



Prifysgol Cymru
Y Drindod Dewi Sant
University of Wales
Trinity Saint David

Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners will ignore illegible script or have it typed and the expense of typing charged to the candidate.
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Complete, fold and seal
(see note 2 below)

SURNAME:

Foley

FORENAME(S):

Adam

1	Student Number	2308380
2	Title of Examination Paper	Mathematics
3	Module Code	ALDA4008
4	Date and Time of Examination	09:00 09/01/2024
5	Examination Venue	Swansea 12 Campus
6	Seat Number	047
7	Number of Answer Books handed in	1

List the numbers of the questions you have attempted	Office Use
Q1	23
Q2	13
Q3	10
Q4	8
Q5	1
Q6	20
Total	75

75%

Good

Question No.

①
④

$$\frac{12p^5qr}{4p^3q^3r^{-2}}$$

$$= \frac{12}{4} p^{(5-3)} q^{(1-3)} r^{(1-(-2))}$$

$$= \frac{3p^2q^2r^3}{1}$$

⑤

$$\textcircled{V} \quad \frac{fg}{16h} \times \frac{8gh^2}{4fh} \times \frac{32fh^3}{12f^3}$$

$$= \frac{(1 \times 8 \times 32)}{(16 \times 4 \times 12)} = \frac{256}{768} = \frac{1}{3}$$

$$= \frac{1}{3} \frac{f^{(1+1)} g^{(1)} h^{(2+3)}}{f^{(1+3)} h^{(1+1)}}$$

$$= \frac{1}{3} \frac{f^2 g^1 h^5}{f^4 h^2}$$

$$= \frac{1}{3} \frac{f^{-2} g^1 h^3}{1}$$

④

Q2

$$\textcircled{i} \quad 5(x+3) = 2(x-8)$$

Expand $\frac{1}{8}$ $5x + 15 = 2x - 16$

$$5x - 2x + 15 = -16$$

$$3x = -16 - 15$$

$$3x = -31$$

$$x = -\frac{31}{3}$$

$$x = -10.33333333$$

8

$$\textcircled{ii} \quad \frac{(x-3)}{4} = x + \frac{(3x+4)}{8}$$

x 8 each?

x4

$$(x-3) = 4(x) + \frac{4(3x+4)}{8}$$

$$x-3 = 4x + \frac{12x+16}{8}$$

$$x-3 = \frac{16x+16}{8}$$

x No don't write like this

$$x-3 = 2x+2$$

Method 1
Sh!

$$\begin{aligned} -3 &= 2x - x + 2 \\ -3-2 &= x \\ -5 &= x \end{aligned}$$

Answer is 8

5

Q3

$$2x + 4y = 6 \quad \text{Eq (1)} \times 5$$

$$3x - 5y = -2 \quad \text{Eq (2)} \times 4$$

$$10x + 20y = 30 \quad \text{Eq (3)}$$

$$12x - 20y = -8 \quad \text{Eq (4)}$$

sub (4) into (3) to find x

$$10x + 20y + 12x - 20y = 30 - 8$$

$$22x = 22$$

$$x = \frac{22}{22}$$

$$x = 1$$

sub x value into Eq (1)

$$2(1) + 4y = 6$$

$$2 + 4y = 6$$

$$4y = 4$$

$$y = \frac{4}{4}$$

$$y = 1$$

24.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\det A = 1 \begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} + 3 \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}$$

$$= 1[(-1 \times 1) - (2 \times 2)] - 2[(2 \times 1) - (2 \times 3)] + 3[(2 \times 2) - (-1 \times 3)]$$

$$= -4 + 4 + 6 = 6$$

$$[A] = 24$$

$$\det A = 6$$

$$A^{-1} = \left(\frac{1}{\det A} \right) \text{adj } A$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$14 = 1[(-1 \times 1) - (2 \times 2)] - 2[(2 \times 1) - (2 \times 3)] + 3[(2 \times 2) - (-1 \times 3)]$$

$$= -5 + 8 + 7(3)$$

$$\det A = 10$$

3

$$A^{-1} = \left(\frac{1}{\det A}\right) \text{Adj } A$$

← transposed, rows become columns

$$A^{-1} = \left(\frac{1}{10}\right) \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 2 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$= \left(\frac{1}{10}\right) \begin{bmatrix} 1 & -2 & 3 \\ -2 & -1 & -2 \\ 3 & 2 & 1 \end{bmatrix}$$

use 11 for Determinant

$$C = \frac{1}{10} \left[\begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} - \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} \right. \\ \left. - \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} \right. \\ \left. + \begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} - \begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} \right]$$

$$\begin{aligned} \text{Row 1} &= [(-1 \times 1) - (2 \times 2)] - [(2 \times 1) - (2 \times 3)] + [(2 \times 2) - (-1 \times 3)] \\ &= 4 + 4 + 7 \\ &= 15 \end{aligned}$$

$$\begin{aligned} \text{Row 2} &= [(2 \times 1) - (3 \times 2)] + [(1 \times 1) - (3 \times 3)] - [(1 \times 2) - (2 \times 3)] \\ &= 4 - 8 + 4 \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{R3} &= [(2 \times 2) - (3 \times -1)] - [(1 \times 2) - (3 \times 2)] + [(1 \times -1) - (2 \times 2)] \\ &= 7 + 3 - 5 \\ &= 5 \end{aligned}$$

$$Rt = 15 + 5 + 0$$

$$= 20$$

$$A^{-1} = \frac{1}{10} \times 20$$

$$= \underline{\underline{2}}$$

Q5

$$x + 2y + 3z = 2$$

$$2x - y + 2z = 5$$

$$3x + 2y + z = 2$$

~~AX = b~~

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 2 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$$

$$x = \frac{1}{\Delta} \begin{vmatrix} 2 & 2 & 3 \\ 5 & -1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{\Delta} (2 \times (-1) \times 1 + 2 \times 2 \times 2 + 3 \times 2 \times 2) = \frac{1}{\Delta} (-2 + 8 + 12) = \frac{18}{\Delta}$$

$$y = \frac{1}{\Delta} \begin{vmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{\Delta} (1 \times 2 \times 1 + 2 \times 3 \times 3 + 3 \times 2 \times 2) = \frac{1}{\Delta} (2 + 18 + 12) = \frac{32}{\Delta}$$

$$z = \frac{1}{\Delta} \begin{vmatrix} 1 & 2 & 2 \\ 2 & -1 & 2 \\ 3 & 2 & 1 \end{vmatrix}$$

Q5

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 2 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$$

$$x = [(1 \times \frac{2}{2}) + (2 \times \frac{5}{2}) + (3 \times 2)] = \underline{12}$$

$$y = [(2 \times 2) + (-1 \times 5) + (2 \times 2)] = \underline{-1}$$

$$z = [(3 \times 2) + (2 \times 5) + (1 \times 2)] = \underline{15}$$

$$x^2 + x + 3x - 3$$

$$Q6 \quad f(x) = \frac{x+2}{x^2+2x-3}$$

$$f(x) = \frac{x+2}{(x+3)(x-1)}$$

$$\frac{x+2}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$$

$$x+2 = A(x-1) + B(x+3)$$

let $x = -3$ to solve for A:

$$-3+2 = A(-3-1) + B(0)$$

$$-1 = A(-4)$$

$$\frac{-1}{-4} = A$$

$$A = 0.25 \text{ or } \frac{1}{4}$$

Make let $x = 3$ and $A = 0.25$ for B .

~~$0.25 = 0.25(3+4) + B$~~

$$1 + 2 = A(0) + B(1+3)$$

$$3 = B(4)$$

$$\frac{3}{4} = B$$

$$B = 0.75 \text{ or } \frac{3}{4}$$

Put A & B into Eq. solve for C .

~~$x = 2$~~

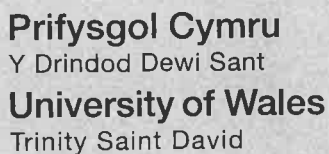
$$\frac{0.25}{x+3} + \frac{0.75}{x-1}$$

$$\frac{f(x)}{3x^2 - 1x + 4} = \frac{0.25x + 0.75}{3x^2 - 1x + 4}$$

$$3x^2 - 1x + 4$$

$= -0.75$ NOT solving here!

(20)



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1	Student Number	2308380
2	Title of Examination Paper	Mathematics
3	Module Code	ACDA4008
4	Date and Time of Examination	15:00 11/01/2024
5	Examination Venue	UWTSD 1Q (amp)
6	Seat Number	031
7	Number of Answer Books handed in	1

[illegible]

(Gweler yr ochr arall am y fersiwn Cymraeg)

Q1

$$\textcircled{i} \quad y = 8x^3 + 3$$

$$\frac{dy}{dx} = 3(8)x^{3-1} + 0$$

$$\frac{dy}{dx} = 24x^2$$

②

$$\textcircled{ii} \quad y = 4x^3 + 10x^{\frac{1}{2}} + \frac{5}{x}$$

$$\frac{dy}{dx} = 3(4)x^{3-1} + \frac{1}{2}(10)x^{\frac{1}{2}-1} + 0$$

$$\frac{dy}{dx} = 12x^2 + 5x^{-0.5}$$

③

$$\textcircled{iii} \quad y = \frac{2}{x^5} + x^3 = 2x^{-5} + x^3$$

$$\frac{dy}{dx} = 5\left(\frac{2}{x^{5-1}}\right) + 3x^{3-1}$$

$$\frac{dy}{dx} = \frac{10}{x^4} + 3x^2$$

①

$$\textcircled{iv} \quad y = \sin 5x$$

$$\frac{dy}{dx} = 5 \sin 5x \quad 5 \cos(5x)$$

Cancelless here!

①

1.

$$(v) \quad y = x^2 \cos x$$

$$\frac{dy}{dx} = \cancel{u \frac{dv}{dx}} + v \cdot \frac{du}{dx}$$

$$u = x^2$$

$$v = \cos x$$

$$\frac{du}{dx} = \cancel{2x} \cdot 2x$$

$$\frac{dv}{dx} = -\sin x$$

$$\frac{dy}{dx} = x^2 \cdot -\sin x + \cos x \cdot 2x$$

$$\frac{dy}{dx} = -x^2 \sin x + 2x \cos x$$

$$\frac{dy}{dx} = -x^2 \sin x + 2x \cos x$$



$$(vi) \quad y = (9x+3)^{10} \Rightarrow \frac{dy}{dx} = 10(9x+3)^9 \cdot 9$$

$$\frac{dy}{dx} = 90x^{10-1} + 0$$

By chain rule

$$\frac{dy}{dx} = 90x^9$$

(vii) $y = (x + 3 \tan(x))$

$$\frac{dy}{dx} = 1 + 3 \sec^2(x)$$

2

(b) $y = \frac{1}{x} + \cos(3x) \Rightarrow -\frac{1}{x^2} - \sin(3x)$

$$\frac{d^4 y}{dx^4} = 4\left(\frac{1}{x^3}\right) - 4 \times 3 \sin 3x$$

$$\frac{d^4 y}{dx^4} = \frac{4}{x^3} - 12 \sin 3x$$

(c) $y = \frac{(2x+1)}{(3+x^2)} \Rightarrow \frac{dy}{dx} = \frac{V \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{V^2}$

$$y = \frac{u}{v}$$

$$v = 3+x^2 \quad u = 2x+1$$

$$\frac{du}{dx} = 2$$

$$\frac{dv}{dx} = 2x$$

$$\frac{(3+x^2)(2) - (2x+1)(2x)}{(3+x^2)^2} = \frac{6+2x^2-4x^2-2x}{(3+x^2)^2}$$

Ans 1

$$\frac{dy}{dx} = \frac{-2x^2 - 2x + 6}{(3+x^2)^2}$$

① if $x = 0$.

$$\frac{-2(0)^2 - 2(0) + 6}{(3 + 0^2)^2} = \frac{6}{9}$$

$$= \frac{2}{3}$$

$$\frac{dy}{dx} = \frac{2}{3} \text{ or } 0.6\bar{6}$$

6

Q2.

②

① $\int (10x + x^3) dx$

$$I = \frac{10x^{1+1}}{1+1} + \frac{x^{3+1}}{3+1} + C$$

$$I = \frac{10x^2}{2} + \frac{x^4}{4} + C$$

$$I = 5x^2 + \frac{x^4}{4} + C$$

3

(i) $\int (2x + 3e^x) dx$

$$I = \frac{2x^{1+1}}{1+1} + 3e^x + C$$

$$I = \frac{2x^2}{2} + 3e^x + C$$

~~ii~~

(ii) $\int (\frac{2}{x} + 4) dx$

$$I = 2\ln x + 4x + C$$

(iv) $\int (x-1) \cdot (3x+4) dx$

$$I = \int (3x^2 + 4x - 3x - 4) dx$$

$$I = \frac{3x^{2+1}}{2+1} + \frac{4x^{1+1}}{1+1} - \frac{3x^{1+1}}{1+1} - 4x$$

$$I = \frac{3x^3}{3} + \frac{4x^2}{2} - \frac{3x^2}{2} - 4x + C$$

$$I = \frac{x^3}{3} + \frac{x^2}{2} - 4x + C$$

Question No.

Q2.

$$\textcircled{V} \int_3^6 (3x^2 - 2x) dx = [x^3 - x^2]_3^6$$

$$= 162$$

~~$$\frac{2(3)x^{2+1}}{2+1} - \frac{2x^{1+1}}{1+1}$$~~

~~$$= \left[\frac{6x^3}{3} - \frac{2x^2}{2} \right]_3^6$$~~

Method Cancel!

you are mixing up $\frac{d}{dx}$ and $\int f(x) dx$

~~$$= \left[\left(\frac{6(6)^3}{3} - \frac{2(6)^2}{2} \right) - \left(\frac{6(3)^3}{3} - \frac{2(3)^2}{2} \right) \right]$$~~

~~$$= 36 - 9$$~~

~~$$I = 25$$~~

$$\textcircled{B} \int_0^1 \frac{e^x}{\sqrt{e^x+1}} dx \quad \text{let } u = e^x + 1$$

$$\int f[g(x)] \cdot g'(x) dx = \int f(u) \cdot du, \quad \text{letting } u = g(x)$$

$$\int_0^1 f(x) dx = [F(x)]_0^1$$

$$= F(1) - F(0)$$

$$\int_0^1 \frac{e^x}{\sqrt{u}} dx$$

$$\int f[\sqrt{e^x+1}] \cdot \sqrt{e^x+1} dx = \int f(e^x) \cdot e^x dx$$

⑦ $\int x \cdot \cos(x) dx$

①

$$\int u \frac{dv}{dx} dx = uv - \int v \cdot \frac{du}{dx} dx$$

$$u = x \quad v = \cos(x) - \sin x$$

$$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = -\sin(x) - \cos x$$

$$\int x \cos(x) dx = x \sin x - \int \sin x \cdot 1 dx$$

$$I = x \sin x + \cos x + C$$

⑧ $\int_1^e \ln x dx$ $I = \ln x - \int 1 \cdot x dx$

$$= [\ln x - \frac{1}{2} x^2]$$

$$u = \ln x \quad v = 1$$

$$\frac{dv}{dx} = 0$$

$$I = [\ln e - e] - [\ln 1 - 1]$$

$$I = 1.7183 + 1$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$I = 2.7183$$

23. (i) $(3+2i)(1-i)$

$$= 3 - 3i + 2i - 2i^2$$

$$4i^2 = 1$$

$$= 3 - i + 2$$

$$= 5 - i \quad \text{Answers here!}$$

(ii) $\frac{3}{(2-i)} \cdot \frac{(2+i)}{(2+i)} = \frac{3(2+i)}{4+2i-2i-i^2} = \frac{6+3i}{5}$

~~3~~

$$= \frac{6+3i}{4+1} = \frac{6+3i}{5} = \frac{2+3i}{5}$$

(iii) $\frac{1}{i^5} = \frac{1}{i^2 \cdot i^2 \cdot i} = \frac{1}{1+1+i} = \frac{1}{2+i}$

(b) $z_1 = \frac{2-j}{2+j}, \quad z_2 = \frac{3+j}{1-2j}$

$$z_1 = \frac{2-j}{2+j} \cdot \frac{(2-j)}{(2-j)} = \frac{4-2j-2j+j^2}{4-2j+2j-j^2}$$

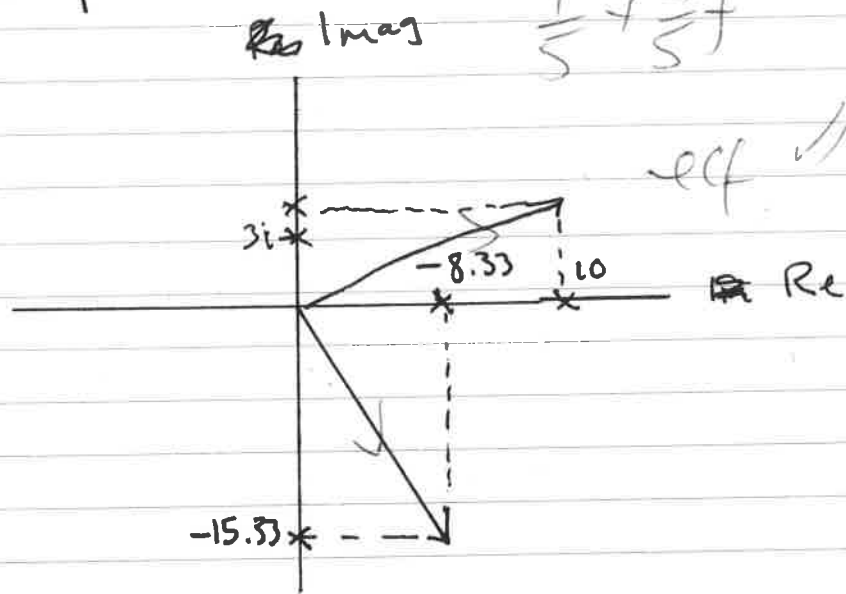
$$= \frac{4-4j+1}{4-1} = \frac{5-4j}{3} = \frac{5}{3} - \frac{4j}{3}$$

$$z = 3 + 6j + j + 2j^2$$

$$= \frac{5 + 7j}{-1}$$

$$= \frac{5}{-1} + \frac{7j}{-1}$$

$$\frac{1}{5} + \frac{2}{5} +$$



$$p = z_1 + 2z_2$$

$$p = \frac{5 - 4j}{3} + 2\left(\frac{5 + 7j}{-1}\right)$$

$$= 1.6667 - 1.333j + -10 + 14j$$

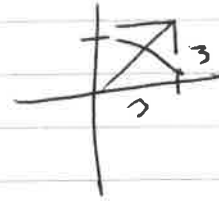
$$z = -8.\dot{3}\dot{3} - 15.\dot{3}\dot{3}j$$

$$(ii) 10 + 3i = 3\left(\frac{5-4j}{3}\right) - \left(\frac{5+7i}{-1}\right)$$

d) $z = (1 + 2i)(1 + i)$

$$z = 1 + i + 2i + 2i^2$$

$$z = 3 + 3i$$



$$\tan^{-1} \frac{3}{3} = \theta = 45^\circ$$

$$R = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$$

$$z = 3\sqrt{2}(\cos 45^\circ + i \sin 45^\circ)$$

②

24. $4^{2x} = 3^{x+1}$

$$(2x) \log 4 = (x+1) \log 3$$

$$\frac{\log 3}{\log 4} = \frac{2x+1}{2x}$$

$$0.792 = \frac{x+1}{2x}$$

$$2x(0.792) = x+1$$

$$1.585x = x+1$$

$$0.585x = 1$$

$$x = \frac{1}{0.585}$$

$$x = 1.71$$

②

ii) ~~$\ln \frac{x^4 y}{z}$~~ $2 \ln x = \ln(4x - 4)$

~~$\ln$~~ $2 \ln x = \log_e \ln \left(\frac{4x}{4} \right)$ ✓

$2 \ln x = \ln x$ ✗

$\frac{2 \ln x}{\ln x} = \underline{\underline{2}}$ (✓)

b. $\log(x^4 y z) = \log x^4 + \log y + \log z$

①

$= 4 \log x + \log y + \log z$

③

i) $\ln \left(\frac{x^4 y}{z} \right) = \log_e \ln(x^4 y) - \log_e \ln z$

$= 4 \log x \cdot 4 \ln x + \ln y - \ln z$

③

③

③

3/4