

POINT OF INFLEXION EXERCISE
SOLUTION

Consider the beam loaded as shown in Fig. 1.

- i. Draw the Shear Force and Bending Moment Diagrams. Values of shear force and bending moment at points A, B, C, D, E, F and G should be provided.
- ii. Determine the position and magnitude of the maximum Bending Moment.
- iii. Determine the position of any point of contraflexure.

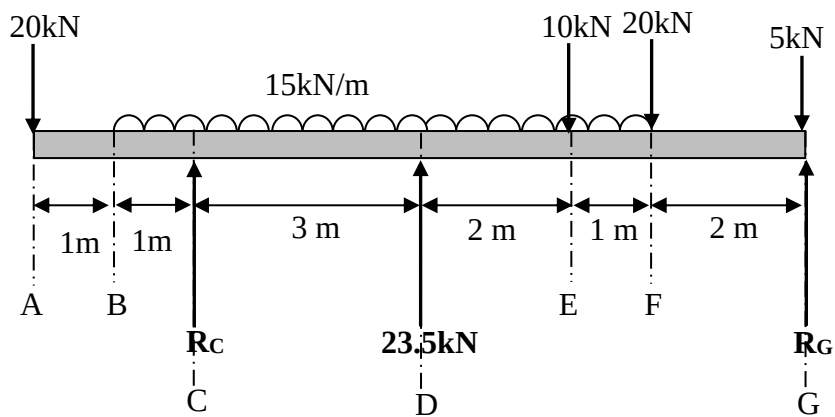


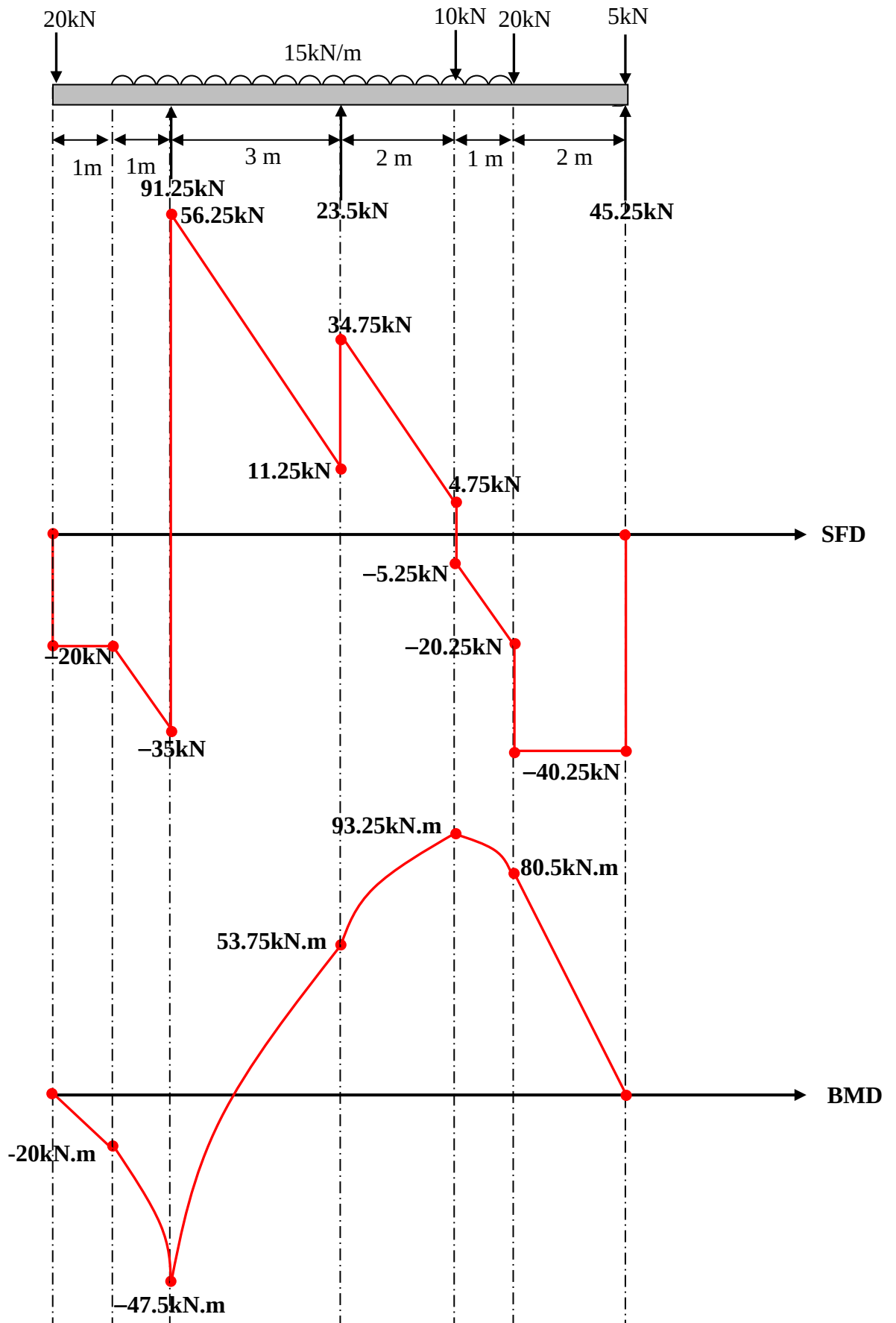
Fig. 1: Beam's applied loads and supports

Calculation of reactions R_C and R_G :

$$\begin{cases} 20 + 15 \times 7 + 10 + 20 + 5 = R_C + 23.5 + R_G \\ M_G(20) + M_G(15 \times 7) + M_G(10) + M_G(20) = M_G(R_C) + M_G(23.5) \end{cases}$$

$$\therefore \begin{cases} 160 = R_C + 23.5 + R_G \\ 20 \times 10 + 105 \times 5.5 + 10 \times 3 + 20 \times 2 = R_C \times 8 + 23.5 \times 5 \end{cases}$$

$$\therefore \begin{cases} R_G = 160 - 23.5 - 91.25 = 45.25 \text{ kN} \\ R_C = \frac{847.5 - 117.7}{8} = 91.25 \text{ kN} \end{cases}$$



At point E the magnitude of the maximum bending moment is,

$$M_E = 45.25 \times 3 - 5 \times 3 - 20 \times 1 - 15 \times 1 \times \frac{1}{2} = 93.25 \text{ kN.m}$$

From the BMD the only point of inflexion lies between C and D (BM at C negative, BM at B positive). If the required point is a distance x from C, then at that point,

$$M = -20(x + 2) - 15(x + 1) \frac{(x + 1)}{2} + 91.25x$$

$$\therefore M = -20x - 40 - 7.5(x + 1)^2 + 91.25x$$

$$\therefore M = 71.25x - 40 - 7.5(x^2 + 2x + 1)$$

$$\therefore M = 71.25x - 40 - 7.5x^2 - 15x - 7.5$$

$$\therefore M = -7.5x^2 + 56.25x - 47.5$$

Thus the bending moment BM is zero where,

$$-7.5x^2 + 56.25x - 47.5 = 0$$

That is,

$$x = \frac{-56.25 \pm \sqrt{56.25^2 - 4 \times (-7.5) \times (-47.5)}}{-7.5 \times 2}$$

$$\therefore x = 0.97 \text{ m or } x = 6.53 \text{ m}$$

Since the second answer can be ignored, the point of inflexion must be situated at 0.97 m to the right of C.