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ADVANCED ENGINE DESIGN

M6X00410

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Q1

$M_{\text{piston assembly}} = 240 \text{ g}$

$M_{\text{con rod}} = 330 \text{ g}$

$M_{\text{shells}} = 20 \text{ g}$

$R = \frac{90}{2} = 45 \text{ mm}$

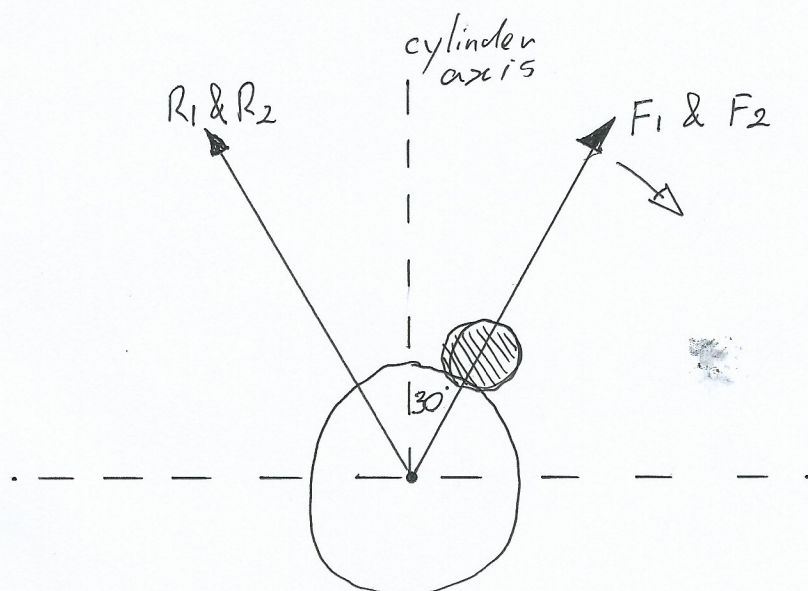
$M_{\text{crankshaft}} = 5.0 \text{ kg}$

Con-rod length = 140 mm

$$M_{\text{rot}} = \frac{2}{3}(330) + 20 = 240 \text{ g} = 0.240 \text{ kg}$$

$$M_{\text{rec}} = \frac{1}{3}(330) + 240 = 350 \text{ g} = 0.350 \text{ kg}$$

(a)



The first order forward rotating vector is the ~~the~~ sum of rotational forces and half of the reciprocating. Reciprocating forces are split into two vectors of each equal angle away from the cylinder axis.

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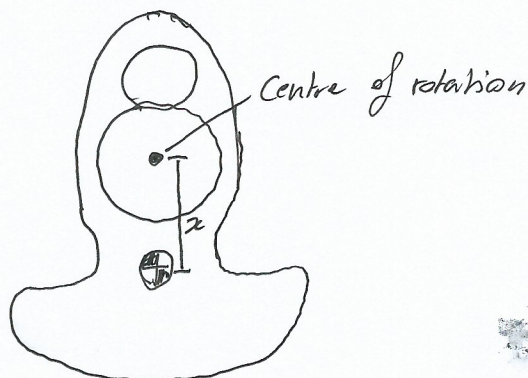
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(a) continued...

$$F_1 = F_2$$

$$\begin{aligned} \therefore \text{Mass moment} &= 2 (M_{\text{rot}} + \frac{M_{\text{rec}}}{2}) R \\ &= 2 (0.24 + \frac{0.35}{2}) 45 \\ &= \underline{\underline{37.35 \text{ kg} \cdot \text{mm}^2}} \end{aligned}$$

(b)



To 100% balance the crankshaft the centre of gravity must be offset from the centre of rotation to counter-act the forward first order mass moment.

$$\therefore M_{\text{crankshaft}} \cdot x = \text{mass moment}$$

$$x = \frac{37.35}{5}$$

$$x = \underline{\underline{7.47 \text{ mm}}}$$

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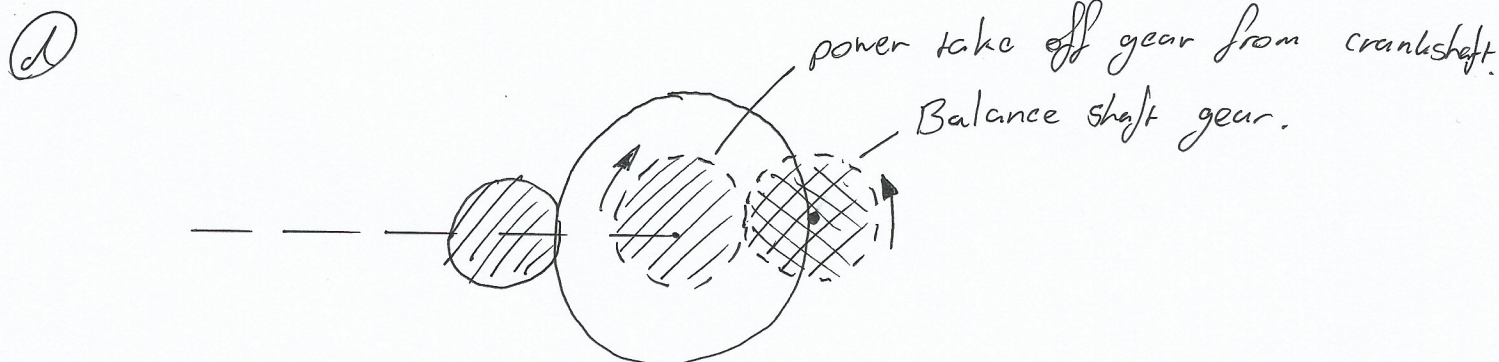
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(c) first order reverse rotating vector mass moment :

See diagram in q.a. ,

$$R_1 = R_2$$

$$\therefore \text{mass moment} = \frac{2(M_{rec})R}{2} \\ = 15.75 \text{ kg} \cdot \text{mm}$$



Gear driven is the preferred method providing reverse direction of rotating with once coupling.

(e) $M_{\text{balance shaft}} = 2.5 \text{ kg}$

To balance first order reverse vector balance shaft COG offset must be :

$$M_{\text{balance shaft}} \cdot y = \frac{2(M_{rec})R}{2} \\ y = \frac{15.75}{2.5} = 6.3 \text{ mm}$$

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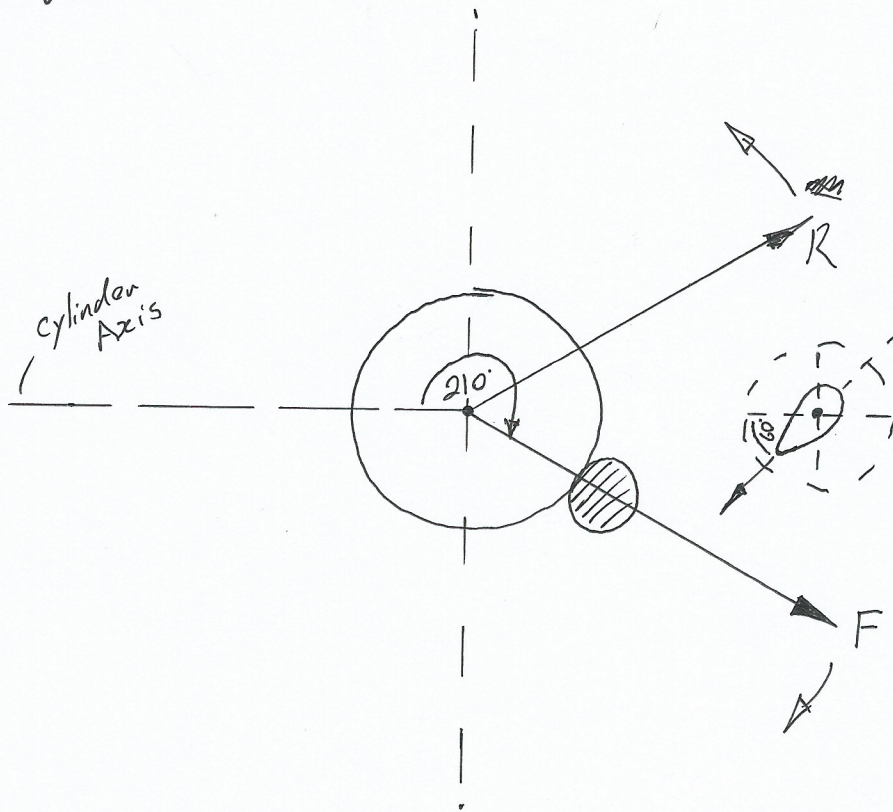
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(f)



(g) For a market where a balance shaft would imply ~~an~~ extra un-wanted costs and complexity it would be most appropriate to adopt an underbalanced strategy to remove vertical resultant forces in ~~an~~ a horizontal configuration. This can be achieved by only balancing the rotational forces and thus, leaving reciprocating forces horizontal.

$$\begin{aligned} \text{Mass moment} &= (M_{rot}) R \\ &= 10.8 \text{ kg. mm} \end{aligned}$$

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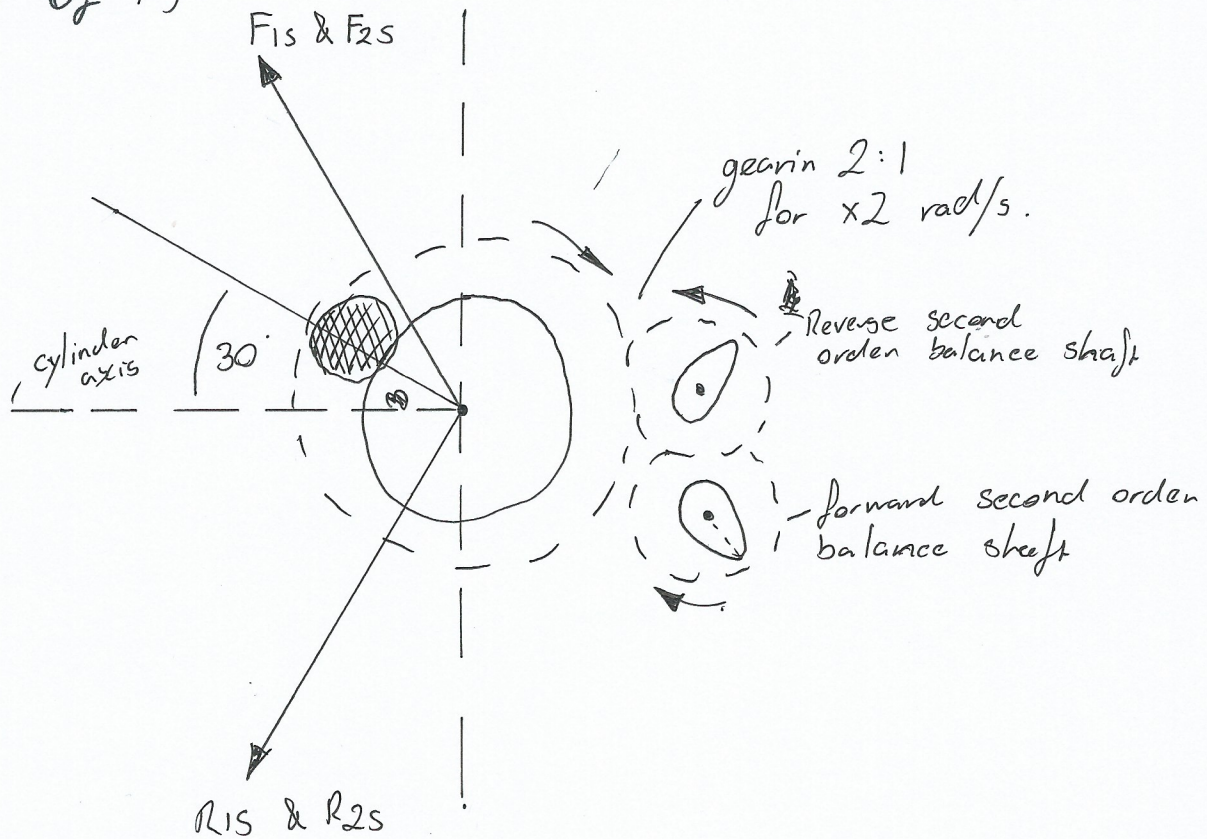
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(b)



Note: $R_{1s} = R_{2s}$

Mass moment for each balance shaft

$$\begin{aligned} &= 2 \left(\frac{1}{2} \frac{R}{L} (M_{rec}) R \right) \\ &= 2 \left(\frac{1}{2} \left(\frac{45}{140} \right) (0.350) (45) \right) \\ &= 2 (2.531) \\ &= 5.063 \text{ kg} \cdot \text{mm} \end{aligned}$$

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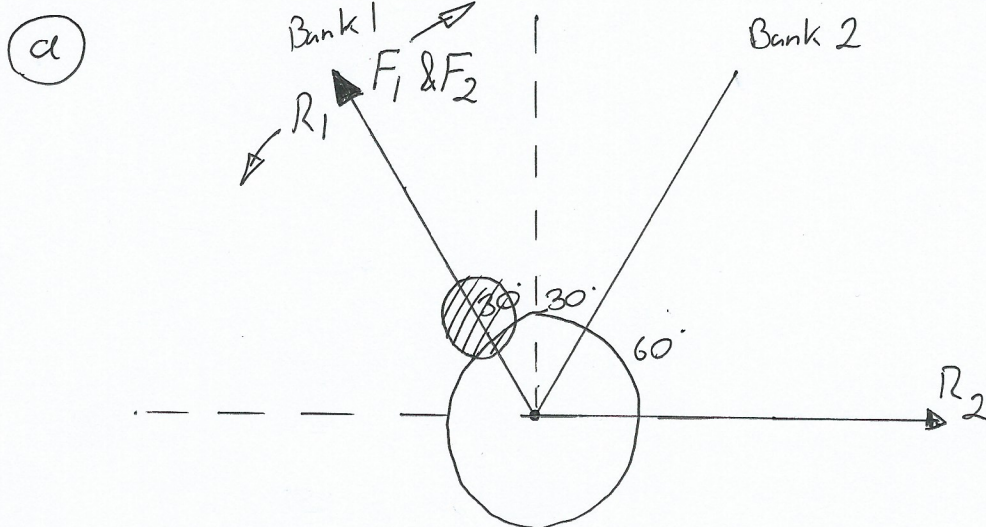
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Q2

$$M_{rot} = \frac{2}{3} (360) = 240g = 0.24 \text{ kg}$$

$$M_{rec} = \frac{1}{3} (360) + 240 = 360g = 0.36 \text{ kg}$$

$$R = 40 \text{ mm}$$



$$F_1 = F_2$$

$$\begin{aligned} \therefore \text{mass moment} &= 2 \left(\left(M_{rot} + \frac{M_{rec}}{2} \right) R \right) \\ &= 2 (16.8) \\ &= 33.6 \text{ kg} \cdot \text{mm} \end{aligned}$$

(b) Refer. to diagram in question 2 a.

$$\begin{aligned} \circ \text{ Resultant} &= R_1 \cos 60 + R_2 \cos 60 \\ &= 1R \end{aligned}$$

$$\circ R_1 = R_2$$

$$\begin{aligned} \therefore \text{mass moment} &= \left(\frac{M_{rec}}{2} \right) R \\ &= \left(\frac{0.36}{2} \right) 40 = 7.2 \text{ kg} \cdot \text{mm} \end{aligned}$$

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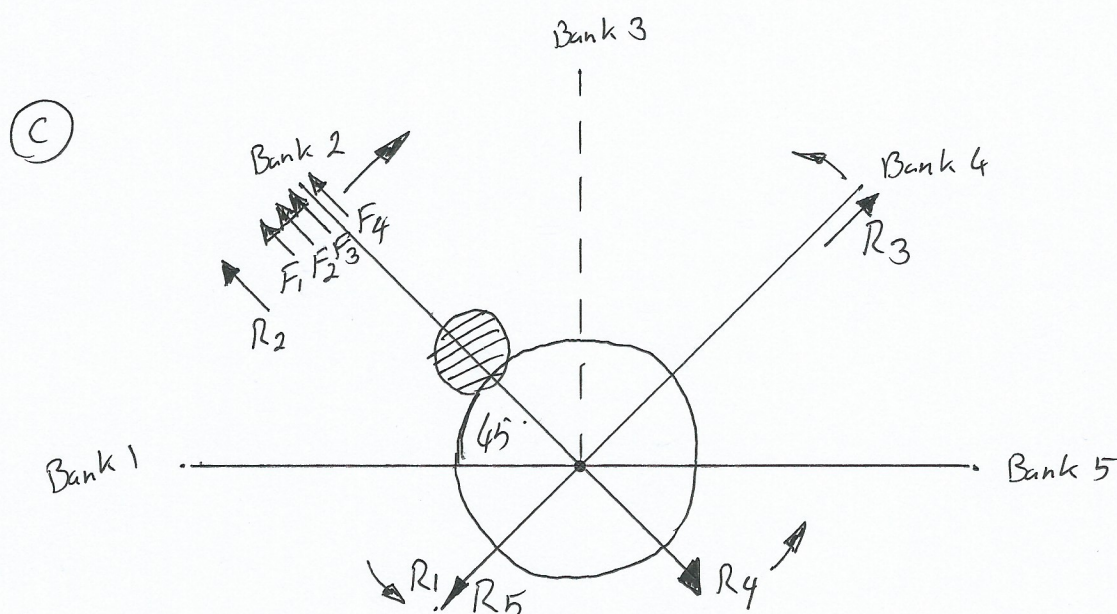
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$$F_1 = F_2 = F_3 = F_4$$

$$\begin{aligned} \therefore \text{mass moment} &= 4 \left((M_{rot} + \frac{M_{rec}}{2}) R \right) \\ &= 4 (33.6)^2 = 4 (16.8) \\ &= 134.4 \text{ kg} \cdot \text{mm} = 67.2 \text{ kg} \cdot \text{mm} \end{aligned}$$

(d) Please see previous diagram for first order reverse vectors.

$$\begin{aligned} \uparrow \text{Resultant} &= R_2 \sin 45 + R_3 \sin 45 - R_1 \sin 45 - R_5 \sin 45 - R_4 \sin 45 \\ &= -0.707 R \end{aligned}$$

Acting downwards the mass moment is

$$\begin{aligned} &= 0.707 ((M_{rec}) R) \\ &= 0.707 (7.2)^2 \\ &= 5.090 \text{ kg} \cdot \text{mm} \end{aligned}$$

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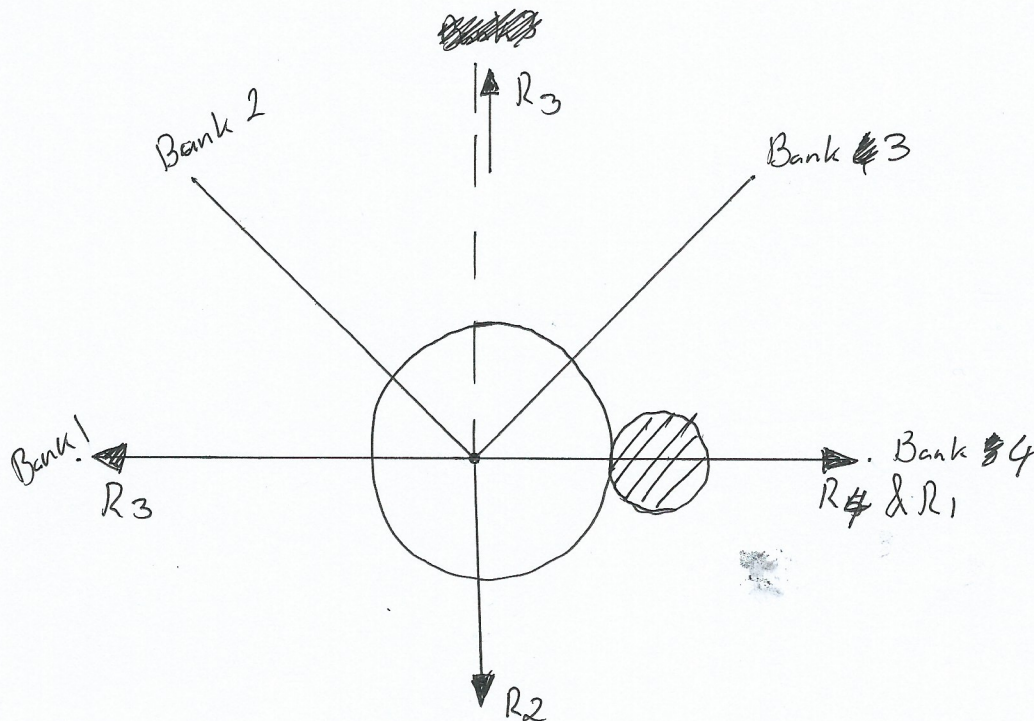
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(e) First order forward rotating vectors would be balanced using the crankshaft countermass. While the ~~reverse~~ or reverse rotating vectors could be balanced with a balance-shaft if so decided. The mass moment is ~~relative~~ relatively small thus it may be decided that ^{soft} engine mount would be good enough.



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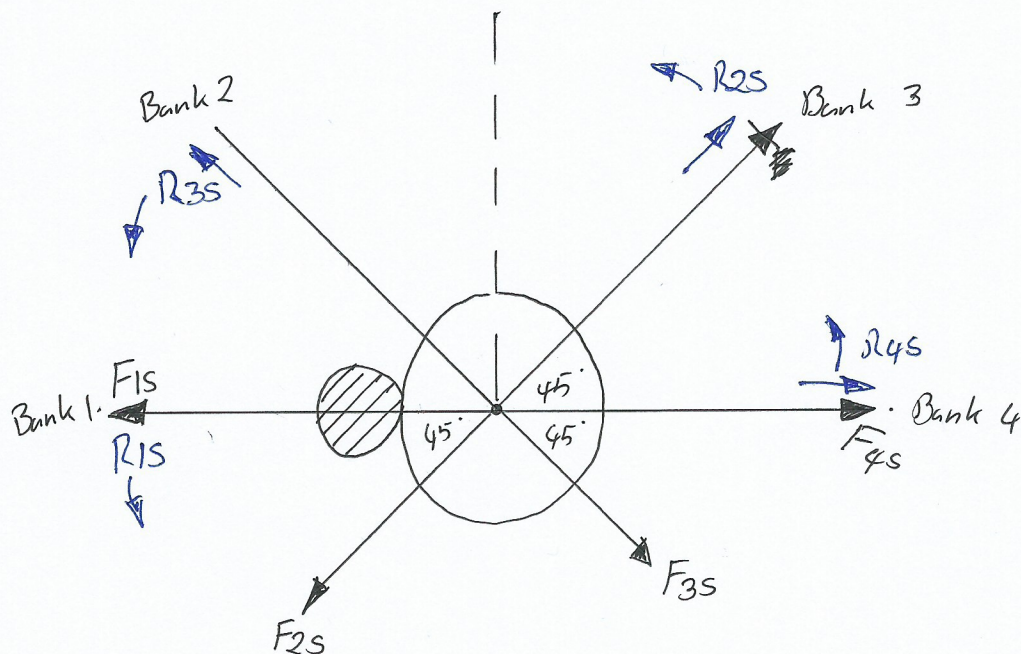
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(g)



$$F_{1s} = F_{2s} = F_{3s} = F_{4s}$$

$$\begin{aligned} \uparrow R_{\text{Resultant}} &= -F_{2s} \sin 45 - F_{3s} \sin 45 \\ &= -1.414 F \end{aligned}$$

$$\begin{aligned} \text{Downward Mass Moment} &= 1.414 \left(\frac{1}{2} \frac{R}{L} (M_{\text{rec}}) R \right) \\ &= 1.414 (0.206) \\ &= \underline{\underline{0.291 \text{ kg} \cdot \text{mm}}} \end{aligned}$$

(h) See diagram above for second order reverse vectors. (blue)

$$R_{1s} = R_{2s} = R_{3s} = R_{4s} = F_{1s}$$

$$\uparrow \text{Resultant} = 1.414 R$$

∴ mass moment is that of the forward second order vector.

$$\text{Mass moment} = 0.291 \text{ kg} \cdot \text{mm}$$

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- (i) It can be seen from the combined forward and reverse second order vector diagram in question 2g that the forces cancel each other and in fact is fully balanced despite rotation. It can be clearly seen. And therefore no second order balance shafts would be required for this configuration.

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Q3

$$M_{rot} = \frac{2}{3} (270) = 180 \text{ g} = 0.180 \text{ kg}$$

$$M_{rec} = \frac{1}{3} (270) + (240) = 330 \text{ g} = 0.330 \text{ kg}$$

$$R = 42.5 \text{ mm}$$

Part One

(a) first order forward
Mass moment at each crankspan = $(M_{rot} + \frac{M_{rec}}{2})R$
 $= (0.180 + \frac{0.330}{2}) 42.5$
 $= \underline{\underline{14.663 \text{ kg} \cdot \text{mm}}}$

(b) ~~crankshaft~~
 $M_{crankspan} = 30 \text{ kg}$ offset = x

$$M_{crankspan} \cdot x = (M_{rot} + \frac{M_{rec}}{2})R$$

$$x = \frac{14.663}{3}$$

$$x = \underline{\underline{4.888 \text{ mm}}} \quad (100\% \text{ local balance})$$

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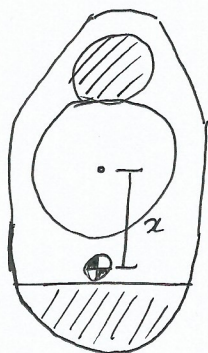
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Part One

(C)



Tungsten bolt on counter web masses can reduce the moment of inertia of the crankshaft as it is about twice as dense as steel.

$$\rho_{\text{steel}} = 7800 \text{ kg/m}^3$$

$$\rho_{\text{tungsten}} = 19300 \text{ kg/m}^3$$

$\therefore$

$$M_{\text{tungsten}} \cdot \alpha_{\text{tungsten}} = M_{\text{steel}} \cdot \alpha_{\text{steel}}$$

$$M_{\text{tungsten}} = 2 M_{\text{steel}} \quad (1)$$

$$\alpha_{\text{tungsten}} = 0.5 \alpha_{\text{steel}} \quad (2)$$

$$\begin{aligned} \text{Moment of inertia of tungsten} &= M_{\text{tungsten}} \cdot \alpha_{\text{tungsten}}^2 \\ \text{Moment of inertia of steel} &= M_{\text{steel}} \cdot \alpha_{\text{steel}}^2 \end{aligned}$$

$$\therefore \frac{M_{\text{tungsten}} \cdot \alpha_{\text{tungsten}}^2}{M_{\text{steel}} \cdot \alpha_{\text{steel}}^2}$$

Through substituting in (1) and (2) we can see,

$$= \frac{2 M_{\text{steel}} (0.5 \alpha_{\text{steel}})^2}{M_{\text{steel}} \cdot \alpha_{\text{steel}}^2}$$

$$= 2 \times 0.5 \times 0.5$$

$$= 0.5$$

That tungsten would cause half the moment of inertia of steel.

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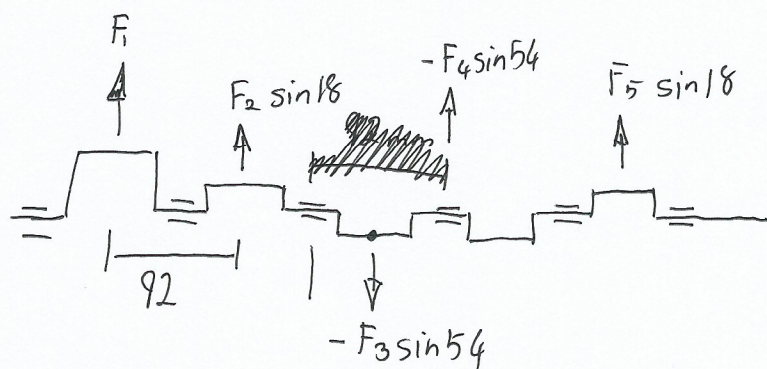
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Part 2

• With cylinder 1 at TDC moments can be taken.



$$M_{||} = \left(\frac{1}{2}F_1 \times 184\right) + \left(\frac{1}{2}F_2 \sin 18 \times 92\right) - \left(\frac{1}{2}(-F_3 \sin 54) \times 92\right) - \left(\frac{1}{2}F_5 \sin 18 \times 184\right)$$

$$F_1 = F_2 = F_3 = F_4 = F_5$$

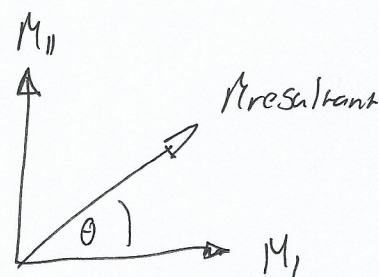
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~~$$M_{||} = 2438.996 + 208.43 + 545.68 - 416.863$$~~

$$M_{||} = 1348.996 + 208.43 + 545.68 - 416.863$$

$$M_{||} = 1686.243 \text{ kg. mm.}$$

$$M_{\perp} =$$



Note: ~~many~~ Out of time.