

Examination Answer Books

12 Pages

Instructions to Candidates

- Please give the information requested in the boxes below.
- Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
- The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
- Write your answers in ink.
- Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
- No part of this book is to be torn out; it must be handed in complete.
- If more books than one are used, fasten them together using the tags available from the invigilator(s).
- Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.

Complete, fold and seal
(see note 2 below)

SURNAME:

Ibrahim

FORENAME(S):

Mohammed

Zeshan

1	Student Number	2224559
2	Title of Examination	Further Engineering Mathematics
3	Module Code	SAFE5001
4	Date and Time of Examination	14/8/24 9:30
5	Examination Venue	Colcynsgar
6	Seat Number	
7	Number of Answer Books handed in	1

List the numbers of the questions you have attempted	
1	16
2	18
3	6
4	15
5	14
Total	69/70

77% = 77

77%

1.

$$a) 4 \frac{dy}{dx} = 8x^2 + 2$$

$$y(0.5) = 2$$

$$\div 4$$

$$\frac{dy}{dx} = 2x^2 + 0.5$$

✓

$$\int \frac{dy}{dx} = \frac{2x^3}{3} + 0.5x + C \quad \checkmark$$

$$y = \frac{2x^3}{3} + 0.5x + C$$

$$2 = 0 + 0 + C$$

$$2 = C$$

$$\therefore y = \frac{2x^3}{3} + 0.5x + 2 \quad \checkmark \quad \checkmark \quad \checkmark$$

10

b) ~~$2x^2 \frac{dy}{dx} + xy = x^3$~~ $x^2 \frac{dy}{dx} + xy = x^3$

$$\div x^2$$

$$\frac{dy}{dx} + \frac{xy}{x^2} = \frac{x^3}{x^2}$$

$$\frac{dy}{dx} + \frac{y}{x} = x \quad \checkmark \quad x=0 \text{ when } y=2 \quad 2$$

$$P = x^{-1} \quad Q = x \quad \checkmark$$

$$\int P dx = \int x^{-1} dx$$

$$= \ln x$$

$$y e^{\ln x} = \int e^{\ln x} x dx$$

$$e^{\ln x} = x$$

~~$xy = \int x^2 dx$~~

$$y = \int x dx$$

~~$xy = \frac{x^3}{3} + C$~~

$$y = \frac{x^2}{2} + C$$

~~$0 \times 2 = 0 + C$~~

$$2 = 0 + C$$

$$C = 2$$

$$\therefore y = \frac{x^2}{2} + C$$

Method correct 2

6

Q1 16

2. $2 \frac{d^2y}{dx^2} + 7 \frac{dy}{dx} + 6y = 0$ $y=3, \frac{dy}{dx}=3$ when $x=0$

Aux eq: $2m^2 + 7m + 6 = 0$ ✓

2

$m = \frac{-7 \pm \sqrt{7^2 - 4(2)(6)}}{2(2)}$

factorises

$(2m+3)(m+2)$

$m = -1.5$ ✓ or -2 ✓ (Real and different) 4

$\therefore y = Ae^{-1.5x} + Be^{-2x}$

$3 = A + B$ ① when $y=3$ and $x=0$ ✓ 2

$\frac{dy}{dx} = -1.5Ae^{-1.5x} - 2Be^{-2x}$ ✓ 4

$3 = -1.5A - 2B$ ② when $x=0$

$A = 3 - B \rightarrow$ Sub into ② ✓

$3 = -1.5(3 - B) - 2B$

$3 = -4.5 + 1.5B - 2B$

$7.5 = -0.5B$

$\times 2$

15

$15 = -B$

$B = -15$ ✓ 4

Sub into ①

$A = 3 - B$ $B = -15$ $\therefore y = -12e^{-1.5x} - 15e^{-2x}$ ✓ Q2 18

$A = 3 - 15 \therefore A = -12$

$A = -12$ $\therefore y = -12e^{-1.5x} - 15e^{-2x}$

3.

x	f	fx	$f(x - \bar{x})^2$
7	5	35	45
8	21	168	84
9	25	225	25
10	20	200	0
11	16	176	16
12	8	96	32
13	5	65	45

$f(x - \bar{x})^2 = (7 - 9.65)^2 \times 5 = 35.01$

3

$$\bar{x} = 10 \quad \bar{x} = \frac{\sum fx}{\sum f}$$

$$\sum fx = 1965 \quad \checkmark$$

$$\bar{x} = \frac{1965}{100}$$

$$\sum f = 100 \quad \checkmark$$

$$\sum f(x - \bar{x})^2 = 247$$

$$\bar{x} = 10.65$$

$$\bar{x} = 9.65 \quad \checkmark \quad 2$$

$$\sigma = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

Method correct 1

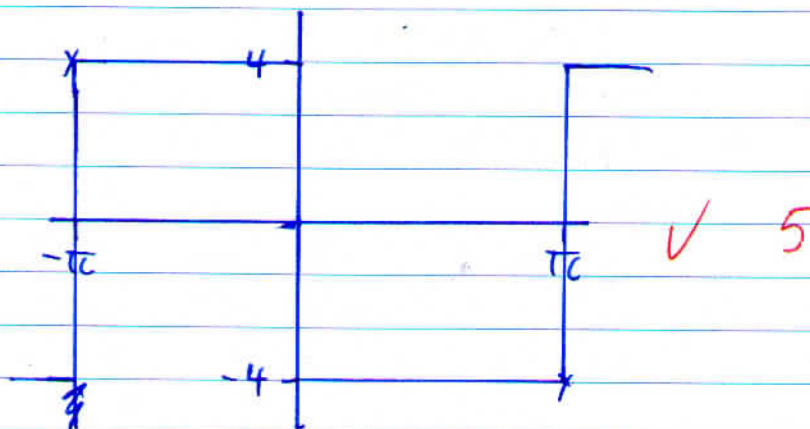
$$\sigma = \sqrt{\frac{247}{100}}$$

$$\sigma = 1.57 \quad \times$$

Q3

6

4. $f(x) = 4$ when $(-\pi < x < 0)$
 $f(x) = -4$ when $(0 < x < \pi)$



$\therefore$ odd function

$a_0 = 0, a_n = 0$

$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin(nx) dx$

$= \frac{2}{\pi} \int_0^{\pi} -4 \sin nx dx$

$= \frac{2}{\pi} \left[\left(\frac{+4 \cos n\pi}{n} \right) - \left(\frac{4 \cos n0}{n} \right) \right]$

$= \frac{2}{\pi} \left[\frac{4 \cos n\pi}{n} - \frac{4}{n} \right]$

$= \frac{2}{\pi} \left[\frac{4 \cos n\pi}{n} - 4 \right]$

$= \frac{2}{n\pi} [4 \cos n\pi] \times \frac{8}{n\pi} (\cos n\pi - 1)$

when $n=1$ $b_n = \frac{2}{\pi} (4 \cos \pi)$
 $b_n = -\frac{8}{\pi}$

$$\text{when } n=3, b_n = \frac{2}{3\pi} (4 \cos 3\pi)$$

$$b_n = \frac{-8}{3\pi}$$

x

Q4

15

5.

a)

$$(i) 2 + t^2 + e^{3t} \quad \checkmark$$

$$L.T = \frac{2}{s} + \frac{2}{s^3} + \frac{1}{s-3} \quad \checkmark \quad 1$$

$$(ii) e^{-2t} \sinh(4t)$$

$$L.T. = \frac{4}{(s+2)^2 + 16} \quad \checkmark \quad 2$$

$$(iii) \frac{2}{s-3}$$

$$I.L.T = 2e^{3t} \quad \checkmark \quad 1$$

$$(iv) \frac{1}{s^2 + 8s + 20} \quad \checkmark$$

$$(s+a)^2 = s^2 + a^2 + 2as$$

$$2a = 8$$

$$a = 4 \quad \checkmark$$

$$\frac{2}{(s+4)^2 + 2^2} \quad \checkmark$$

$$\therefore (s+4)^2 = (s+4)(s+4) = s^2 + 8s + 16$$

$$I.L.T = \frac{1}{2} e^{-4t} \sinh(2t) \quad \checkmark \quad 4t \quad \times$$

b) $\frac{dy}{dt} + 12y = 8$

$y = 2$ when $t = 0$

~~82-20~~

$$s\bar{y} - y_0 + 12\bar{y} = \frac{8}{s}$$

✓

2

$$s\bar{y} - 2 + 12\bar{y} = \frac{8}{s}$$

$$s\bar{y} + 12\bar{y} = \frac{8}{s} + 2$$

$$\bar{y}(s+12) = \frac{8}{s} + \frac{2s}{s}$$

$$\bar{y}(s+12) = \frac{8+2s}{s}$$

$$\bar{y} = \frac{8+2s}{s(s+12)}$$

$$\frac{8+2s}{s(s+12)} = \frac{A}{(s+12)} + \frac{B}{s}$$

Method correct
math error made

$$8+2s = As + B(s+12)$$

$\frac{10}{13}$ for part b

Let $s = 0$

Let $s = -12$

$$8 = 12B$$

$$B = 0.67$$

$$8 - 2(-12) = -12A + B(0)$$

$$32 = -12A$$

$$A = -2.67$$

$$\therefore \bar{y} = \frac{-2.67}{s+12} + \frac{0.67}{s} \quad (\text{Answer on next page})$$

$$I.L.T(f) = -2.67e^{-12f} + 0.67$$

Q5

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