

Maths paper - at home

Woke!

$\ln = \log$, okay?
My calc uses $\ln = \log$

1 $y = \frac{x^2}{2x+1} = (x^2)/(2x+1) \rightarrow \frac{d}{dx} \left(\frac{x^2}{2x+1} \right)$

$\frac{2x(x+1)}{(2x+1)^2} \leftarrow \frac{2x}{2x+1} - \frac{2x^2}{(2x+1)^2}$

2 $y = \sqrt{x^2+1} \rightarrow (x^2+1)^{0.5} \rightarrow \frac{d}{dx} ((x^2+1)^{0.5})$

$\frac{x}{\sqrt{x^2+1}}$ or $\frac{x}{(x^2+1)^{0.5}}$

3 $y = 3x^{1.5} + \frac{2}{x^2} - 6x = (3x)^{1.5} + (3/(x^2)) - (6x)$ (both are the same)

$\rightarrow \frac{d}{dx} \left((3x)^{1.5} + \frac{2}{x^2} - 6x \right)$

$7.194232^{0.5} - \frac{6}{x} - 6$

4 $y = \sec^3 4x = (\sec^3) 4x$

5 $y = (x + \sin 2x) = (x + (\sin(2x))) \rightarrow \frac{d}{dx} (x + \sin(2x))$

$2(\cos(2x)) + 1$

6 $y = \frac{1-10x}{(2x-1)^2} = (1-(10x))/((2x-1)^2) \rightarrow \frac{d}{dx} \left(\frac{1-10x}{(2x-1)^2} \right)$

$\frac{20x+6}{(2x-1)^3}$

Q2

$$a) \int \frac{2x}{(4+3x^2)} dx = \int (2x)/(4+(3x^2)) dx \rightarrow \frac{1}{3} \ln(3x^2+4) + C$$

$$2 \int \frac{x}{3x^2+4} \rightarrow \frac{1}{3} \int \frac{1}{u} du \quad \begin{matrix} [u=3x^2+4] \\ [du=6x dx] \end{matrix}$$

$$\frac{\ln(u)}{3} + C$$

B)

$$\int_0^4 (e^{0.5x}) dx \rightarrow [u = \frac{x}{2} \quad du = \frac{1}{2} dx]$$

$$2 \int_0^2 e^u du$$

$$2e^u \Big|_0^2$$

$$2(e^2 - 1)$$

$$\boxed{12.7781} \quad (\text{if feels hard...})$$

$$c) \int_0^4 \frac{5x+13}{(2x+1)(x+4)} dx = ((5x+13)/((2x+1)(x+4))) dx$$

$$\left[\begin{matrix} u=2x+1 \\ du=2dx \end{matrix} \right] \leftarrow 3 \int \frac{1}{2x+1} dx + \int_0^4 \frac{1}{x+4} dx$$

$$\frac{3}{2} \int_1^9 \frac{1}{u} du + \int_0^4 \frac{1}{x+4} dx \rightarrow \frac{3 \log(u)}{2} \Big|_1^9 + \int_0^4 \frac{1}{x+4} dx \rightarrow \left[\begin{matrix} s=x+4 \\ ds=dx \end{matrix} \right] \rightarrow \int_4^8 \frac{1}{s} ds$$

$$(3.1p) \\ 3.91p \approx$$

$$\boxed{\log(54)}$$

$$\log(27)$$

$$+ \log(2) \leftarrow \log(x) \Big|_4^8 \leftarrow \log(1)$$

B)

$$\int(x) = \frac{4}{x(1+4\ln(x))^2} dx \rightarrow ((4)/(x(1+4\ln(x))^2)) dx$$

$$\int \frac{4}{x(4\log(x)+1)^2} dx$$

$$4 \int \frac{1}{x(4\log(x)+1)^2} dx$$

$$[v = \log(x) \quad dv = \frac{1}{x} dx]$$

$$4 \int \frac{1}{(4v+1)^2} dv$$

$$[s = 4v+1, ds = 4dv]$$

$$\int \frac{1}{s^2} ds$$

$$-\frac{1}{s} + C$$

$$-\frac{1}{4v+1} + C$$

$$\frac{1}{-4\log(x)-1} + C$$

c) $f(x) = \log(\operatorname{cosec}(x) - \cot(x))$ show $f'(x) = \operatorname{cosec}(x)$

show that $\frac{d}{dx}(\log(\operatorname{cosec}(x) - \cot(x))) = \operatorname{cosec}(x)$

$$\frac{d}{dx} \log(\operatorname{cosec}(x) - \cot(x)) = \frac{\frac{d}{du} \log(u)}{\frac{du}{dx}} \left[u = \operatorname{cosec}(x) - \cot(x) \right]$$

$$\left[\frac{d}{du} \log(u) = \frac{1}{u} \right]$$

$$\frac{\frac{d}{dx}(\operatorname{cosec}(x) - \cot(x))}{\operatorname{cosec}(x) - \cot(x)} = \frac{\left(-\frac{d}{dx}(\cot(x)) \right) + \frac{d}{dx}(\operatorname{cosec}(x))}{\operatorname{cosec}(x) - \cot(x)}$$

$$\left[\begin{array}{l} u = x \\ \frac{d}{du} f(\cot(u)) = -\operatorname{cosec}^2(u) \end{array} \right]$$

$$\frac{\operatorname{cosec}^2(x) \left(\frac{d}{dx}(x) \right) + \frac{d}{dx}(\operatorname{cosec}(x))}{-\cot(x) + \operatorname{cosec}(x)}$$

$$\frac{\operatorname{cosec}^2(x) + \left(-\cot(x) \operatorname{cosec}(x) \left(\frac{d}{dx}(x) \right) \right)}{-\cot(x) + \operatorname{cosec}(x)}$$

$$\operatorname{cosec}(x)$$

!! it worked!

Simplify to

B) $y = \frac{1}{x} + \cos 2x$ find $\frac{d^5 y}{dx^5}$

$y = (\frac{1}{x}) + \cos(2x) \parallel \frac{d^5 y}{dx^5}$

(find 5th derivative?)
if find 5th derivatives
then...

$(\frac{1}{x}) + \cos(2x) \rightarrow \text{first}$

$-\frac{1}{x^2} + (-2\sin(2x)) \quad (d^1)$

$\frac{2}{x^3} - 4\cos(2x) \quad (d^2)$

$-\frac{6}{x^4} + 8\sin(2x) \quad (d^3)$

$\frac{24}{x^5} + 16\cos(2x) \quad (d^4)$

$-\frac{120}{x^6} - 32\sin(2x) \quad (d^5) \xrightarrow{d^5} \text{answer (?)}$

Q3

$$1) \left(\frac{1+i}{1-i} \right)^3 = ((1+i)/(1-i))^1 \cdot 3 = \begin{pmatrix} -1 \\ 0-i \end{pmatrix} \quad (i^2 \times i = -1i = -i)$$

or

$0-i$

$$2) 3/((1+i)(2+i)) \rightarrow \frac{3}{(3i+1)} \rightarrow \frac{3(-3i+1)}{(3i+1)(-3i+1)} \rightarrow \frac{3(-3i+1)}{10}$$

~~$\frac{3}{10}(-3i+1)$~~ or $\frac{3}{10}(1-3i)$

$$3) \frac{(1+i)}{1-i} - ((1+2i)(2+2i)) + \frac{3-i}{1+i}$$

↓

$$((1+i)/(1-i)) - ((1+2i)(2+2i)) + ((3-i)/(1+i)) \quad \text{What? ... aha...}$$

So...

$$\begin{aligned} & \left(\frac{(1+i)(1+i)}{(1-i+i)(1+i)} \right) - (2i+1) \times 2(1+i) + \frac{-i+3}{-i+1} \\ & \quad \downarrow \quad \quad \quad \downarrow \quad \quad \quad \downarrow \\ & \frac{(1+i)^2}{2} \quad \quad \quad 2(3i-1) \quad \quad \quad \frac{(-i+3)(1+i)}{(-i+1)(1+i)} \Rightarrow \frac{2(i+2)}{2} \\ & \quad \downarrow \quad \quad \quad \downarrow \quad \quad \quad \downarrow \\ & 1 \quad \quad \quad - (6i-2) + 1+2 \quad \quad \quad i+2 \\ & \quad \quad \quad \downarrow \\ & \quad \quad \quad -4i+4 \\ & \quad \quad \quad \downarrow \\ & \quad \quad \quad 4(-i+1) \end{aligned}$$

$$2 + \frac{4}{i-1} + (3\sqrt{3} - i) + \frac{(1+i)(1+i)}{1} = \frac{\sqrt{3} + (-2-i)}{1}$$

$$2i^2 - 2i$$

$$r(\sqrt{3}) \quad (-2-i) \rightarrow$$

B)

$$Z = (4+7i)(3-2i) \rightarrow 13(2+i)$$

$$r = 13\sqrt{5}$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right) \quad (\text{Radians})$$

C)

$$1+3i \text{ and } 1+i$$

$$(1+3i)(1+i) = 2(-1+2i)$$