

Advanced Physics (OME) Exam

1a.

If mass into shockwave from undisturbed fluid is:

$$\rho_1 D \times \Delta t \times A = \Delta m_1$$

and mass out of shockwave into disturbed fluid is

$$\rho_2 (D - u_2) \times \Delta t \times A = \Delta m_2$$

∴ if $\Delta m_1 = \Delta m_2 = \Delta m$ = Conservation of Mass
then:

wrong ref frame

$$\text{Conservation of Mass} = \underline{\rho_1 D = \rho_2 (D - u_2)}$$

b.

when $D = a + bu$ and $a = 1.45 \text{ km s}^{-1} = 1450 \text{ m s}^{-1}$ and $b = 1.99$

$$u = 500 \text{ m s}^{-1}$$

$$D = 1450 + (1.99 \times 500) \\ = 2445 \text{ m s}^{-1}$$

where $(P_2 - P_1) = \rho_1 D u_2$ and $\rho_1 = 1000 \text{ kg m}^{-3}$, $D = 2445 \text{ m s}^{-1}$
and $u_2 = 500 \text{ m s}^{-1}$

$$P_1 = 0$$

∴

$$P_2 = \rho_1 D u_2 = 1000 \times 2445 \times 500$$

$$= 1222500000 \text{ Pa} \quad (3)$$

$$\therefore \text{Pressure} = \underline{1.22 \text{ GPa to 2d.p.}}$$

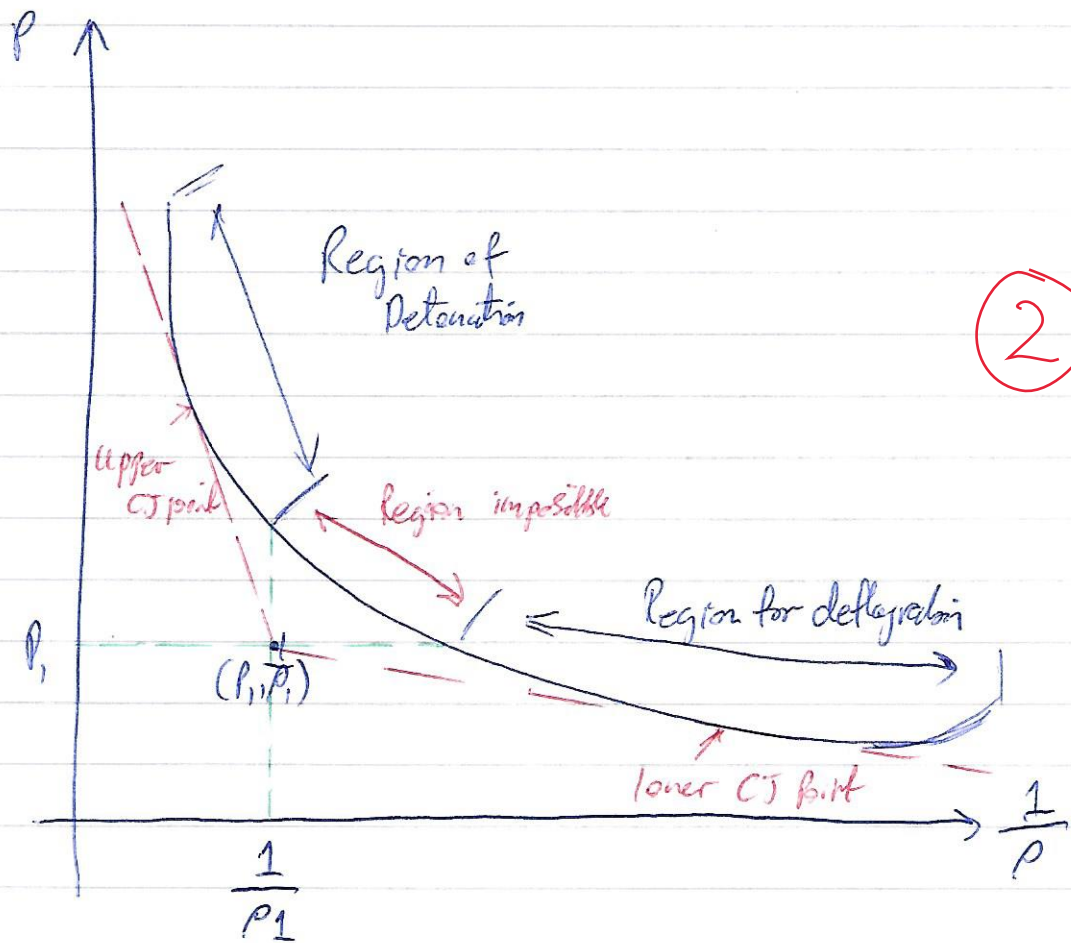
$$\text{If } \rho_1 D = \rho_2 (D - u_2) \text{ then } \rho_2 = \frac{\rho_1 D}{(D - u_2)} = \frac{1000 \times 2445}{2445 - 500} = 1257 \dots$$

All must
exponent
about

$$\underline{\text{Specific volume}} = \frac{1}{\rho_2} = \frac{1}{1257 \dots} = \underline{7.96 \text{ m}^3 \text{ kg}^{-1} \text{ to 2d.p.}}$$

(1)

c) + (D)



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Detonation = Super Sonic Shock wave, velocity of order 2000 ms^{-1}

Deflagration = Sub Sonic Shockwave velocity in order of $1 - 100 \text{ ms}^{-1}$

1

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Q2

- (a) Cavity indicated that at no net moment (dynamic stresses should be equal giving rise to only 6 out of 7 values as 3 were equal to one another.

Also Cavity said that the outputs stiffnesses in the stiffness matrix all had contributing strains from all 6 objects as the 6×1 matrix is multiplied by the 1×6 matrix for stress and strain to give stiffness. i.e. 6 components of stress and linearly related to 6 components of strain. (1)

bi)

If $E = 300 \text{ GPa}$ and $\nu = 0.3$

The stiffness matrix is as follows.

multiplier

$$\begin{bmatrix} 0.7 & 0.3 & 0.3 & 0 & 0 & 0 \\ 0.3 & 0.7 & 0.3 & 0 & 0 & 0 \\ 0.3 & 0.3 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4 \end{bmatrix}$$

ii)

follow through:

$$\begin{aligned} \sigma_{xx} &= (0.7 \times 0.05) + (0.3 \times 0.1) + (0.3 \times 0.05) \\ &= 0.08 \times \frac{300 \times 10^9}{(1+0.3)(1-0.3)} \\ &= \underline{\underline{6.12 \times 10^{11} \text{ Nm}^{-2}}} \end{aligned}$$

c ii)

The Deviatoric Stress matrix is the Stress matrix with Hydrostatic Stress subtracted.

$$\text{i.e. Stress matrix} - \frac{1}{3(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})}$$

(2)

ii) The material behaviour represented by the Deviatoric Stress matrix is Shear Modulus.

(1)

$\frac{9}{15}$

Q3

a. Ideal Gas Law $\Rightarrow PV = nRT$, when $P = 2.5 \times 10^6 \text{ Pa}$, $V = 0.4 \text{ m}^3$
 $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$, $T = 40^\circ\text{C} = 313.16 \text{ K}$

$$\therefore n = \frac{PV}{RT} = \frac{(2.5 \times 10^6 \text{ Pa})(0.4 \text{ m}^3)}{(8.31)(313.16)}$$

$$= 384.2666479 \text{ moles}$$

$$= \underline{\underline{384.27 \text{ moles to 2d.p.}}}$$

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b.

(i) Isothermally $PV = \text{Constant} \therefore P_1 V_1 = P_2 V_2$

$$\therefore P_2 = \frac{P_1 V_1}{V_2} \quad \text{when } P_1 = 5 \text{ MPa} = 5 \times 10^6 \text{ Pa}, V_2 = 0.15 \text{ m}^3$$

$$V_1 = 0.2 \text{ m}^3$$

$$= \frac{(5 \times 10^6)(0.2)}{0.15}$$

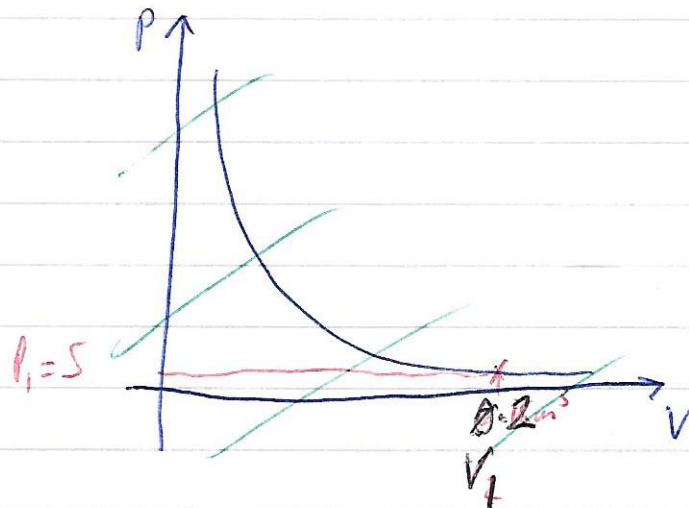
$$= 6666666.667 \text{ Pa}$$

$$= 6.67 \times 10^6 \text{ Pa to 2d.p.}$$

$$= \underline{\underline{6.67 \text{ MPa to 2d.p.}}}$$

3

b(ii)



$$V_1 = 0.15 \text{ m}^3 \quad P_1 = 6.67 \text{ MPa to 2 d.p.}$$

$$V_2 = 0.2 \text{ m}^3 \quad P_2 = ?$$

When $PV^\gamma = \text{constant}$, $P_1 V_1^\gamma = P_2 V_2^\gamma$

$$\therefore P_2 = \frac{P_1 V_1^\gamma}{V_2^\gamma}$$

$$= P_1 \left(\frac{V_1}{V_2} \right)^\gamma$$

Ans!:

$$\gamma = \frac{C_p}{C_v}, \text{ when } C_p = 919.8 \text{ J K}^{-1} \text{K}^{-1} \text{ and } C_v = 657 \text{ J K}^{-1} \text{K}^{-1}$$

$$\gamma = \frac{919.8}{657} = 1.4$$

If $P_1 = 6.67 \text{ MPa to 2 d.p.}$, $V_1 = 0.15 \text{ m}^3$ and $V_2 = 0.2 \text{ m}^3$

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma = (6.67 \times 10^6) \left(\frac{0.15}{0.2} \right)^{1.4}$$

$$= 4456506.145 \text{ Pa}$$

$$= 4.46 \times 10^6 \text{ Pa}$$

$$= \underline{\underline{4.46 \text{ MPa to 2 d.p.}}}$$

b(iii)

Adiabatic means $PV^\gamma = \text{constant}$

$$\therefore P_1 V_1^\gamma = P_2 V_2^\gamma \therefore \frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^\gamma$$

As gas is ideal gas $PV = nRT$

$$\therefore V = \frac{nRT}{P}$$

$$\frac{P_1}{P_2} = \left(\frac{\frac{nRT_2}{P_2}}{\frac{nRT_1}{P_1}} \right)^\gamma = \frac{nRT_2}{P_2} \times \frac{P_1}{nRT_1} = \frac{nRT_2 P_1}{nRT_1 P_2}$$

$$\frac{P_1}{P_2} = \left(\frac{T_2 P_1}{T_1 P_2} \right)^\gamma \quad \text{Taking logs gives:} \quad \gamma \log \left(\frac{T_2 P_1}{T_1 P_2} \right) = \log \left(\frac{P_1}{P_2} \right)$$

divide by γ and take inverse log of both sides give:

$$\frac{T_2 P_1}{T_1 P_2} = \text{Inverse log} \left(\frac{1}{\gamma} \log \left(\frac{P_1}{P_2} \right) \right)$$

where $T_1 = 300K$, $T_2 = ?$, $P_2 = 4.46 \text{ MPa to 2dp}$, $P_1 = 6.67 \text{ MPa to 2dp}$

$$T_2 = \frac{T_1 P_2 \text{ Inverse log} \left(\frac{1}{1.4} \log \left(\frac{P_1}{P_2} \right) \right)}{P_1}$$

$$= \frac{300 \times (4.46 \times 10^6) \text{ Inverse log} \left(\frac{1}{1.4} \log \left(\frac{6.67 \times 10^6}{4.46 \times 10^6} \right) \right)}{6.67 \times 10^6}$$

$$\underline{T_2 = 265.40K \text{ to 2dp}}$$

(2)(7)

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4

Q4.

- a) The first Law of Thermodynamics:
(1st LTD)

3

$$dQ = dU + dW$$

Heat energy into the System $\nearrow$

$\nearrow$ Increase in Internal energy

$\nwarrow$ Energy (Work done) By the gas
(-ve denotes work done on the gas.)

b)

As work is done on the gas the 1st LTD becomes

$$dQ = dU - dW$$

As the compression is adiabatic $dQ = 0$

$$\therefore dW = dU, \text{ where } dU = mc\Delta T$$

$$\therefore \text{ when } dW = 250 \text{ J, } m = 10 \text{ g} = 0.01 \text{ Kg and } c = 500 \text{ J K}^{-1}$$

$$\text{If } dW = mc\Delta T$$

$$\Delta T = \frac{dW}{mc} = \frac{250}{(0.01)(500)}$$

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$$\underline{\underline{\Delta T = 50 \text{ K temperature change}}}$$

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Q4

- c) As gas absorbs 15J of energy (ΔdQ) and does 7J of work (dW), The 1st LTD is as follows:

$$dQ = dU + dW$$

Which becomes:

$$15J = dU + 7J$$

$$\therefore dU = 15J - 7J$$

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Change in Internal Energy = 8J

d)

Where change in entropy (ΔS) of a reversible process is given by:

$$\Delta S = \frac{\Delta Q_{rev}}{T} \quad \text{and} \quad \Delta Q_{rev} = mc \Delta T$$

$$\Delta S = \frac{mc \Delta T}{T}$$

from $T_1 = 20^\circ C = 293.16K$
to $T_2 = 80^\circ C = 353.16K$

Becomes:

$$\Delta S = mc \int_{T_1}^{T_2} \frac{1}{T} dT$$

$$\Delta S = mc [\ln T]_{T_1}^{T_2}$$

$$\Delta S = mc [\ln T_2 - \ln T_1]$$

$$= mc \times \ln \left(\frac{T_2}{T_1} \right)$$

when $m = 7 \text{ kg}$, $C = (4.2 \times 10^3) \text{ J kg}^{-1} \text{ K}^{-1}$
 $T_2 = 353.16 \text{ K}$, $T_1 = 293.16 \text{ K}$

$$\Delta S = 7.0 \times (4.2 \times 10^3) \times \ln \left(\frac{353.16}{293.16} \right)$$

$$= 5474.358703 \text{ J K}^{-1}$$

change in.
entropy of $\Delta S = 5474.36 \text{ J K}^{-1}$

$$15/15$$

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