversation

Pupils play a hidden object game in pairs.

Pupils make a wall between them with a book - one pupil can see a shape and their partner has to find out what it is by asking questions about the shape. As they get good at this, challenge the pupils to put two or three two-dimensional shapes down on their side of the wall, again through asking questions their partner has gathered enough information to be able to draw the shapes and position them relative to each other. With 3D shapes, one partner builds a simple construction. From a collection of shapes that include copies of the hidden shapes, the other partner has to ask questions to enable them to create a duplicate construction.

Assessment

- Are pupils using 'guessing' questions ('Is it a square?') or are they using 'property' questions ('Are all the sides the same length?')?
- Can pupils use the vocabulary of shapes and their properties accurately?

Public talk

Keep a collection of 2D and 3D shapes on display and frequently play 'What's my shape?' One pupil secretly thinks of one of the shapes, their peers have to find out what it is by asking questions.

Variations

The range and types shapes can be added to over time.

SHAPE TREASURE HUNT

Foundational inquiry.

Level

Key Stage 1.

Direct object of learning

Compare common 2D and 3D shapes and everyday objects.



Indirect object of learning

Problem solving: use the vocabulary of position.

Tools

Everyday classroom objects. Mathematical vocabulary.

Prepare a collection of slips of paper each bearing the name of a 2D or 3D object and an image of that shape. Make sure there are examples of these shapes around the class, either in the form of everyday items or geometrical shapes placed around the room. There needs to be at least three examples of each shape in the room, for example, three square-based pyramids.

Prepare a pupil record sheet with space at the top for their name and room to paste one of the slips of paper with a shape name and image. The bulk of the record sheet has a two by three table:

Where I found my object	Where I found my object	Where I found my object
I think the object is	I think the object is	I think the object is

Task**Public talk**

Discuss with the pupils what they know about how to describe the position of things - write up terms and phrases on the board, such as; in front of, behind, next to, under, on top of, beside, and so forth.

Explain what the task is:

Everyone will be given a slip of paper with the name of a shape on it, and an image of the shape. They are to stick this to the top of their record sheet. They have to find three objects in the classroom that they think are examples of their shape. On their record sheet, they describe where they found their object and write instructions for finding each one in the box labeled 'where I found my object'. They do not fill in the name of the object.

Model this by taking, for example, circle as the shape, and choosing the class clock as an object with this shape. Work with the pupils on writing a description of where the clock is, without naming it. For example: it is high up on the wall, to the left of the whiteboard.

When they have found three objects and described the whereabouts of each one, they will swap their sheet with a partner. They then go off and try to find the objects recorded by their partner on the sheet they have just been given. When they think they have found each object, they write its name in the box below the description of its whereabouts. Getting back together with their partner, they check to see if they had identified the same objects.

Private conversation

As the pupils are looking for objects, check that they are correctly identifying the shape they were given. Help them refine their descriptions of the objects' locations.

Assessment

- Can pupils recognise two-dimensional shapes when they are embedded within three-dimensional objects; for example, seeing a circle as the bottom of a cup?
- Can they use the language of position to help others find their shape; for example, on top of the bookshelf, next to the pens?

Public talk

Put the pupils into new pairs. Ask them to 'think-pair-share' and talk about:

'What was the easiest object to find? Why was it easy?'

'What was the hardest object to find? Why was it hard?'

Get some pupils to share what they were discussing with the class.

Variations

The range of shapes to look for can be varied, or made more specific, for example, an equilateral triangle or a 'long thin rectangle'.

Source

Based on an activity from Gilpin, S. (2002), *Shapes, Shapes Everywhere. A Second Grade Geometry Unit Aligned with Mathematics Standards and Content Knowledge: A Compendium of Standards and Benchmarks for K-12 Education* (3rd ed.).

CAN YOU MAKE . . . ?

Foundational inquiry.

Level

Lower Key Stage 2 but adaptable to Upper Key Stage 2.

Direct object of learning

Compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes.

Indirect object of learning

Reasoning: justify and explain findings about properties of two-dimensional shapes, including when conditions are impossible.

Tools

Lengths of string (around 2m) knotted to form a loop.

Task

Public talk

Both parts of this task need to be done in a space where pupils have plenty of room to move about.

Ask the pupils to wander around. Call out a number - they have to move quietly into groups of that size (it does not matter if one or two groups have one more or one fewer members). Then call out the name of a 2D shape - the group has to use bodies to show that shape. After they have made their shapes and you've invited one or two groups to remain in position while the other pupils look at them and see how they made the shape. They go wandering again before you call out a different size grouping and new shape. Keep the match between the group size and the shape a less than obvious one, for example, ask groups of four to create a pentagon or groups of five to create an isosceles triangle.

Private conversation

Finish with the pupils in groups of four. Each group collects a loop of string. They have to make various shapes with their string - the instructions can either be given orally or put up on the board.

- Make a shape with three equal sides.
- Without measuring, make the sides as accurately equal as you can. Keeping the sides equal, can you transform it into a different shape?
- What is the name of your shape?
- Make an isosceles triangle. Make it as accurately as you can.
- How can you convince someone that your triangle has two equal sides?
- Make a shape with four equal sides.
- Make the sides as equal as possible.
- Keeping the sides equal, can you transform it?
- Make a shape with four equal angles.

- Keeping the angles equal, can you transform it?
- Make a shape with exactly one right angle.
- Make a shape with exactly two right angles.
- Make a shape with exactly three right angles.
- Can you make a quadrilateral with exactly one line of symmetry?
- How can you convince someone that your shape has just one line of symmetry?
- Is there any other quadrilateral you can make with exactly one line of symmetry?
- What about a quadrilateral with exactly two lines of symmetry?
- Three lines of symmetry?
- Exactly four lines of symmetry?

Assessment

- How do pupils go about making the sides of their shapes as equal as possible? Can they find a way to divide the loop of string equally into thirds?
- As they transform their shapes, what do pupils notice stays the same? What do they notice changes?

Public talk

Look at some of the shapes that were created. Were any challenges impossible? How do they know they were impossible?

Variations

Pupils could work in pairs with small loops of cotton or fine string and make written records of the shapes they create. They could also work with straws and pipe cleaners or geostrips to create shapes.

Source

Adapted from Nrich's 'Stringy Quads', available online at <http://nrich.mathematics.org/2913>.



RIGHT ANGLES EVERYWHERE

Practice and consolidation.

Level

Lower Key Stage 2.

Direct object of learning

Identify right angles, recognise that two right angles **make** a half turn, three make three quarters of a turn and four a complete turn; identify whether angles are **greater** than or less than a right angle.

Indirect object of learning

Fluency: identify right angles and use the language of acute and obtuse for angles greater or lesser than a right angle.

Tools

Scrap A4 paper.

Task

Public talk

Make sure everyone has enough space to move about in.

With all pupils standing and facing front, check that everyone is clear about which direction they need to turn if you ask them to turn to their right or left. Also if you ask them to turn clockwise or anticlockwise that they are clear about this distinction. Establish that making a quarter turn is the same as being asked to turn through a right angle. And that a half turn is equivalent to turning through two quarter turns or two right angles and so forth.

Give a variety of turning instructions involving one, two, three or four right angles and a mix of instructions to turn left or right, clockwise or anticlockwise. For example:

Make a quarter turn to your left.

Turn through two right angles in a clockwise direction.

Turn anticlockwise through three right angles.

As the pupils become confident in this, ask them, before making a turn, to point and predict the direction they will end up facing.

Then challenge them to tell you what instruction would bring them back to facing the front. Can anyone come up with a different instruction that will also bring everyone back facing the front?

Make a quarter turn to the left.

'What instruction will bring us all back facing front?'

'Yes, turn through three right angles to the left.'

'Another?'

'Ok, make a quarter turn to the right?'

And so on.

Give everyone a piece of used A4 paper and show them how to make a right angle tester. First they fold the paper in half - any fold line will do. Then they fold that crease back to meet itself - the corner formed is a right angle (see Figure 7.3).

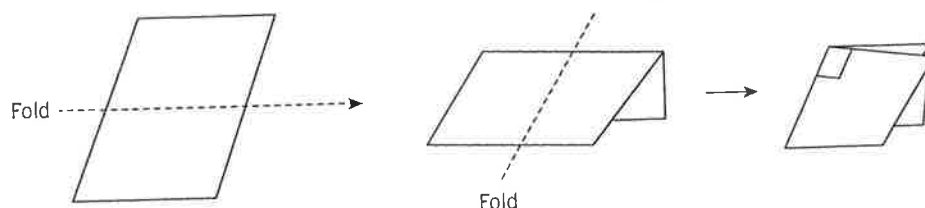


Figure 7.3

Private conversation

Send the pupils off in pairs to find and list various angles around the room: at least five should be very close to being right angles, at least five should be angles less than a right angle and at least five should be greater than a right angle.

As the pupils are looking for and listing their angles, go round and introduce the language of acute and obtuse angles appropriately.

Assessment

- Can pupils talk about the connection between finding right angles and quarter turns?
- Can they use the language of acute and obtuse to describe angles less than or greater than a right angle?

Public talk

Can pupils help each other see the angles they found by describing where they are but not revealing the actual object?

Variation

Show pupils how to make an angle indicator. Take two circles of thin card in two different colours. Cut one radius on each card and slot them together. By turning the circles different angles can be shown (see Figure 7.4).

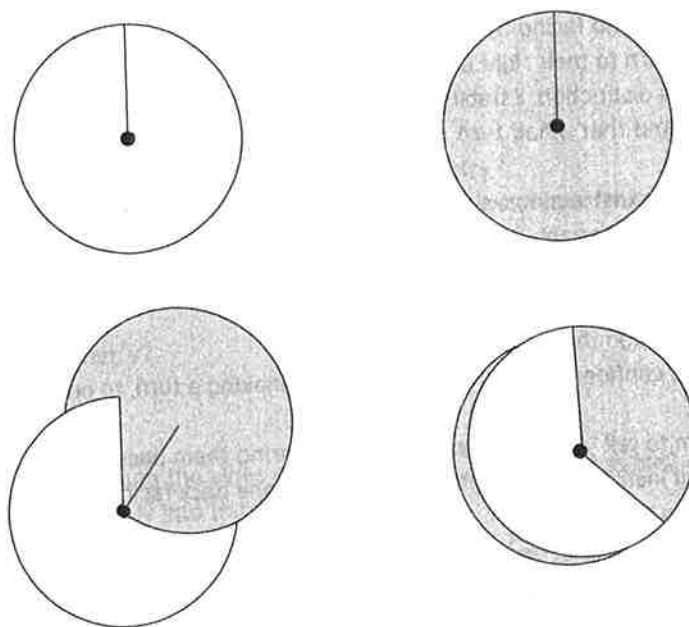


Figure 7.4



ALL KINDS OF QUADRILATERALS

Extension challenge.

Level

Upper Key Stage 2.

Direct object of learning

Distinguish between regular and irregular polygons based on reasoning about equal sides and angles.

Indirect object of learning

Reasoning: identify properties of quadrilaterals and which properties are dependent on others.

Tools

Thin card, rulers, pens, scissors.

Preparation

Groups each need a large (on A2 sized paper) copy of a grid like this one, for all kinds of quadrilaterals:

Pairs of parallel sides	Pairs of equal sides		
	0	1	2
0			
1			
2			

Task

Public talk

Go over how the grid has to be interpreted. Point out that it is 'pairs' of equal sides; so one pair of equal sides means exactly that: two equal sides, no more or less. Similarly, a shape with one pair of parallel sides cannot have two pairs of parallel sides.

Talk about the properties that a shape will have in some of the cells in the grid - for example the shapes in the middle of the grid (1, 1) have to have exactly one pair of equal sides and one pair of parallel sides (this does not have to be the same pair of sides).

The challenge is for groups to create as many different quadrilaterals as they can to go in the nine cells of the grid. If a cell can contain more than one type of quadrilateral, then they should make more than one. Not every cell has a possible quadrilateral, but do not tell the pupils this in advance.

Private conversation

Working in small groups, give pupils a large version of the grid and materials from which to construct quadrilaterals.

As the pupils are creating quadrilaterals, are they able to name them?

Are there any cells that pupils think can contain more than one type of quadrilateral?

This question focuses the pupils' attention on the fact that some quadrilaterals that we give different names to can still share common properties. For example a non-rectangular parallelogram and a rectangle both satisfy the condition of two pairs of parallel sides and two pairs of equal sides.

What about a square? Does that count as having two pairs of equal sides, given that all four sides are equal?

Are any of the cells impossible?

When pupils begin to think some cells have no solutions they usually want confirmation that they are right in this conjecture. Resist telling them whether they are correct or not but bring the class together to share their results and discuss the reasons for why they think quadrilaterals cannot be created for some cells.

The main point with this task is not so much the creating of the shapes as working with the pupils on developing the quality of their arguments for why certain shapes belong in particular cells and why some are impossible to find.

Assessment

- Can pupils accurately name the shapes they have created?
- Can pupils explain why they think some combinations are impossible?

Public talk

Put a large version of the grid up on the board. Pupils offer quadrilaterals to put in the cells. How many names can they give to each quadrilateral?

What about the empty cells? Can anyone convince the class as to why these cannot be made?

Variations

An extension of this activity is to replace the label 'pairs of parallel sides' with 'numbers of right angles' (that is, shapes with precisely no, one or two right angles). The basic structure of a 3×3 grid can be adapted to other contexts - for example putting properties of numbers along the axes - a multiple of three, less than 100, odd - along one axis. A multiple of five, prime, two-digit - along the other axis.

Source

This activity comes from Fielker (1983) *Removing the Shackles of Euclid*.



TREE SORTS

Extension challenge.

Level

Upper Key Stage 2.

Direct object of learning

Compare and classify geometric shapes based on their properties.

Indirect object of learning

Reasoning: explore the naming of shapes and how these names are based on properties of lines and angles.

Tools

A selection of 2D shapes.

Task

Public talk

Introduce the idea of tree sorts by inviting four pupils to the front to play 'Guess who'. Tell the pupils that you are secretly thinking of one of these four volunteers at the front and the class have to ask questions with 'yes' or 'no' answers to find out whom you are thinking of.

For example, 'Are they wearing trainers?' Or, 'Do they wear glasses?' Talk about what a good question is, looking for those that separate the group of four into two groups of two and then split the pairs.

Using appropriate questions draw a tree sorting diagram on the board and then invite pupils to take turns in secretly thinking of one of the four and have questions directed to them to find out who their mystery person is (see Figure 7.5).

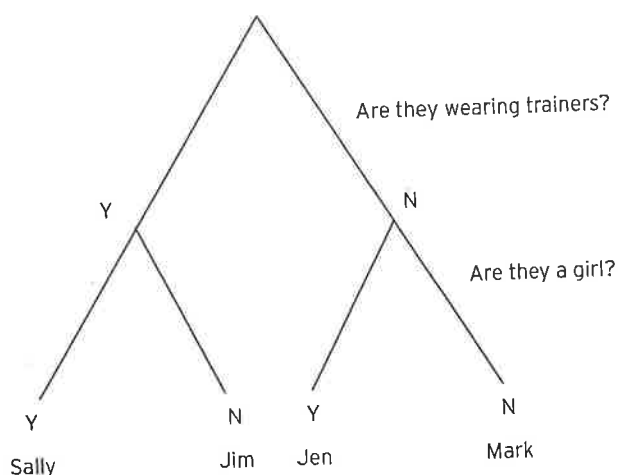


Figure 7.5

Private conversation

Working in pairs or small groups, give the pupils a collection of two-dimensional shapes to create their own tree sorting diagrams. Eight or sixteen shapes are a good number to start with as pupils can try to think of questions that repeatedly halve the collection.

They draw up their tree diagrams on a large sheet of paper and take it in turns to secretly think of a shape and check that their sorting diagram works by getting other pupils to work through the questions.

Assessment

- Can pupils pose questions that involve the properties of shapes, not just their names?
- Can pupils pose questions that separate their shapes into relatively equal groups, rather than just separating off one or two shapes?

Public talk

Display one or two of the pupils' sorting diagrams on the board. Check that they 'work' by inviting individuals to secretly think of one of the shapes and others to use the questions on the tree diagram to find out what the shape is.

Discuss what the pupils think are good questions - questions that help split groups of shapes. How do these questions relate to the naming of shapes?

Variations

One obvious variation is to work with three-dimensional shapes. The basic idea of a tree sort can be used with other mathematical objects, such as a collection of numbers. It can also be used across the curriculum to sort and classify objects, for example, leaves or insects. Basic yes/no questioning is at the heart of all modern technology - computers work on logic of on/off - pupils could explore this on the Internet.

Progression in position and movement

Learning about geometrical properties involves looking at shapes, comparing them with other shapes and identifying and describing key features. Understanding position of shapes in space is about the placing of shapes relative to something else. In *Shape treasure hunt*, pupils work on this with the everyday language of 'behind', 'above', 'next to' and so forth. But mathematical conventions have been developed to make the describing of position more precise, with the coordinate grid as the main convention that pupils meet in primary school. *Place the cubes* introduces pupils to using a simple grid: putting coloured cubes in squares and describing their position helps young learners to embody the idea of describing position in terms of along and up and link this to everyday language such as 'in front of', 'next to', 'behind'. The game helps pupils to observe, visualise and describe positions.

The task *Be seated* extends this into a more formal introduction to coordinates. By the end of primary school pupils are expected to be working with coordinates in the four quadrant grid that includes positive and negative numbers: *Plot the quadrilateral* provides an inquiry into this.

One of the challenges in coming to understand position and movement is how to capture something that is transitory - for example the movement of a shape in space - through the static medium of marks on paper. *Clap and tap* presents pupils with such a challenge: can they find a way to record a rhythmic pattern in such a way that someone else can produce it?

Angles can be thought about in two ways - as an object - this is what a right angle looks like - and as a process - a right angle is the result of an amount of turn that is equivalent to a quarter of a full turn. Pupils explore this connection through dynamically making angles in *Turn about*.

As pupils move into the later primary years they can begin to identify, describe and represent the positions of a shape following reflections, as suggested in *Mirror, mirror*.

PLACE THE CUBES

Foundational inquiry.

Stages

Key Stage 1.



Direct object of learning

Describe position, direction and movement.

Indirect object of learning

Problem solving: describe and communicate position.

Tools

Grids.

Task*Public talk*

Explain to the class how the game is played modelling it through playing a sample round with a pupil.

Playing in pairs, each pupil needs a 3×3 grid (of squares, each about $5 \text{ cm} \times 5 \text{ cm}$). The top of the grid needs to be marked with a symbol so that they know which way to orientate the grid. Each pupil has four different coloured cubes (matched to their partner's cubes). The pair also needs a book to stand up as a screen so that they cannot see each other's grid.

The player to go first places her cubes on four different squares of the grid. Her partner then asks where to put cubes on his grid so that the arrangement matches. The pupil who put the cubes out describes where to place the cubes of particular colours. The players check to see if their completed grids match and then swap roles.

Private conversation

Once the learners are more confident suggest to them that they play the game on a 6×6 grid. This makes it more difficult to specify different positions on the grid without some system. Encourage the pupils to develop a way of labeling the squares, perhaps with colours along one side and letters or numbers on the other.

Assessment

- Can pupils follow instructions you give them for placing a cube on the grid?
- Do pupils use language such as 'along', 'up', 'next to' and so forth appropriately?

Public talk

Discuss with the class the different systems that pairs came up with to describe the position of the cubes on the grid. Work with a particular system, say, letters along the horizontal axis and numbers up the vertical axis. Introduce and work with the convention of always saying the letter (the horizontal axis) followed by the number (the vertical axis).

Variations

Learners can play a simple version of battleships - placing their four cubes on the grid and taking it in turns to guess where their partner's cubes are positioned - an answer of green means they have found a cube (and it is removed from the grid). Yellow means the square named has a side adjacent to a square with a cube in it.

Source

Adapted from an activity in Van de Walle (2007), *Elementary and Middle School Mathematics: Teaching Developmentally* (6th ed.).

CLAP AND TAP

Foundational inquiry.

Stages

Key Stage 1.

Direct object of learning

Order and arrange combinations of mathematical objects in patterns and sequences.

Indirect object of learning

Problem solving: devise notational systems.

Tools

Coloured cubes or different shapes such as polydron. Coloured pens and large sheets of paper.

Task

Public talk

Start a simple rhythm using claps and taps (both hands on your knees), such as 'clap, clap, tap, clap, clap, tap, clap, clap, tap ...'

Invite the class to join in. When everyone has got the pattern, stop and set the challenge: can the learners go off and find a way to record the pattern so that someone else would be able to make it? Tell them they can use paper and pens to record it in some way or if they prefer that there are objects they could use.

Bring the class back together to look at the various ways the pupils solved the problem. Which representations do the class think were particularly helpful?

Set the further challenge of pairs going off and making up a new pattern of their own. They have to record their pattern and then swap with another pair to see if their recording works. Can the other pair make the same pattern from the representation they are given?

Private conversation

As pupils are working on the problem, those who have found a solution could be asked to include a new sound or action, such as a stamp. Can they extend their method of recording to include this new third sound?

Assessment

- Can pupils describe the rule behind their pattern?
- Can they predict what will be, say, the 20th sound or action in their pattern?

Public talk

Put some of the learners' records up on the board. Can the class follow the instructions thus presented for creating a sound pattern?

Create a sound pattern with a repeat of five, such as 'clap, clap, tap, stamp, stamp, clap, clap, tap, stamp, stamp, clap, clap, tap, stamp, stamp ...'

After the class has joined in and got a good sense of rhythm, number the elements in the pattern, so that the 1st is a clap, the 2nd a clap, the 3rd a tap and so forth. Can the pupils say what sound the 10th will be? What about the 30th, the 39th? The 100th, 101st?

Variations

Learners could produce a visual pattern of images or objects and then make up a sound pattern to go with it. In what ways are the different sound patterns made up from the same visual pattern alike? In what ways are they different?

Source

Adapted from an activity in *Mathematics Their Way* by Mary Baratta-Lorton (1976).

**TURN ABOUT**

Practice and consolidation.

Stages

Lower Key Stage 2.

Direct object of learning

Recognise angles as a description of a turn.

Indirect object of learning

Fluency: describe angles using the language of right angle, acute, obtuse and reflex.

Tools

Angle measurers made out of strips of card, about 2-3 cm wide, fixed together at one end with a split pin - learners can make these themselves. Although usually made with two strips of card the same length, this can lead pupils to thinking that angles always have to be represented with lines of equal length and that the length of these lines has something to do with the size of the angle, so make the arms of the angle strips different lengths (see Figure 7. 6).

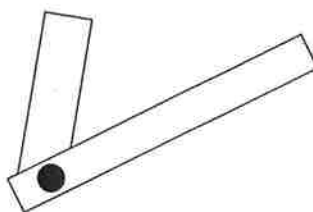


Figure 7.6

Preparation

Images of a number of angles to display on the whiteboard. Sets of four cards labelled obtuse, acute, reflex and right angle (learners could make these themselves).

Task**Public talk**

Ask the pupils to stand up. Get them to physically model turning a certain amount of degrees by following your instructions.

Turn three right angles anticlockwise.

Rotate 90° to your left.

Introduce or recap on the terms obtuse, acute, reflex and right angle.

Private conversation

Pairs of pupils share an angle strip: they take it in turns to create an angle and their partner has to select the correct card to label it as obtuse, acute, reflex or right angle. The learners then find examples of each type of these angles around the classroom.

Public talk

Project up on the board images of angles - learners have to hold up the card indicating what type of angle it is.

Variations

Introduce the idea of a right angle being 90° and of half of this being 45° . Pupils make angles as before but have to decide what number of right angles (to the nearest half right angle) they think the angle is, and work out how many degrees this would be.

Source

Adapted from www.bbc.co.uk/schools/teachers/ks2_lessonplans/mathematics/angles.shtml.

BE SEATED

Foundational inquiry.

Stages

Lower Key Stage 2.

Direct object of learning

Describe positions on a 2D grid as coordinates in the first quadrant.

Indirect object of learning

Fluency: read, write and use pairs of coordinates, for example (2, 5).

Tools

Space for an embodied experience.

Task

Public talk

Arrange 16 chairs in four rows, a short distance apart, all facing towards the class.

Establish which one of the chairs is designated as being in the first row and the first column, and so referred to as the chair in position 'one, one'.

Which chair do the pupils think you are referring to when talking of the one in position 'three, two' - is this the chair in row three, column two or the one in column three, row two?

Establish the convention that the position is described by going along first to find the column that the chair is in and then 'up' that column to the row specified by the second number. Don't expect the pupils to immediately be correct in their use of language here - it is confusing that you need to go along the first row to find out which column you are in!

Ask four or five pupils to take up seats. In turns, can they and the 'observers' work out which column and row their particular chair is in and describe their position using this ordered pair of numbers (you'll need to judge when you think it is appropriate to introduce the language of coordinates)?

Invite other pupils to take up a chair by giving them appropriate instructions. For example:

'Joao, please go and sit in the chair in column four, row two.'

'Robyn, will you go and sit in chair three, four?'

Pupils not yet sitting can give similar instructions to others to take up seats. When most of the seats are full, give various instructions to carry out. For example,

- 'Pupils sitting in 2, 2 and 4, 3 please stand up and swap places.'
- 'Everyone in row three please stand up.'

Again, once the idea of this is clear, pupils can give the instructions.

When certain pupils have been asked to stand, for example everyone in column two, ask them, in turn to say their individual position (coordinates).

Jot these down on the board. What do the pupils notice that they all had in common (in this case a first coordinate of 2)? Can anyone explain why this has happened?

Pose challenges such as:

- Which pupils need to stand up so that their second coordinate is 4?
- Which pupils need to stand up so that both their coordinates are the same?
- Which pupils need to stand up so that their second coordinate is larger than their first?

Private conversation

In pairs, pupils play a treasure hunt game: Each player has a 10×10 grid - they label this with horizontal and vertical axes, the squares being numbered in each direction from 1 to 10. Working so that their partner cannot see where they are playing them, each player 'hides' eight pieces of treasure on their grid by marking eight squares. They take it in turns to guess where their partner's treasures are positioned - an answer of green means they have found a treasure (and scored a point). Yellow means the square named has a side adjacent to a square with treasure in it. Orange means the squared named shares a corner with a square containing treasure. Red means the square named is not near one with treasure in it.

Public talk

Put a large 'treasure map' on the board, overlaid with a coordinate system. Invite pupils to identify various locations on the map using the coordinate system.

Variations

Once learners are confident, they can label the grid lines instead of the squares, thus meeting the idea of the 'origin' at (0, 0).

Source

Adapted from an activity in Van de Walle (2007), *Elementary and Middle School Mathematics: Teaching Developmentally* (6th ed.).



MIRROR, MIRROR

Extension challenge.

Stages

Upper Key Stage 2.

Direct object of learning

Identify, describe and represent the position of a shape following a reflection.

Indirect object of learning

Reasoning: use a 2D grid and coordinates in the first quadrant to locate a shape following a reflection parallel to an axis.

Tools

Large grid paper, cubes (with sides the same size as the grid squares), mirrors, cardboard 'frames' - card rectangles about the same size as the mirrors with the middle cut out to leave a frame to look through, paper clips.

Task*Public talk*

Explain the challenge. Working in pairs pupils take it in turns to set up a challenge for their partner: this involves first of all deciding where the mirror is going to be placed on the grid - the positioning has to be parallel either to the x- or y-axis. As the mirror is going to be moved in and out of position, the placing of it is marked with paperclips on the edge of the grid.

The same person places some cubes on the grid between the mirror line and the axis it is parallel to. (The cubes are placed singly on the grid - not stacked on each other, see Figure 7.7.)

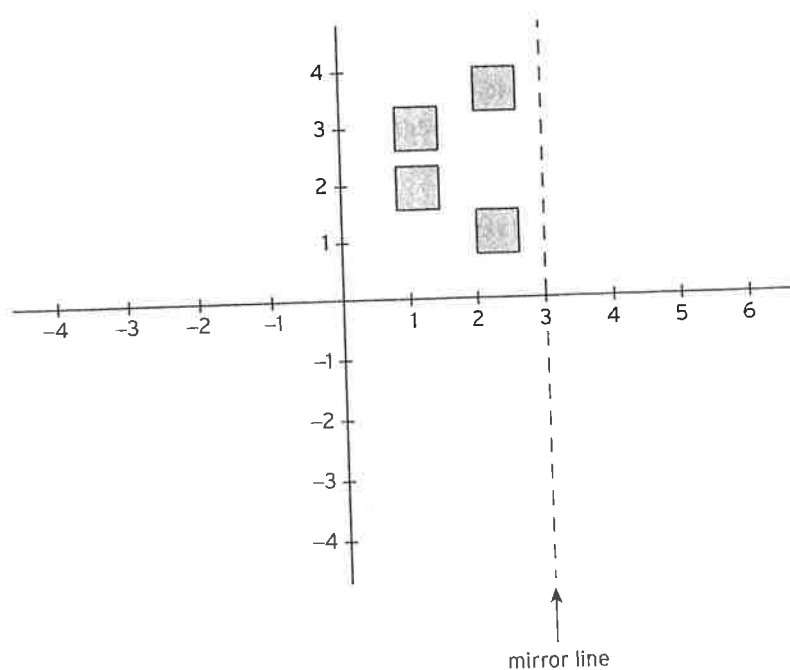


Figure 7.7

Their partner places the frame along the mirror line - they hold the frame in position, upright along the mirror line and looking from the side of the frame that has the cubes on it places cubes on the other side of the frame in the position they would expect to see them in the mirror. When they are confident that the cubes have been placed in the right position, they replace the frame with the mirror, lifting it in and out of position to check if the cubes are correctly placed and moving them if not.

Pairs swap over and create a new set up.

Private conversation

As the learners become confident in setting up a situation and accurately positioning the cubes in the reflected position, ask them to record the coordinates of the cubes and the positions of each reflected cube.

Assessment

- Do pupils notice any patterns in the coordinates recorded? Can they describe these?
- Are they able to predict any of the coordinates of the reflected cubes?

Public talk

Put up a coordinate grid on the board and draw in a mirror line parallel to the y-axis.

Mark up three or squares between the axis and mirror line. Invite learners to come up and write down the coordinates of each of these marked squares.

Can anyone predict what the coordinate of the one of the squares reflected in the mirror line is going to be? Can they explain how they think they know what the coordinates are going to be?

Work on where the reflected squares need to be located and check if any of the predictions about the coordinates are correct. Repeat this with a mirror line parallel to the x-axis.

Variations

Learners work with grids and shading in squares, marking a mirror line (always parallel to one of the axes). Before shading in the reflection of these squares, they write down the coordinates of the squares they shaded and their predictions for the coordinates of the reflections.

**PLOT THE QUADRILATERAL**

Extension challenge.

Stages

Upper Key Stage 2.

Direct object of learning

Describe positions on the full coordinate grid (all four quadrants).

Indirect object of learning

Reasoning: explore the effect of translating simple shapes on the coordinate plane and reflecting them in the axes.

Tools

Squared paper.

Task*Public talk*

This sequence of activities develops from creating and using coordinates through to exploring translations, reflections and rotations involving all four quadrants.

Explain the first part of the task:

Using only the digits -5 , -2 , 1 and 3 the pupils, in pairs, list all the possible coordinate pairs that can be made (there are 16 in all, including $(1, 1)$, $(-2, -2)$ and so forth and the reversed pairs such as $(3, -5)$ and $(-5, 3)$).

Private conversation

As the pupils are working on this, if any of them are unconfident of working with negative numbers, make all the digits positive.

Public talk

Set up this challenge:

Selecting only from the coordinates just created, can the pupils choose sets of four that when plotted on a coordinate grid and joined produce these quadrilaterals:

- square
- rectangle (that is not a square)
- parallelogram (that is not a rectangle)
- rhombus (that is not a square)
- trapezium.

They record the coordinates for each of their quadrilaterals.

Private conversation

As the pupils are working on this, talk about whether they can predict any of the coordinates of a shape, given only two or three of the coordinates. For example, if (2, 5) and (6, 1) are the coordinates of diagonally opposite corners of a square, what are the other two coordinates?

Assessment

- Do the pupils notice any relationships in the coordinates of any particular shape?
- Do any pupils realise that shapes might be made that do not have sides parallel to the axes of the coordinate grid?

What do the pupils notice about the coordinates of, say, squares? If (2, 3) is one corner of a square and the square has a length of side of five units, what is the coordinate of the diagonally opposite vertex ((7, 8))? If you think the learners can deal with it, these could be expressed algebraically for example, if (a, b) is one corner of a square with side length d, then (a + d, b + d) is the coordinate of the opposite vertex.

Starting with a simple translation ask learners to explore the effect of this on the coordinates of their quadrilaterals. For example, you might give them the translation, 'Move five squares to the left and three squares up'. They apply this to one or two of their quadrilaterals, writing down the original coordinates and then the coordinates after the translation.

Can they identify a relationship between the coordinates before and after the translation? Can they then predict what the coordinates of a quadrilateral will be before carrying out the translation, checking their prediction by performing the translation?

Private conversation

Working together, can the pupils come up with a statement that expresses in general terms the effect of the translation on the coordinates? For example:

'Moving five squares left means subtracting 5 from the first coordinate and moving three squares up means adding 3 to the second coordinate.'

Pupils make up their own rules for translations, try these out and see if they can express the general rules. As they are doing this, watch out for pupils who seem ready to use the language of x-coordinate and y-coordinate instead of first and second.

Public talk

Talk about the rule that the learners have found for translations. These could be expressed algebraically for example, translating vertex (a, b) to (a - 3, b + 4).

Variations

Simple reflections - reflecting in the y-axis or reflecting in the x-axis - can be similarly explored.

Finally the pupils can play with what happens with rotations about the origin, clockwise and anticlockwise through 90° or 180°.