

International Journal of Mathematical Education in Science and Technology

ISSN: 0020-739X (Print) 1464-5211 (Online) Journal homepage: <http://www.tandfonline.com/loi/tmes20>

Using calculators for assessing pupils' conceptualization on place-value

Ioannis Papadopoulos

To cite this article: Ioannis Papadopoulos (2013) Using calculators for assessing pupils' conceptualization on place-value, International Journal of Mathematical Education in Science and Technology, 44:4, 523-544, DOI: [10.1080/0020739X.2012.756549](https://doi.org/10.1080/0020739X.2012.756549)

To link to this article: <https://doi.org/10.1080/0020739X.2012.756549>



Published online: 21 Jan 2013.



Submit your article to this journal [↗](#)



Article views: 475



View related articles [↗](#)

Using calculators for assessing pupils' conceptualization on place-value

Ioannis Papadopoulos*

School of Primary Education, Aristotle University of Thessaloniki, Thessaloniki, Greece

(Received 20 June 2012)

In this paper a two-stage research study is described focused on problem solving relevant to place-value and on the use of the operations within the calculator environment. The findings show that in this specific environment and via appropriate tasks teachers are provided with a context to better understand what year 5 or 6 pupils know or do not know about place-value and how they are able to apply their understanding of place-value and the use of operations in this particular context.

Keywords: calculator; place-value; misconceptions

1. Introduction

The usage of calculators in the classroom setting is not something new. Professional organizations such as the National Council of Teachers of Mathematics (NCTM) suggest the introduction of calculators into school mathematics even from the early years of schooling, arguing that 'when calculators are used effectively in the classroom, they can enhance pupils' understanding and use of numbers and operations' [1]. On the other hand, there exists a scepticism concerning, for example, the legitimacy and benefits of the calculator or the use of calculators as a procedural tool rather than as a learning aid [2]. Reviewing the relevant research literature it can be seen that calculator use is, to a great extent, considered a manipulative and motivational tool aiming to serve as a means of exploring numbers and tackling computations, which means developing children's concept of number sense. However, in this study, there is a shift in the way the calculator is used. Instead of as a teaching tool, the calculator is now adopted by the teacher as a way to uncover pupils' conceptual understanding with emphasis on misconceptions and errors related to place-value in primary school. Knowing the mathematical errors and misconceptions of young pupils can provide teachers with an insight into pupils' thinking. This allows teachers to adapt the focus of the teaching and learning process in the classroom. Therefore, it is important that calculators can be seen as tools to assist teachers in detecting such errors and misconceptions and at the same time to focus on the pupils' actual conceptions. The present study took place in two phases. In the first, considered a pilot study, a series of tasks concerning place-value in the context of calculator usage was presented to 3rd graders aiming to detect certain misconceptions, errors, and indications of limited or deeper understanding related to place-value. In the second phase, 5th and 6th graders were involved. Given that these misconceptions follow primary school pupils across the grades, the research question was whether the above mentioned technique is reliable and valid, and one that can be used

*Email: ypapadop@otenet.gr

successfully in different grades. Would it be possible to detect similar findings for older pupils using the same tool and the same sequence of tasks?

The paper is organized as follows. The next two sections will cover literature findings concerning the usage of calculators in mathematics classrooms and the issue of misconceptions related to place-value. Afterwards, a section discusses the setting of the study including a description of the tasks and some methodology issues. The findings of the study are presented in the next section and at the same time these findings are examined and commented in detail. The paper ends with some conclusions and implications for future research.

2. Calculators in primary school

Two major, comprehensive research projects in Britain and Australia highlight potential usage of calculators in the classroom: *The Calculator-Aware-Number (CAN)* and the *Calculators in Primary Mathematics*, respectively. A lot of information about the former can be found in the work of Kenneth Ruthven [3–6] whereas for the latter the proper source is the work of Susie Groves [7–10].

For the CAN project a research team worked with teachers in a number of schools for three years to develop an approach for teaching and learning numbers in primary school. Among the principles that governed this effort, were:

- (1) The availability of calculators letting the pupils to decide when to use them.
- (2) Avoiding teaching the traditional column methods of calculation since the calculators would be used for computations which could not be done mentally.

Ruthven [3] examined the use of mental, written and calculator strategies of numerical computation by 6th grade pupils. He found that the CAN project employed in one group encouraged pupils to develop informal methods of calculation and to use calculators to explore numbers and execute demanding computations. In another work, Ruthven, Rousham and Chaplin [4] examined the long-term influence of the CAN project on the mathematical attainment of pupils (10–11 year-old) in primary school. They used as a starting point the claim that the early exposure to a wide range of numbers and number types, which is characteristic of the CAN approach to number, contributes to the development of place-value concepts. Thus, five variables were formed from blocks of test items: symbolizing numbers, ordering numbers, rounding numbers, extending patterns and formulating patterns. They concluded that this early exposure to a CAN approach made no longstanding contribution to the achievement of place-value concepts. Additionally, Ruthven and Chaplin [5] examined the idea of calculator as a cognitive tool analyzing how a sample of primary education pupils with differing degrees of experience of a CAN curriculum, tackled a realistic number problem. What they highlighted was the importance of developing pupils' skills in making effective use of a calculator beyond merely making simple computations. They also highlighted the need to help pupils grasp the relationship between mathematical concepts and their operationalization in the machine. It seems also that there is a consistent trend towards greater use of calculators by lower-attaining pupils, especially among the males. However, in one of his current papers, Ruthven [6] reviewed the effort for crafting a CAN curriculum and examined pedagogical strategies based on Diagnose-Explain-Reinforce and Observe-Predict-Surpass sequences, as also the wider use of calculators for purposes of Computation-Implementing, Result-Checking, Trial Improving and Structure-Modelling. He mentioned that the way in which the calculator is used to structure a task is crucial. More

specifically the task must be structured in a way that could help the teacher to diagnose the extent to which pupils have grasped the underlying mathematical concept. Therefore, further similar tasks can be used to make clear where further teaching is needed. This exact exemplification of the Diagnose part of calculator use became the starting point for approaching the calculator as a context to understand what pupils know or do not know and how they are able to use it.

The Australian *Calculators in Primary Mathematics* project was a long-term investigation into the effects of the introduction of calculators on the learning and teaching of primary mathematics. The project focused on pupils from kindergarten up to the 4th grade. The findings showed that four major ways of using the calculators emerged: as a recording device, as a counting device, as a computational tool and as an object to explore [7–9]. The use of calculators provided a rich mathematical environment for children to explore and promoted the development of number sense by removing previous restrictions on the types of numbers children use. More specifically, analyzing the interviews of grade 3 and 4 pupils, they found that children with long-term experience of calculators performed better than children without such experience, exhibited better knowledge of number, particularly place-value, made more appropriate choices of calculating device and were better able to interpret their answers when using calculators. Some years later, the teachers in whose classrooms the above mentioned project took place, participated in another study focused not only on the effect of calculator use on children's learning of mathematics, but also on their teachers' classroom practice. The teachers identified many changes in their mathematics teaching – common changes included mathematics teaching becoming more 'open-ended'; increased knowledge of children's conceptual development; more discussion and sharing of mathematics; and more use of inquiry and problem solving [10].

It should be noted that there are two important differences between Ruthven's research and the research of Groves and her colleagues. First, the participating pupils in the former study were older and many of them had more longstanding experience with calculators. Second and perhaps more important, the pupils in the British project had gained that experience under rather more typical conditions than those of a special development project.

Even though we have referred to these two mainstays in the field, this does not mean that they are the only studies dealing with the usage of calculators. Additional studies showed that appropriate usage of calculators: (i) improves pupils' computational skills [11] and influences positively their mathematical achievement [12]; (ii) helps pupils by stimulating their problem solving thinking [13]; (iii) enables pupils to make conjectures and generalizations in relevance to numbers and operations [14]; (iv) develops understanding of the structure of the computational algorithms that are most often taught and learned by rote with little conceptual understanding [15]; (v) facilitates making inverse operations for subtraction and division [16]; (vi) facilitates teachers' effort to help pupils understand mathematics [17]; and vii) tackling realistic problems with a calculator leads to children encountering, and having to make sense of, negative and decimal numbers much earlier than envisaged in the established curriculum sequence [18].

Obviously, there is also some scepticism about the use of calculators. Mackey [19] suggests that there is an astonishing switch from solving simple problems (i.e., *methods*) to caring only about answers (i.e., things that appear in the display of a calculator). Klein and Milgram [20] suggest that calculator use led mathematics to neglect the contribution of written algorithms in developing mathematical concepts. Fisher and Stephens [21] found that the presence of calculators, even with guided instruction, may not alone be the driving force needed to significantly improve the achievement of pupils far below average. Such raise the question: When should calculators be used? Goldenberg [22] presents some

principles for thinking about technology use including among others the *Purpose Principle* and the *Fluent Tool Use Principle*. The first suggest allowing calculator use when computational labour can get in the way of the lesson. So, when the aim of the lesson is to teach how to perform the computation, then the use of calculators is a rather bad idea. The second principle suggests that ‘touching’ several calculator tools without really mastering them may lead to do more harm than good. The reservations mentioned above frame a widespread view that calculators have to be used only after the pupils have learned how to do the relevant mathematics without them. Otherwise, pupils would become dependent on them as a crutch instead of as a valuable tool [23].

Despite the great number of studies that deal with the usage of calculators in primary school mathematics, the studies focused on the usage of broken calculators are rather rare. Leatham, Lawrence and Mewborne [24] claim that Broken Calculator problems can be seen as open-ended since there are multiple (or alternative) problem-solving strategies (see also [25]) and multiple correct answers and they allow pupils to demonstrate their sense of operations as well as their ability to choose operations that will accomplish given mathematical objectives. Disabling keys can be used in order to alter the level of difficulty and to assess understanding of mathematical concepts.

3. The issue of place-value and the relevant misconceptions and errors

An essential component of teachers’ competence in mathematics teaching is their knowledge about pupils’ misconceptions. Therefore, an understanding of most pupils misconceptions and effective strategies is important so as to help pupils avoiding them. This means that teachers try to teach in a way that helps pupils avoid misconceptions and at the same time they have different approaches to deal with those misconceptions that do arise. Once a misconception is detected then it is up to the teacher to decide upon the proper strategy to be used. This could mean re-teaching the concept and in that case the teacher must decide which part of the concept needs more attention. But this could also mean causing a cognitive conflict, thus forcing the pupil into a situation that is in contrast with his/her initial understanding hoping that he/she will re-evaluate his/her beliefs [16].

Research studies and experience with young pupils reveal that many of them have difficulties in learning and applying place-value in mathematics.

Ross [26] describes four properties of place-value understanding:

- Additive property. The whole number represents a quantity which is the sum of the values represented by the individual digits.
- Positional properties. The position of each digit in the whole numeral determines the quantity represented by it.
- Base-ten property. As we move from right to left the values of the positions increase in powers of ten.
- Multiplicative property. For finding the value of an individual digit one has to multiply the face value of the digit by the value assigned to its position.

There is a plethora of evidence that pupils face great difficulties in correctly understanding place-value, especially in early grades [27]. As measured by digit-correspondence tasks (known as ‘face value’ tasks), even in the fourth and fifth grades no more than half the pupils demonstrate an understanding that the ‘5’ in ‘25’ represents five of the objects and the ‘2’ the remaining 20 objects. In the same spirit Kamii [28] found that the proportion of children who say that the ‘1’ in ‘16’ means ten is generally 0% at the end of 1st grade, 33%

at the end of 3rd grade and 50% at the end of 4th grade. Fuson [29] describing the difficulties children have in relation to place-value, refers to *sequence-multiunit conceptual structures*. These structures require sequence pattern skills of skip-counting by tens, hundreds and thousands. Sequence pattern skills can be learned by rote just based on the sound patterns in the sequence. Thus, a pupil can learn the decade word list (ten, twenty, thirty, forty, etc.) as a memorized list without any understanding that the words refer to one ten, two tens, three tens and so on, or that each word is ten more than the previous quantity. In the same paper, Fuson [29] mentions the *concatenated single-digit conceptual structure*. In this case pupils regard multidigit number marks as though they are concatenated single digit numbers placed side by side. Thus, the digit 4 represents the same quantity whether it is in the ones, tens, or hundreds place (314, 341 and 431 respectively). Bednarz and Janvier [30] found that many children *don't see the usefulness of grouping*. Some of them form groupings but only in order to count a collection (by ten, by five, . . .). Few children see the validity of doing and undoing groupings (partitioning) to solve multidigit number problems. Another difficulty connected to place-value is the case of seeing multidigit numbers as unitary collections [31]. It is a way of global correspondence between multidigits and numbers. Multidigit numbers are to be 'read together' and correspond to some particular quantity. For example, 15 is to be read together as a collection of 'fifteen' units. Another topic that is difficult for many elementary school children is arithmetic involving zero. It is known that pupils' view of zero may contain important misconceptions such as 'zero is not a number' or 'zero is nothing'. The fact is that similar misconceptions can be found not only among pupils but among teachers also. Wheeler and Feghali [32] studying the case of zero as 'place holder' focused on preservice teachers' understanding of zero. Results showed that teachers had an inadequate knowledge about zero. They were reluctant to accept zero as an attribute for classification and they were confused as to whether or not zero is a number.

There are other studies dealing with place-value, but presenting all of them in detail is beyond the aim of the paper. However, we can mention some additional misconceptions and errors connected to place-value: There are *counting errors* such as errors in naming the next multiple of ten in a counting sequence (10, 20, 30, 50, 60, . . .) and mistakes in the new number at a change of decade (63, 62, 61, 50, . . .). There are errors in naming and writing symbols for numbers and errors in applying values to digits such as reversing tens and ones.

4. Setting of the study: description and methodology

The study aimed to answer the question: Can the broken calculator be used as a valid and reliable way to help teachers to uncover their pupils' conceptual understanding of place-value, ranging from its problematic acquisition to its deep understanding?

For the purpose of the study the usage of web-based 'broken' calculator problems was considered the most appropriate approach. Judah Schwartz initially published the Broken Calculator software in 1989 under the title 'What Do You Do with a Broken Calculator'. (A web-based Flash version can be found at http://seeingmath.concord.org/broken_calculator/; see Figure 1). Broken calculator problems are essentially puzzles in which the pupils must figure out how to perform certain tasks on a calculator that has one or more non-working keys. Such activities can be considered open-ended since there are multiple solution strategies and multiple correct answers. In this sense such activities encourage pupils to demonstrate their understanding (or lack of understanding) in informative ways. For the teacher, this means that these incorrect solutions will demonstrate pupils' sense of

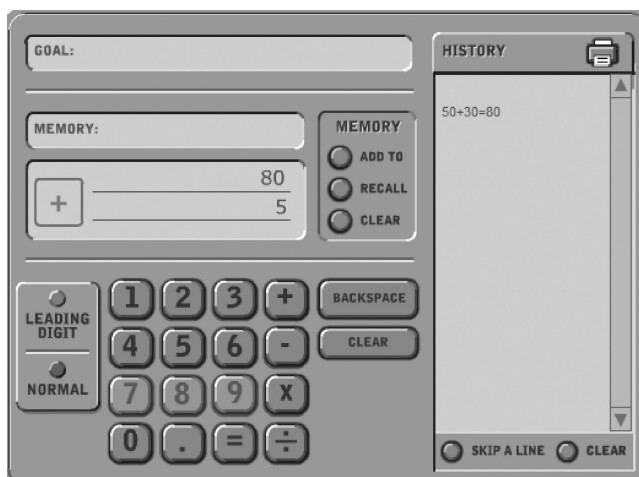


Figure 1. Web-based broken calculator.

number and computational abilities as well as their understanding of place-value and their sense of operations [24]. According to Goldenberg [22] such activities ‘*help pupils focus on and analyze the structure and elements of arithmetic and gain skills along with understanding, rather than have the calculator replace their thinking*’. Schwartz [15] explains that the partial availability of the keys forces pupils to devise alternative ways of doing what the missing tools would ordinarily do. This serves as a way for accessing the pupils’ mathematical understanding.

The study took place in a primary public school in Thessaloniki, Greece during two consecutive years. This suggests a two-stage program. The first stage was in essence a pilot study and the participants were 19 pupils attending the third grade (9 year-old). This aimed to evaluate the effectiveness of the usage of calculator as a way for detecting misconceptions and errors related to place-value. The tasks were posed to them after they completed the unit about the 3-digit numbers, which means that they had spent time becoming familiar with these numbers and on using operations and solving problems using these numbers. More specifically, according to the official curriculum after finishing this unit the pupils must be able: to read and write 3-digit numbers, to move from oral to written form of numbers and vice versa, to distinguish units, tens and hundreds and their place-value, to easily apply numbers in everyday situations and finally to write numbers in their extended form. The results of this pilot study were considered a first step towards considering the calculator as a way to see pupils’ conceptualization of place-value and they were presented to the CME-10 conference in Poland [33]. In the next year the study was broadened so as to include pupils from the 5th and 6th grades (11–12 year-old) in the same school. The aim of this stage was to investigate the consistency and reliability of this approach. Given the prevalence and persistence of misconceptions by age, it was important to verify that the calculator is still a valid and effective vehicle that allows primary school teachers to detect pupils’ misconceptions and errors about place-value. Therefore, in this second stage 42 pupils of the 5th grade (two classes) and 45 pupils from the 6th grade (two classes) participated. The tasks were presented to the pupils at a constant rate of one task per week for each class. Pupils were familiar with the use of the calculator as a means for carrying out long and complicated operations since there are such tasks included in their textbooks.

Task-1

Press the keys to get the number 432. Add or subtract any number so that 3 is replaced by 0.

Task-2

On your calculator the only keys working are 0, 1, and the keys of the four operations. Make any calculation you want to get the number 5000.

Task-3

On your calculator the keys 7, 8 and 9 are broken. Make any calculation you want to get the number 78. Do the same for seven hundred seventy seven.

Task-4

Keys 9 and + are broken. Explain how to get 90. Do the same for 109. Do the same for 99.

Task-5

On your calculator the key 8 is broken. Explain how you can make the subtraction $800-130$.

Task-6

The only keys working on your calculator are 1, 0, +, -. Make any calculation you want to get the number 345.

Task-7

The only keys working on your calculator are 2, 5, +, -. Make any calculation you want to get the number 205.

Figure 2. The broken calculator tasks.

Thus, they knew how the calculator buttons operate when all the buttons are available. However, the idea of the broken calculator was completely new and unknown.

The tasks are presented in Figure 2.

Pupils were first asked to construct on a piece of paper the keystroke sequence to accomplish the tasks before using the original keys of the calculator in order to have evidence of the way they are thinking about place-value. Thinking in terms of pressing calculator keys is important since – as will be shown later – this would serve for detecting certain misconceptions, difficulties and errors related to place-value. At the same time, this demand for thinking in terms of pressing keys is in itself a difficult task since it is in essence a demand for a change in register. It is a kind of different language for expressing their own strategy in order to obtain the asked number. This was the first part of the activity before working in the actual environment of Schwartz's web-based broken calculator. In this second part the pupils were asked to apply their written strategy to the web-based calculator environment. The first part was necessary for detecting certain misconceptions, errors or a limited-deeper understanding. The second one aimed to lead pupils to a cognitive conflict since the application of their strategy did not result in the desired number. This second part is not our focus since our main aim is to highlight the role of the calculator as a way to uncover pupils' conceptual understanding.

The tasks were carefully designed to reveal the above mentioned errors, misconceptions and level of understanding as will be shown later.

Task-1

The requirement that the number 3 be replaced by zero means that one has to subtract 30, 31, 32, add 68 (e.g., the complement of 32 in relation to 100) or add 70–77. However,

knowing that many pupils consider tens digits to represent single units and not multiples of 10, one might expect to see mistaken actions towards this goal (*face value constructs*) such as subtracting 3 or adding 7.

Task-2

The critical element of this task is that the target number includes three zeros. This is connected to the known issue of '*zero as placeholder*'. These zeros are obtained through certain arithmetical operations. However, it seems that very often pupils put down the zeros following a pattern. For them, zero means nothing and the way of writing the wanted number is rather a memorized rule (e.g., zero has to be there, thousands mean three digits, etc) rather than a real understanding of zero in place-value numeration. This is why their effort is limited to getting the non-zero digits of the number.

Task-3

The required number is 78, given that the 7 and 8 keys are broken. Disabling these keys could reveal an effort to separately obtain these two numbers in the calculator environment, thus indicating that (for some pupils) tens digits represent single units and not multiples of 10 ones (*face value construct*). Additionally, giving the second number of the task as a number-word involving in various ways the number 7 would possibly reveal instances of mistaken number representations. There could be a weak association between written symbols and number words. Finally, supposing that pupils will mainly use subtraction to obtain the wanted numbers it could be possible to track cases of '*counting sequence errors*' (e.g., errors at a change of decades or hundreds).

Task-4

The aim of the task was to support evidence obtained from previous tasks since it deals with the same place-value difficulties. On the one hand the usage of number 9 highlights possible errors on change of decades or hundreds (*counting sequence errors*). On the other hand the presence of zero in the number 90 poses the question of *zero as placeholder*.

Task-5

This task aims to give extra evidence to the issue of *zero as placeholder* in combination with the case of *face value construct* (e.g., first obtaining the number 8 and then writing down 2 zeros next to it).

Task-6

The required number is 345 given that 3, 4, and 5 keys are broken. In this case the aim was to see whether the *face value construct* has a global effect (e.g., whether it applies to tens as well as to hundreds).

Task-7

This task could provoke both the *face value construct* and *zero as placeholder* errors. Moreover, this task has a higher level of difficulty since the handling of the available combinations of the allowed keys demanded a higher level of skills. Thus, it was expected to require and reveal a deeper understanding of place-value.

Ending this section it should be noted that each of the tasks had the potential to reveal limited or advanced understanding of place-value as will be discussed in the next two sections. This is connected to the fact that critical keys were broken and subsequently the pupils had to exploit their knowledge of place-value to obtain alternative approaches.

The pupils' worksheets were collected and their responses were classified according to two criteria: a) whether a specific misconception or error was underlying it and b) whether a correct answer could be considered an indication of either a surface or deeper understanding of place-value.

Table 1. Instances of 'zero' difficulty.

Task-2	$\boxed{1} + \boxed{1} + \boxed{1} + \boxed{1} + \boxed{1} = \boxed{5} \boxed{0} \boxed{0} \boxed{0}$ (obviously pressing the zero key would give 0 instead of 5000) Gr.3 (3/19), Gr.5 (3/41), Gr.6 (5/45)
Task-4	$\boxed{6} + \boxed{3} = \boxed{9} \boxed{0}$ Gr.6 (1/45)
Task-5	$\boxed{4} + \boxed{4} = \boxed{8} \boxed{0} \boxed{0}$ (obviously 0 instead of 800) Gr.3 (1/19), Gr.5(1/41) $\boxed{2} * \boxed{4} = \boxed{8} \boxed{0} \boxed{0}$ (obviously 0 instead of 800) Gr.6 (1/43)

5. Results

The analysis of the data collected allowed us to distinguish three types of answers. The first type includes answers showing certain misconceptions and errors related to place-value. The second type includes answers showing limited understanding of place-value, while the third type includes answers indicative of a deeper understanding of place-value.

5.1. Type 1. Misconceptions and errors relevant to place-value

5.1.1. Zero is not place holder

As a digit, zero operates at three levels: i) to indicate there are zero components of a given place-value; ii) as a place holder to show the value of surrounding digits; and iii) to indicate the accuracy of measurement [34]. The use of zero as a placeholder is a source of difficulties for young pupils. It appears to confuse them [35]. Part of this difficulty perhaps has been caused by considering zero as nothing and this difficulty afflicts both pupils and teachers equally. For the former, Horne and Livy [36], working on a project focussed on numeracy in the early years of schooling, found that numbers including the digit zero proved the most difficult. For the latter, Wheeler and Feghali [32] as well as Chick and Baker [16] found that teachers also have inadequate knowledge of zero.

Task-2 could reveal the handling of numbers that include zero as 'following a pattern'. Actually many pupils use digits mechanically and with some degree of automaticity. For example, in order to make the sequence 100, 200, 300, . . . one has just to follow the pattern: *Increase the first digit by one and then just place two zeros*. In the specific task, 5000 was the desired number. So, according to the above mentioned pattern the pupils just had to get the number 5 and then write down three zeros. This could be done easily working on paper. It has been noticed that pupils often write the numbers 1, 2, 3, etc, down in a column and then just put the required number of zeros next to each of these numbers. So, the lack of knowledge concerning the arithmetic operations that result in 5000 would go unnoticed. But the situation is different when thinking in terms of pressing calculator keys. In this case the pupils had to reveal the way they think so as to achieve the three zeros and subsequently the specific number. In Table 1, the instances of the pupils' work that indicate this specific misconception are summarized.

It is clear that in Task-2 the main effort of the pupils was to achieve the number 5. In Task-4, one of the three desired numbers was 90. There was one pupil in Grade-6 who followed a similar process in trying to obtain the number 9. Finally, the same line of thought was found in Task-5. The pupils were asked to make the subtraction 800-130. Thus, they first had to somehow get the number 800 since the $\boxed{8}$ key was not available. Hence, they focused on how to alternatively obtain the number 8, either by addition or multiplication, thinking that it would then be easy to get 800 by simply pressing the zero key twice.

Table 2. Instances of face value construct.

Task-1	432 − 3 = (429 instead of 402) Gr.3 (3/19), Gr.5 (2/41), Gr.6 (2/45)
Task-3	1 + 1 + 1 + 1 + 1 + 1 + 1 = 7 4 + 4 = 8 Gr.5 (1/41)
Task-6	1 + 1 + 1 = 3 1 + 1 + 1 + 1 = 4 1 + 1 + 1 + 1 = 5 Gr.3 (1/19), Gr.5 (1/41)
Task-7	2 − 2 = 0 Gr.3 (1/19)

5.1.2. Face value construct

Acquiring correct place-value means that the pupils understand the value represented by each digit of a multidigit number. Therefore, the value represented by a digit is the product of the ‘face value’ [31] of the digit and the value associated with the place of the digit in relation to the point of reference (i.e., the ones places). However, for pupils, very often the tens digits represent single units and not multiples of 10 ones [37]. This is described in the work of Ross [38] as the *face value stage* of place-value understanding and signifies a critical misconception of multidigit numbers that may be difficult to detect. Fuson and Briars [39] refer to this construct as *concatenated single digits*. So, the question is whether children understand that in the number 25, the number 2 represents twenty objects rather than 2 (thus representing a two digit number as two sets of units). We will present examples from four different tasks to show how the pupils use the number’s digits as single units (Table 2).

In Task-1 pupils had to replace the number 3 (tens) in 432 by 0. The process they proposed makes it obvious that the number 3 was considered as representing units and this is why the pupils decided to subtract 3 instead of 30. This is an indication of ignorance of the arithmetic relationship that exists between the digits of a number. In a similar way, in Task-3, some pupils trying to get the number 78 worked separately for the digits 7 and 8 assuming that these numbers are not connected through a very concrete arithmetic relationship. On the contrary it seemed that these numbers were considered as just two independent ones that are placed one next to the other to form a new number. Slightly different is the case of Task-7. It involves a 3-digit number and all keys are available but one. The number 205 requires the presence of zero but the relevant key is broken. Thus, the pupils thought that the only thing they needed to do was to find a way to get zero since the

2

 key is available. Then, according to them it would be easy to get 205 by having

2

0

5

. In another case the rationale was to first get the 200 and then by adding 5 to get 205. So, the idea was to follow the previous process for getting zero, then to repeat it (to get the two zeros) and then it would be easy to take 200 by putting the digits

2

0

0

 one next to the other.

5.1.3. Reversing values of tens and ones

In this case the digits are reversed when writing numbers. This is similar to what Steinle and Stacey [40] call *reverse thinking*, a misconception found rarely within their research. Pupils with this misconception regard numbers such as 52 and 25 as identical. Ultimately, understanding these two numbers as identical means that pupils do not take place-value of the digits into consideration. Indeed, we found only one instance of this misconception concerning a pupil in Grade 3 and in Task-3. The pupils were asked to get the number 78

Table 3. Instances of counting sequence errors.

Task-3	$\boxed{800} - \boxed{3} = 777$ (instead of 797) Gr.6 (1/43)
	$\boxed{1000} - \boxed{333} = 777$ (instead of 667) Gr.5 (1/42), Gr.6 (1/43)
	$\boxed{700} - \boxed{33} = 777$ (instead of 667) Gr.6 (1/43)
Task-4	$\boxed{80} - \boxed{1} = 99$ (instead of 79) Gr.3 (1/19)
	$\boxed{200} - \boxed{1} = 109$ (instead of 199) Gr.3 (1/19)
	$\boxed{200} - \boxed{90} = 109$ (instead of 110) Gr.3 (1/19)
	$\boxed{200} - \boxed{110} = 109$ (instead of 90) Gr.5 (1/42)
	$\boxed{200} - \boxed{189} = 109$ (instead of 11) Gr.6 (1/43)

given that the keys $\boxed{7}$, $\boxed{8}$ and $\boxed{9}$ were broken. This is the process proposed by the pupil:

$$\boxed{50} + \boxed{30} = 80 + \boxed{5} + \boxed{2} = 87 \text{ (instead of 78)}$$

According to this type of misconception the desired number 78 is considered as identical to 87. Digits' places are considered independently without regard for values of places relative to each other.

5.1.4. Counting sequence errors

Counting sequence errors occur when pupils use counting approaches to work out answers for questions involving multidigit numbers. Four types of such errors have been recorded by Price [41] as the most common ones: (i) Mistakes in the new number at a change of tens place or change in the number of hundreds. The problem is for example on naming the next or the previous decade in a counting sequence (e.g., 60, 71, 72, ... or 63, 62, 61, 50, 49, 48, ...), (ii) Omitting numbers (e.g., 31, 41, 61, 71, ...), (iii) Using an incorrect increment or decrement when asked to count forward or backward by 10 (e.g., 452, 562, 672, 892) and (iv) Omitting the ones part of each number (e.g., asking to count by 10 from 263: 270, 280, 290, ...). In Table 3, some typical examples that can be considered counting sequence errors are presented.

5.1.5. Mistaken number representations (that reflect the base-10 numeration system)

To be successful in understanding place-value, it is necessary for the pupils to possess certain mental skills such as the correct recalling of the numbers' names. Good number sense is important for the pupils to grasp the meaning of mathematical questions and for working out potential answers. There exists an association between the written symbols and the number words which are alternative representations of numerical quantities. By the age of 10 most children have mastered both notations for numbers: verbal numerals such as *seven hundred seventy seven* and Arabic numerals such as 777. Pupils also have been taught at this age to translate numbers from the one form to the other (many researchers call this *transcoding*). Studies by Power and Martello [42] and Seron and Fayol [43] showed that even from early grades pupils generally make no errors in transcoding number words into single or two-digit numerals and only when dealing with more complex numbers such as 3-digit ones errors may appear more often. However, there is also evidence that transcoding between number words and Arabic numerals causes problems to pupils especially when different arithmetical abilities exist in the classroom [44]. So, it is a very challenging task to

Table 4. Mistaken number representations in Task-3.

Task-3

600	+	103	+	4	=	707	Gr.3 (1/19)												
100	+	100	+	100	+	100	+	100	+	100	+	100	+	4	+	3	=		
707	Gr.3 (1/19)																		
600	+	110	+	60	=	770	Gr.3 (1/19)												
500	+	200	+	50	+	20	=	770	Gr.5 (1/42)										
300	+	300	+	100	+	50	+	20	=	770	Gr.5 (1/42)								
600	+	100	+	66	+	4	=	770	Gr.6 (1/43)										
2	+	2	+	2	+	1	*	100	=	700	Gr.3 (1/19)								
40	+	30	+	1	+	1	+	1	+	1	+	1	+	1	+	1	=	77	Gr.5 (1/43)
35	+	42	=	77	Gr.6 (1/43)														
10	+	10	+	10	+	10	+	10	+	10	+	10	=	70	Gr.6 (1/43)				
100	+	50	+	20	=	170	Gr.5 (1/42)												

help pupils to reach the stage of learning numbers and their names. The way pupils respond to specific tasks can indicate a limited possession of the above mentioned association.

In Task-3 the pupils were asked to get the number 777. However, for some of them getting 777 was the same as getting 707 or 770 or 700 or 77 or 170. It is interesting that even though these kinds of errors could be expected for younger learners, there were also similar instances in Grades 5 and 6 (see Table 4).

5.2. Type 2. Limited understanding of place-value – Unitary concept of multidigit numbers

The ‘unitary construct’ is considered one of the first steps for understanding the base-ten numeration system. Unitary construct refers to recognizing from early schooling the symbols for the numbers 1 to 9 and associating each one of these symbols with one number which can represent the total number of objects in a group. This construct is an important step for younger children but sometimes can lead to a limited conception especially for older pupils that multidigit numbers are collections of single units. Thus, for example, pupils see 25 not as 2 tens and 5 ones but rather as 25 ones. Researchers refer to this situation with various names. For example, Ross [38] refers to it as *whole numeral stage* and Fuson [29] as *collected multiunits*.

Some examples of our pupils’ work are presented later showing limited understanding of place-value. The way the pupils worked leads to the correct result. However, the path they followed indicates that they did not understand place-value in depth (Table 5).

In Task-6 the pupils wanted to get the number 345. Among the responses there were instances that show limited understanding of place-value (Table 5). The first instance for Task-6 is a pure one of the unitary concept for the number 345. The pupil did not see the 345 as 3 hundreds 4 tens and 5 ones but as 345 ones (Figure 3). However, it is worth paying attention to the two other instances. The pupils’ work is in the same spirit. The difference is that in these examples the pupils worked at two levels. The first one is based on collections of units (10s and 100s) rather than on single units whereas in the second there is a clear presence of the unitary concept for the remaining 45. Similar approaches were also detected in other tasks. In Task-2, a Grade-3 pupil got 5000 by adding consecutive 100s (collections of 100s instead of 5000 units). In Task-3 the desired numbers were 78

Table 5. Examples showing unitary concept of multidigit numbers.

Task-6	$\boxed{1} + \boxed{1} + \boxed{1} + \boxed{1} + \boxed{1} + \dots + \boxed{1} = \mathbf{345}$ Gr.6 (1/42) $\boxed{10} + \boxed{10} + \dots + \boxed{10} + \boxed{1} + \boxed{1} + \dots + \boxed{1} =$ (the tens for getting 300 and the ones for getting 45) Gr.5 (1/41), Gr.6 (1/42) $\boxed{100} + \boxed{100} + \boxed{100} + \boxed{1} + \boxed{1} + \dots + \boxed{1} =$ Gr.3 (1/19), Gr.6 (1/42)
Task-3	$\boxed{10} + \boxed{10} + \dots + \boxed{10} + \boxed{1} + \boxed{1} + \dots + \boxed{1} = \mathbf{78}$ (the tens for getting 70 and the ones for getting 8) Gr.5 (1/41), Gr.6 (5/43) $\boxed{100} + \boxed{100} + \dots + \boxed{100} + \boxed{10} + \boxed{10} + \dots + \boxed{10} + \boxed{1}$ $+ \boxed{1} + \dots + \boxed{1} = \mathbf{777}$ (the hundreds for getting 700, the tens for 70 and the ones for 7) Gr.6 (6/43)
Task-7	$\boxed{5} + \boxed{5} + \boxed{5} + \dots + \boxed{5} + \boxed{5} = \mathbf{205}$ Gr.3 (1/19) $\boxed{255} - \boxed{5} - \boxed{5} - \dots - \boxed{5} = \mathbf{205}$ Gr.5 (4/40), Gr.6 (2/42)

and 777. It is interesting that the number of instances that show limited understanding of place-value was significantly increased in the upper grades. Both examples for Task-3 are based on collections of units. For 78 the main process is characterized by groups of 10s whereas for 777 by groups of 100s (for 700) and 10s (for 70). This process was also found not only in consecutive additions but in subtractions as well. In Task-7 the desired number was 205. Various approaches were recorded for obtaining this number. Two of them fit this section. In the first instance 205 is seen as collection of 5s, whereas in the second, 205 is obtained by subtracting consecutive 5s from 255.

5.3. Type 3. Answers showing understanding of place-value

Usually, the aim of a diagnostic tool in mathematics education is to find evidence for pupils' difficulties concerning a particular topic. However, it is important that the efficiency of a diagnostic tool ranges from the problematic acquisition of a concept to the deep understanding and exploitation of the taught knowledge. In this section we present some of the pupils' responses that show a refined understanding of place-value (Table 6).

In Task-1 all of the correct answers for the pilot phase (Grade-3) and for the majority of the pupils in 5th and 6th grades were based on subtracting 30 from 432. There were also some answers based on subtracting numbers other than 30, which indicate a good number sense. However, what is interesting is that in the pilot phase (3rd grade) none of the pupils



Figure 3. Unitary concept of multidigit numbers.

Table 6. Instances showing understanding of place-value.

Task-1	432 − 29 = Gr.5 (1/41)
	432 − 331 = Gr.6 (1/45)
	432 + 68 = Gr.5 (1/41), Gr.6 (2/45)
Task-3	100 − 22 = Gr.5 (2/42)
	3 * 25 + 3 = Gr.5 (1/42)
	1000 − 223 = Gr.5 (2/42)
Task-4	442 + 335 = Gr.6 (1/43)
	180 / 2 = 90 Gr.5 (1/42), Gr.6 (2/43)
	18 * 5 = 90 Gr.3 (1/19), Gr.6 (1/43)
	45 * 2 = 90 Gr.6 (2/43)
	3 * 33 = 99 Gr.3 (1/19)
	4.5 * 2 + 100 = 109 Gr.5 (1/42)
	54.5 * 2 = 109 Gr.6 (1/43)
	218 / 2 = 109 Gr.6 (2/43)
	113 − 4 = 109 Gr.6 (1/43)
	120 − 11 = 109 Gr.6 (1/43)
Task-7	255 − 55 + 5 = Gr.5 (1/40), Gr.6 (4/42)
	222 − 5 − 5 − 5 − 2 = Gr.5 (2/40)
	222 − 22 + 5 = Gr.5 (1/40)
	52 + 52 + 52 + 52 + 2 − 5 = Gr.3 (1/19), Gr.6 (2/42)
	225 − 25 + 5 = Gr.3 (1/19)
	255 − 25 − 25 = Gr.3 (1/19)

used addition (e.g., adding 70) even though the [+] key was available. In the second phase there were only 3 pupils who decided to use addition as a way to obtain the desired zero.

For Task-3 the most common answers were based on the idea $60 + 10 + 4 + 4 =$ (or other quite similar answers) for getting 78 and on the idea $600 + 100 + 60 + 10 + 6 + 1 =$ (or again other similar answers) for getting 777. However, a few answers were recorded that indicated a higher level of thinking, as can be seen in Table 6. These answers reached the desired number sooner, using subtractions or additions of higher level of difficulty and in some cases used a combination of operations (multiplication and addition) which implied knowledge of the order of operations.

In Task-4 the most popular approach for the whole population was $100 - 10 =$ (for 90), $100 - 1 =$ (for 99), and $110 - 1 =$ (for 109). It is obvious that these approaches share two main elements: the usage of 100 and the subtraction. The approaches that were considered as advanced ones involved other operations than subtraction. But the most important is that in this task we found examples of pupils using decimal numbers.

In Task-6 the pupils had to get the number 345. The most common answer was: $100 + 100 + 100 + 10 + 10 + 10 + 10 + 1 + 1 + 1 + 1 + 1$. On the other hand, the most impressive answer was given by one 5th grade pupil and three 6th grade pupils:

111

+

111

+

111

+

11

+

1

=

Table 7. Instances showing parallel processes.

Task-3		Core idea	
78	777		
35 + 35 + 4 + 4	350 + 350 + 35 + 35 + 4 + 3	$3.5 + 3.5 = 7$	
50 + 20 + 5 + 3	500 + 200 + 50 + 20 + 5 + 2	$5 + 2 = 7$	
11 * 6 + 11 + 1	111 * 6 + 111		
60 + 10 + 5 + 3	600 + 100 + 60 + 10 + 5 + 2	$6 + 1 = 7$	
30 + 40 + 4 + 4	300 + 400 + 30 + 40 + 4 + 3	$3 + 4 = 7$	
66 + 11 + 1	666 + 111		
60 + 16 + 2	600 + 160 + 6 + 1		
100 - 22	1000 - 223		
50 + 24 + 4	500 + 240 + 30 + 3 + 4		
30 + 30 + 10 + 6	300 + 300 + 100 + 60 + 10 + 1	$3 + 3 + 1 = 7$	
+ 2	6 + 1		

Table 8. Quantitative data concerning misconceptions/errors.

	Grade-3	Grade-5	Grade-6
<i>Instances of 'zero' difficulty (Tasks 2, 4, 5)</i>	6/57 (10.52%)	4/123 (3.25%)	7/129 (5.42%)
<i>Face-value construct (Tasks 1, 3, 6, 7)</i>	5/76 (6.57%)	4/164 (2.43%)	2/172 (1.16%)
<i>Counting sequence errors (Tasks 3, 4)</i>	3/38 (7.89%)	2/82 (2.43%)	4/86 (4.65%)
<i>Mistaken number representations (Task 3)</i>	4/19 (21.05%)	4/41 (9.75%)	3/43 (6.97%)

In Task-7 the pupils had to get the number 205. This task was of higher level of difficulty since the handling of the available combinations of the allowed keys demanded a higher level of skills in order to get 205. Additionally, it was not obvious how to obtain a 3-digit number such as 205. However, again there were many different answers revealing a broad understanding of place-value.

Finally, the analysis of the collected data particularly for Task-3 had as a consequence a remarkable pattern concerning the line of thought followed by the same pupil so as to get the two required numbers 78 and 777. There was an analogy in the processes for getting these numbers as the examples in Table 7 show.

For some of the cases it is clear that in each pair the same core idea is hidden behind the operations for getting 70 and 700 as can be seen in the last column to the right.

Ending this section, it may be helpful to present a summary of some quantitative data concerning the distribution of the three levels of place-value understanding among the three grades. The percentages represent the number of certain answers per total numbers of answers for the given tasks.

Data in Table 8 depict the situation for each grade for the totality of the misconceptions/errors. The percentages are greater for the third grade and smaller for the fifth and sixth ones. This may be attributed to the fact that third graders have limited experience on place-value compared to their older classmates. Grade 5 and 6 pupils have the chance to elaborate and repair their incorrect conceptions of place-value during the years after 3rd grade. Moreover, it seems that the percentages of mistaken number representations are

Table 9. Quantitative data concerning limited-deeper understanding of place-value.

	Grade-3	Grade-5	Grade-6
Limited understanding (unitary concept of multidigit numbers)	2/57 (3.50%)	6/123 (4.87%)	16/129 (12.40%)
Deeper understanding of place-value	5/76 (6.57%)	13/164 (7.92%)	20/172 (11.62%)

greater than the percentages of the remaining three misconceptions. This is the same for all the grades, but still the numbers continue to decrease as we move from 3rd grade to 6th grade.

The numbers in Table 9 show a rather expected picture. Moving from 3rd grade to 6th grade the percentages for limited understanding increase and this could be considered as the first step for older pupils towards resolving the misunderstanding and acquiring a more compact knowledge of place-value. The percentages concerning a solid understanding of place-value are also better for 5th and 6th graders. This could be attributed to their better knowledge of decimal numbers as well as to their more compact knowledge in handling the four operations. This is confirmed by the fact that only 5th and 6th grade pupils employed multiplications and divisions in their approaches.

6. Discussion

In the first part of the results the pupils' answers revealed five different misconceptions/errors related to place-value. This first sub-section started with answers that indicated how zero is perceived by the pupils. Very often, in order to make a sequence of numbers that include zero, the pupils handle these numbers as 'following a pattern': *Increase the first digit by one and then just place zeros*. All the instances presented in Table 1 show that the pupils got the desired thousands (5), hundreds (8) or tens (9) and then, according to the above mentioned pattern, they pressed the zero key three or two consecutive times (for 5000 and 800) or one time (for 90), respectively. This meant that for them: i) Zero was not related to tens, hundreds and thousands, and ii) it did not require a specific arithmetical operation to obtain 5000 or 800 or 90. One can argue that these results could be due to pupils' lack of understanding of the logic and function of the calculator or to their lack of practice. So, since this will be an option for almost all of the findings, it is proper to present our thesis at this point: (i) Several of the test items were linked and consequently the consistency of similar responses across them does make it plausible that a misconception is operative. (ii) The pupils were familiar with the way the calculator functions since there were tasks in their textbooks asking to use it. But it is different when some keys are broken. Then it is not only a matter of how the machine functions but rather of how to exploit the acquired mathematical knowledge relevant to the specific topic. (iii) The pupils continued to wonder when they came to apply online their approach and found that there was a contradiction between what they expected as a result and what they received on their screen. They were convinced about the propriety of their process and this is why they were confused with the result. For them it was rather a matter of how it happened to take a mistaken result than of knowing how to work in the calculator environment.

The second misconception referred to the face value construct, that is the fact that the value represented by each digit of a multidigit number is the same as the product of its 'face value' and the value associated with the place of the digit in the number (i.e., the ones or tens place). But it is not rare to see pupils who consider tens digits as representing single

units and not multiples of 10 ones. According to our data (see Table 2) it became apparent that the pupils made an effort to transfer this mistaken ‘knowledge’ from 2-digit numbers to 3-digit ones. Thus, an overgeneralization of this ‘rule’ is a 3-digit number given that all the necessary keys are broken as it happens in Task-6. All the digits of the 3-digit number were considered separately as being just concatenated single digits or sets of units without any arithmetical relationship among them.

All the cases show clearly that in the pupils’ mind digits are not carrying a certain value that is associated with their placement in the number. These objects do not really represent groups (multiples) of ten or hundred units.

The third misconception refers to reversing values of tens and ones (i.e., reverse thinking). This is a rare error and the only instance in this study was the example of the numbers 78–87. Obviously, it is not an easy task to distinguish between a misconception and an erroneous response due to some other factors such as misremembering or miscalculating. Indeed, it could have happened that the pupil did carelessly read the problem statement and this is why he/she tried to get 87 instead 78. Thus, it is important for the teacher to be aware of such cases so as to include in his/her design of the test some mechanisms that would verify that these errors were not just typo mistakes or misleading but real conceptual errors. The most typical verification procedure could be for the teacher to call the specific pupil for an interview or to ask him/her to solve more problems similar to the one that revealed the error. In any case it is something that has to be noticed by the teacher since it may hide a noticeable misconception.

Then it was the counting sequence errors, i.e., errors in the numbers when changing decades or hundreds during a counting approach. All the examples presented in Table 3 share a common characteristic. They are based on subtraction and thus a change in decades or hundreds is necessary. In Task-3 and in order to get the number 777 some 5th and 6th graders found difficulties in changing decade or hundred. However, another explanation that is more or less connected to place-value could also be accepted. This is based on a mistaken subtraction: $10 - 3 = 7$ (with borrowing 1), $10 - 3 = 7$ (with borrowing 1), $8 - 1 = 7$. Similarly, in the second example, it could be $10 - 3 = 7$, three times separately, or two times for the last example keeping 7 unaltered. This is something that can be later tested and verified. Similar results were found in Task-4. The incorrect numbers in bold (i.e., 777, 99 and 109) were the expected results that the pupils wrote in their worksheets and thought that would be displayed on calculator’s screen. All the instances presented in Table 3 indicate that there may have been a problem in finding the decade or hundred when counting down starting from a specific number.

Finally, there were detected instances that revealed mistaken number representations, i.e., inability of the pupils to conceive the association that exists between the written symbols and the number words. The mistaken representations that were presented in Table 4 could be attributed to a lack of place-value understanding. These pupils were not able to use features of the base-ten numeration system so as to represent accurately certain quantities by written numerical symbols. However, one can say that this may be the result of misremembering or an accidental error and obviously, this option is a valid one. But, again it is easy for the teacher to verify the origin of these errors.

The second part of the results section was relevant to the limited understanding of place-value. The pupils’ answers were based on the unitary concept of multidigit numbers, i.e., multidigit numbers were seen as collections of single units. Moreover, in some cases the pupils broadened this concept and instead of considering single units they considered collections of units. Thus, for a three-digit number, they applied three times the same concept, one for each digit. The target number was 777. Instead of using 777 single units

they used 7 collections of 100 units, 7 of 10 units and 7 single ones. For getting 205 in task-7, instead of 205 single units, a 3rd grade pupil considered a sequence of collections of 5's. This is in itself an interesting aspect of control that shows a progressive movement towards a deeper understanding of the concept of place-value.

Finally in the third part the pupils' answers were indicative of a substantial mathematical knowledge concerning place-value. For Task-1, it would be sufficient to subtract 30 to get zero. This was done by the majority of the pupils. However, some of them preferred either to use subtractions or additions of higher level of difficulty (i.e., $432 - 29$, $432 + 68$). For Task-3, it was the combination of operations (multiplication and addition) that indicated deeper knowledge of the order of operations. For Task-4, it has to be mentioned that there was not any restriction as far as the kind of the numbers is concerned. On the contrary, it would be expected, especially for 5th and 6th graders, to include other numbers than natural ones in their approaches. However, almost all of them resorted to the natural numbers. Therefore, the fact that some of the pupils decided to involve decimals for getting 109 was considered especially important and indicative of an exploitation of the mathematical knowledge taught to them. Finally, it would be appropriate to make an additional comment on Task-3. As Table 7 showed, there was a pattern in the pupils' line of thought for getting the numbers 78 and 777. More precisely, there was an analogy in the processes for getting these numbers. This meant that the pupils were able to transfer the knowledge acquired through the first part of the task to the second. Moreover, this knowledge was not transferred verbatim. They had to modify it so as to get another number but expressing this number in a way that would fit in the process adopted earlier. This is in itself a significant mathematical action that could be included on the advanced understanding of place-value.

Ending this section and despite that the aim of the paper was to present the broken calculator as a teacher's assistant to detect the pupils' conceptual understanding of place-value, it is worth adding a few lines about how such tasks (or other similar to them) can improve understanding of place-value. First of all, pupils must be inventive in their problem solving. In the standard algorithms they have just to follow rules and the steps are quite simple since they are based on their knowledge of single-digit addition or multiplication facts. However, this is not the case when working in the broken calculator environment. The way of thinking is now beyond the mechanical application of certain steps. Pupils are forced now to explore relationships between numbers and exploit their knowledge of mathematics in order to reach new strategies. Given, for example, that in Task-5 pupils have to find an alternative way to build the minuend 800 (since the 8 key is disabled) they are forced to use their knowledge of number relationships and operations. In Task-7 the asked number is 205 and the zero key is broken. This helps pupils to focus on and analyze the structure and elements of arithmetic and to gain skills and deepen understanding instead of letting the calculator replace their thinking. They work around the limits set by the disabled keys or functions which results in deepening their number sense. At the same time, when pupils work in this environment they experience a contradiction between what they expected as a result and what they see on their computer screen. This is a consequence of the pupils' misconceptions and could launch a 'repair' process.

7. Conclusions

Knowing the common mathematical errors and misconceptions of young pupils can provide teachers with an insight into pupils' thinking. This allows teachers to adapt the focus of the teaching and learning process in the classroom. But what does it mean to have a 'misconception'? Normally, this would be defined in terms of someone following in a

regular way a pattern of thinking that either runs counter to what is mathematically accepted or that applies an accepted pattern but in an overgeneralized way. So, in order to infer that someone holds a misconception it is necessary to have at least one and preferably both of the following types of evidence: (a) consistent use of the misconceived pattern across a range of cases; (b) some form of trace of the thinking behind responses that provides a fuller indication of the underlying pattern. This will ensure distinguishing between a misconception and an erroneous response due to other accidental factors (misremembering, miscalculation). Calculators should be seen as tools to assist teachers in detecting such errors and misconceptions. The fact is that the attitudes of the teachers against calculator are varying. There are teachers who prefer not to use a calculator and this reluctance is connected with their liking for number work and thinking out number problems, their limited confidence in the alternative written and mental modes of calculating in the calculator environment, their doubts about the legitimacy of using a calculator or their beliefs about the negative influence of the calculator on understanding and learning numbers [2]. Conversely, there are teachers who prefer using calculator because they do not enjoy number work; they accept its legitimacy and its facility, reliability and speed of the calculator mode of calculation. In general, it seems that experience is a factor in teachers' beliefs about the cognitive affects of using calculators [45]. In a recent study [46] it was found that pre-service teachers were inclined to regard calculators as a crutch in mathematics learning whereas for practicing teachers calculators were seen as a catalyst for enhancing pupils' mathematical achievement. Even among those who are in favour of the usage of calculators there are contrasting views between their own use of calculators and their views of appropriate use of calculators in the elementary school classroom [47].

The main idea of this paper is to see calculators as valuable allies of the teachers in their effort to help their pupils to learn mathematics. More specifically, the aim was to see whether broken calculators could become valid and reliable allies in order to uncover what pupils know or do not know about place-value.

- (1) Carefully designed tasks especially for the calculator environment were used to allow the teacher to understand the extent to which pupils have grasped the mathematical meaning of the topic (place-value in our case) and consequently where further teaching is needed. The pupils' responses (considered as indicative of their thinking) combined with the tasks' design (when it is considered from a 'testing' perspective), taken together, would form one kind of validity. The set of the tasks has content validity through the connections of the items to various ideas from the review and also has some form of face validity since the items provide a sampling of 'correct' and 'incorrect' responses which in their analysis connect to keystroke sequences related to various place-value ideas.
- (2) Given that the pilot study gave insight into the way pupils think about place-value it was important to test the reliability of the usage of calculators. So, the second phase involved older pupils and it seemed that the usage of the same collection of items succeeded in uncovering similar misunderstandings and errors and at the same time showed that these pupils have a deeper understanding of place-value.
- (3) Finally, it seems that it is not just the calculator that is of importance. The proper design of the tasks and the teachers' awareness of the known misconceptions and errors relevant to the topic are equally needed. The former would allow teachers to collect evidence not only about what the pupils do not know but actually what they know and how they know it. The latter would make teachers able to develop an understanding of the known misconceptions relevant to place-value so as to relate

a pupil's way of solving a task to a specific misconception. Hence, this would result in discovering an individual pupil's misconceptions which are identical with known ones already established in the literature.

In the light of the information that the teacher is able to extract from pupils' responses about the nature of their misconceptions, the teaching process is revised and retested. In this study, the design of the tasks was based on the pedagogical point that it is useful to ask pupils to do a task without the full complement of tools ordinarily available for doing that task [15]. This could be considered a way of revealing the depth of pupils' understanding. This approach, based on calculator usage, allowed us to detect some of the misconceptions concerning place-value: zero is not place holder; face value construct; reversing values of tens and ones; counting sequence errors; and mistaken number representations. At the same time we had the chance to record pupils with limited understanding of place-value or pupils who showed a performance that ensures an exploitation of the acquired mathematical knowledge of place-value. So, it seems that the usage of calculators can be broadened. The major view for calculators considers them as tools for speeding up arithmetic computations or for ascertaining the accuracy of those computations. There also exist studies proposing calculators for exploring patterns and discovering more about mathematical concepts and even more for investigating mathematical relationships. However, the purpose of this paper is to add and demonstrate another role for the calculator: a facilitator, making the teachers able to see pupils' conceptual understanding in a multimodal environment which would give them a better way of understanding what their pupils know or do not know and how they are able to use it. We hope that as teachers have started progressively to adopt the perspective of the usage of technology in their classrooms, they would be better able to incorporate calculators in their daily practice. This would give them the chance to assess the usefulness of technology as a tool for modifying their lesson so as to respond to the pupils' needs and level of knowledge.

References

- [1] National Council of Teachers of Mathematics. Computation, calculators, and common sense. Position Statement [Internet]. Reston, VA: National Council of Teachers of Mathematics; 2005. Available from: <http://www.nctm.org/about/content.aspx?id=6358>
- [2] Ruthven K. Pupils' views of number work and calculators. *Educ. Res.* 2005;37(3):229–237.
- [3] Ruthven K. The use of mental, written and calculator strategies of numerical computation by upper primary pupils within a 'Calculator-aware' number curriculum. *Brit. Educ. Res. J.* 1998;24(1):21–42.
- [4] Ruthven K, Rousham L, Chaplin D. The long-term influence of a 'calculator aware' number curriculum on pupils' mathematical attainments and attitudes in the primary phase. *Res. Pap. Educ.* 1997;12(3):249–281.
- [5] Ruthven K, Chaplin D. The calculator as a cognitive tool: upper-primary pupils tackling a realistic number problem. *Int. J. Comput. Math. Learn.* 1997;2(2):93–124.
- [6] Ruthven K. Towards a calculator-aware mathematics curriculum. *Mediterranean J. Res. Math. Educ.* 2009;8(1):111–124.
- [7] Stacey K, Groves S. Calculators in primary mathematics. Paper presented at: The Research Pre-session of the National Council of Teachers of Mathematics. 72nd Annual Meeting; 1994 April; Indianapolis, IN.
- [8] Groves S. Developing number sense in a calculator rich environment - implications for classroom practice. Proceedings of a Working Group at the Eighth International Congress on Mathematical Education. Nottingham: Shell Centre for Mathematical Education; 1997. p. 73–89.
- [9] Groves S, Stacey K. Calculators in primary mathematics: exploring number before teaching algorithms. In: Morrow LJ, Kenney MJ, editors. The teaching and learning of algorithms in school mathematics. Reston, VA: Yearbook of the N.C.T.M.; 1998. p. 120–129.

- [10] Groves S, Dale J. Using activity theory in researching young children's use of calculators. Proceedings of AARE 2004 International Education Research Conference. Melbourne: Australian Association for Research in Education; 2005. p. 1–11.
- [11] Hembree R, Dessart D. Research on calculators in mathematics education. In: Fey JT, editor. *Calculators in mathematics education*. Reston, VA: Yearbook of the N.C.T.M.; 1992. p. 22–31.
- [12] Polly D. Modeling the influence of calculator use and teacher effects on first grade pupils' mathematics achievements. *J. Comp. Math. Sci. Teach.* 2008;27(3):245–263.
- [13] Campbell PF, Stewart EL. Calculators and computers. In: Jensen R, editor. *Early childhood mathematics: NCTM research interpretation project*. New York: Macmillan; 1993. p. 251–268.
- [14] Charles R. Calculators at the elementary school level? Yes, it just makes Sense! *Math. Educ. Dialogues.* 1999;2(3):11.
- [15] Schwartz J. Can technology help us make the mathematics curriculum intellectually stimulating and socially responsible? *Int. J. Comput. Math. Learn.* 1999;4:99–119.
- [16] Chick H, Baker M. Investigating teachers' responses to pupil misconceptions. Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education, Vol. 2. Melbourne: PME; 2005. p. 249–256.
- [17] Stigler J, Hiebert J. *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: Free Press; 1999.
- [18] Ruthven K. Teachers, technologies and the structures of schooling. Proceedings of the 5th Congress of the European Society for Research in Mathematics Education; Larnaca, Cyprus; 2007. p. 52–67.
- [19] Mackey K. Do we need calculators? *Math. Educ. Dialogues.* 1999 May/June, p. 3.
- [20] Klein D, Milgram J. The role of long division in the K-12 curriculum; 2000 [cited 2010 Oct 23]. Available from: <http://www.csun.edu/~vcnth00m/longdivision.pdf>
- [21] Fisher J, Stephens L. Effects of using calculators on mathematical achievement for remedial seventh grade pupils. *Int. J. Math. Educ. Sci. Technol.* 1992;23(5):745–748.
- [22] Goldernberg EP. Thinking (and talking) about technology in math classrooms. *Issues Math. Educ.* 2000:1–8.
- [23] Dick T. The continuing calculator controversy. *Arith. Teach.* 1988;35(8):37–41.
- [24] Leatham K, Lawrence K, Mewborn D. Getting started with open-ended assessment. *Teach. Child. Math.* 2005;11(8):413–419.
- [25] Lu J, Rose R. Seeing math through multimedia case studies. *@Concord.* 2003;7(1):4–5.
- [26] Ross S. Place value: problem-solving and written assessment. *Teach. Child. Math.* 2002;8(7):419–425.
- [27] Ross S. Children's acquisition of place-value numeration concepts: the roles of cognitive development and instruction. *Focus Learn. Probl. Math.* 1990;12(1):1–17.
- [28] Kamii C. Place value: an explanation of its difficulty and educational implications for the primary grades. *J. Res. Child. Educ.* 1986;1(2):75–86.
- [29] Fuson K. Conceptual structures for multiunit numbers: implications for learning and teaching multidigit addition, subtraction, and place-value. *Ethics Behav.* 1990;7(4):343–403.
- [30] Bednarz N, Janvier B. A constructivist approach to numeration in primary school: results of a three year intervention with the same group of children. *Educ. Stud. Math.* 1988;19(3):299–331.
- [31] Sinclair A, Scheuer N. Understanding the written number system: 6 year olds in Argentina and Switzerland. *Educ. Stud. Math.* 1993;24(2):199–221.
- [32] Wheeler M, Feghali I. Much ado about nothing: preservice elementary schools teachers' concept of zero. *J. Res. Math. Educ.* 1983;14(3):147–155.
- [33] Papadopoulos I. Using simple arithmetic calculators as a diagnostic tool on place-value. Proceedings of the Conference on Children's Mathematical Education. Rzeszow: Uniwersytetu Rzeszowskiego; 2010. p. 290–299.
- [34] Stacey K. Travelling the road to expertise: a longitudinal study of learning. Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education, Vol. 1. Melbourne: PME; 2005. p. 19–36.
- [35] Cockburn A, Parslow-Williams P. Zero: understanding an apparently paradoxical number. In: Cockburn A, Littler G, editors. *Mathematical misconceptions*. London: Sage; 2008. p. 7–22.
- [36] Horne M, Livy S. Young children developing place value understandings. Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education, Vol. 3. Prague: PME; 2006. p. 313–320.

- [37] Hiebert J, Wearne D. Links between teaching and learning place value with understanding in first grade. *J. Res. Math. Educ.* 1992;23(2):98–122.
- [38] Ross S. Parts, wholes and place value: a developmental view. *Arith. Teach.* 1989;36:47–51.
- [39] Fuson K, Briars D. Using a base-ten blocks learning/teaching approach for first- and second-grade place value and multidigit addition and subtraction. *J. Res. Math. Educ.* 1990;21:180–206.
- [40] Steinle V, Stacey K. A longitudinal study of pupils' understanding of decimal notation: an overview and refined results. *Proceedings of the 27th Annual Conference of the Mathematics Education Research Group of Australasia*, Vol. 2. Townsville: MERGA; 2004. p. 541–48.
- [41] Price P. Year 3 pupils' place-value misconceptions: another look at MAB. *Annual conference of the Mathematics Education Research Group of Australasia*, Vol. 2; 1998. p. 452–459.
- [42] Power R, Dal Martello MF. The dictation of Italian numerals. *Lang. Cogn. Proc.* 1990;5:237–254.
- [43] Seron X, Fayol M. Number transcoding in children: a functional analysis. *Brit. J. Dev. Psychol.* 1994;12:281–300.
- [44] Loosbroek E, Dirx G, Hulstijn W, Janssen F. When the mental number line involves a delay: the writing of numbers by children of different arithmetical abilities. *J. Exp. Child Psychol.* 2009;102:26–39.
- [45] Fleener J. A survey of mathematics teachers' attitudes about calculators: the impact of philosophical orientation. *J. Com. Math. Sci. Teach.* 1995;14(3):53–70.
- [46] Carter J. Incorporating calculators into primary school mathematics: prospective and practicing teachers' beliefs about their role. *Proceedings of the 13th Asian Technology Conference in Mathematics*. Blacksburg, VA: Mathematics and Technology, LLC; 2008. p. 266–275.
- [47] Walen S, Williams S, Garner B. Pre-service teachers learning mathematics using calculators: a failure to connect current and future practice. *Teach. Teach. Educ.* 2003;19(4):445–462.