

**Complete, fold and seal**  
(see note 2 below)

**SURNAME:** CAWTE

## 12 Pages

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
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8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

|   |                              |                        |
|---|------------------------------|------------------------|
| 1 | Student Number               | 2313853                |
| 2 | Title of Examination Paper   | MATHEMATICS<br>PAPER 1 |
| 3 | Module Code                  | ACDA4008               |
| 4 | Date and Time of Examination | 09/01/2024<br>09:00    |
| 5 | Examination Venue            | IA 310                 |
| 6 | Seat Number                  | 038                    |

|   |                                  |   |
|---|----------------------------------|---|
| 7 | Number of Answer Books handed in | 1 |
|---|----------------------------------|---|

[illegible]

98%

Excellent

Q1: i)  $6x - 3x + 2x + 2y - 9y - 5y - 5z + 3z + 8z$   
 $= 5x - 12y + 6z$

ii)  $x(x+3) + -x(x-4)$   
 $x^2 + 3x - (x^2 - 4x)$   
 $= x^2 + 3x - x^2 + 4x = 7x$

iii)  $2p^2qr(4pq^3)(6qr^2) = 48p^3q^5r^3$

iv)  $\frac{12p^5qr}{4p^3q^2r^{-2}} = 3p^2q^{-2}r^3$

v)  $\frac{fg^8gh^2(32fh^3)}{16h(4fh)(12f^3)} = \frac{256f^2g^2h^5}{61768f^4h^2}$   
 $= \frac{1}{3}f^{-2}g^2h^3$

5

5

5

5

5

Q2:

i)  $5(x+3) = 2(x-8)$

$5x+15 = 2x-16$

$3x+15 = -16$

$3x = \cancel{(-16)} \cancel{+15} (-16) + (-15)$

$3x = -31$

$x = \frac{-31}{3}$

~~Ans~~

ii)  $\frac{(x-3)}{4} = x + \frac{(3x+4)}{8}$

$2(x-3) = 8x + 3x + 4$

$2x-6 = 8x+3x+4$

$2x-6 = 11x+4$

$-6 = 9x+4$

$-10 = 9x$

$x = \frac{-10}{9}$

8

12

Q3:

$$\textcircled{1} \quad 2x + 4y = 6 \xrightarrow{\times 3} 6x + 12y = 18$$

$$\textcircled{2} \quad 3x - 5y = -2 \xrightarrow{\times 2} 6x - 10y = -4$$

$$\begin{array}{r} 6x + 12y = 18 \\ - \quad 6x - 10y = -4 \\ \hline \end{array}$$

$$22y = 22$$

$$y = 1$$

$$2x + 4(1) = 6$$

$$2x + 4 = 6$$

$$2x = 2$$

$$x = 1$$

$$x = 1$$

$$y = 1$$

10

Q4:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

multiply by  
inverse

$$M = \begin{bmatrix} 1(-1-4) & 2(-6)^+ & (4--3) \\ 2(-6) & (1-9)^+ & (2-6) \\ (4--3) & (2-6)^+ & (-1-4) \end{bmatrix}$$

need to use

Signs!  
multi each element

$$C = \begin{bmatrix} -5 & 4 & 7 \\ 4 & -8 & 4 \\ 7 & 4 & -5 \end{bmatrix}$$

$$M = \begin{bmatrix} (-5) & (-4) & 7 \\ (-4) & (-8) & (-4) \\ 7 & (-4) & (-5) \end{bmatrix}$$

Explain  
why!

$$C^T = \begin{bmatrix} -5 & 4 & 7 \\ 4 & -8 & 4 \\ 7 & 4 & -5 \end{bmatrix}$$

$$|A| = 1(-5) + 2(4) + 3(7) = 24$$

$$A^{-1} = \frac{1}{24} \begin{bmatrix} -5 & 4 & 7 \\ 4 & -8 & 4 \\ 7 & 4 & -5 \end{bmatrix}$$

(5)

(9)

Q5:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 2 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$$

$$A X = b$$

$$\Rightarrow X = A^{-1} b$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{5}{24} & \frac{4}{24} & \frac{7}{24} \\ \frac{4}{24} & -\frac{8}{24} & \frac{4}{24} \\ \frac{7}{24} & \frac{4}{24} & -\frac{5}{24} \end{bmatrix}$$

(From Q4)

$$A^{-1} A X = A^{-1} b$$

$$\underline{X} = \underline{A^{-1} b}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{5}{24} & \frac{4}{24} & \frac{7}{24} \\ \frac{4}{24} & -\frac{8}{24} & \frac{4}{24} \\ \frac{7}{24} & \frac{4}{24} & -\frac{5}{24} \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2\left(-\frac{5}{24}\right) + 5\left(\frac{4}{24}\right) + 2\left(\frac{7}{24}\right) \\ 2\left(\frac{4}{24}\right) + 5\left(-\frac{8}{24}\right) + 2\left(\frac{4}{24}\right) \\ 2\left(\frac{7}{24}\right) + 5\left(\frac{4}{24}\right) + 2\left(-\frac{5}{24}\right) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$x = 1$$

$$y = -1$$

$$z = 1$$

Q6:

$$f(x) = \frac{x+2}{x^2+2x-3}$$

$$x^2+2x-3$$

$$(x+3)(x-1)$$

$$\begin{array}{r} 3 \\ 1 \end{array} \begin{array}{r} 2 \\ -1 \end{array} \begin{array}{r} -1 \\ -3 \end{array}$$

$$\frac{x+2}{(x+3)(x-1)} = \frac{A}{(x+3)} + \frac{B}{(x-1)}$$

$$x+2 = A(x-1) + B(x+3)$$

$$x \neq 1$$

$$(1)+2 = 0 + B(1+3)$$

$$3 = 4B$$

$$B = \frac{3}{4}$$

$$x = -3$$

$$(-3)+2 = A(-3-1) + 0$$

$$(-1) = (-4)A$$

$$\frac{1}{4} = A$$

$$\frac{x+2}{(x+3)(x-1)} = \frac{\left(\frac{1}{4}\right)}{(x+3)} + \frac{\left(\frac{3}{4}\right)}{(x-1)} = f(x)$$

$$= \frac{1}{4(x+3)} + \frac{3}{4(x-1)}$$

This is a  
very good  
effort Sam  
well  
Done!

20



## 12 Pages

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|   |                                  |                        |
|---|----------------------------------|------------------------|
| 1 | Student Number                   | 2313853                |
| 2 | Title of Examination Paper       | MATHEMATICS<br>PAPER 2 |
| 3 | Module Code                      | ACDA4008               |
| 4 | Date and Time of Examination     | 11/01/2024<br>15:00    |
| 5 | Examination Venue                | IQ 310                 |
| 6 | Seat Number                      | 039                    |
| 7 | Number of Answer Books handed in | 1                      |

| List the numbers of the questions you have attempted | Office Use |
|------------------------------------------------------|------------|
| Q1                                                   | 30         |
| Q2                                                   | 29         |
| Q3                                                   | 27         |
| Q4                                                   | 10         |
|                                                      |            |
|                                                      |            |
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|                                                      |            |
|                                                      |            |
|                                                      |            |
| Total                                                | 96         |

96%  
Excellent

Q1:

$$a) \frac{dy}{dx} = 24x^3 \quad (2)$$

$$ii) \frac{dy}{dx} = 12x^2 + 5x^{-1} \quad (3)$$

$$iii) \frac{dy}{dx} = -10x^{-6} + 3x^2 \quad (3)$$

$$iv) \frac{dy}{dx} = 5 \cos(5x) \quad (2)$$

$$v) \text{ Product rule: } u = x^2 \quad v = \cos x$$

$$du = 2x \quad dv = -\sin x$$

$$(x^2)(-\sin x) + (\cos x)(2x) = -x^2 \sin x + 2x \cos x$$

$$\frac{dy}{dx} = -x^2 \sin x + 2x \cos x$$

$$\text{or } x(-x \sin x + 2 \cos x)$$

$$vi) y = (9x+3)^{10} \quad u = 9x+3 \quad \frac{du}{dx} = 9$$

$$y = u^{10} \quad \frac{dy}{du} = 10u^9$$

$$10u^9 \times 9 = 90u^9$$

$$= 90(9x+3)^9$$

$$\frac{dy}{dx} = 90(9x+3)^9$$

$$b) y = \frac{1}{x} + \cos 3x = x^{-1} + \cos 3x$$

$$\frac{dy}{dx} = -x^{-2} + 3 \sin 3x$$

$$\frac{d^2y}{dx^2} = 2x^{-3} + 9 \cos 3x$$

$$\frac{d^3y}{dx^3} = -6x^{-4} + 27 \sin 3x$$

$$\frac{d^4y}{dx^4} = 24x^{-5} + 81 \cos 3x$$

$$c) y = \frac{2x+1}{3+x^2}$$

$$u = 2x+1 \quad du = 2$$

$$v = 3+x^2 \quad dv = 2x$$

$$y' = \frac{v du - u dv}{v^2} = \frac{(3+x^2)(2) - (2x+1)(2x)}{(3+x^2)^2}$$

$$\frac{6+2x^2-4x^2-2x}{(3+x^2)^2} = \frac{-2x^2-2x+6}{(3+x^2)^2}$$

$$\text{At } x=0: \frac{-2(0)^2-2(0)+6}{(3+(0)^2)^2} = \frac{2}{3}$$

$$vii) y = x + 3 \tan x$$

$$\frac{dy}{dx} = 1 + 3 \sec^2 x \quad (2)$$

(5)

(3)

(4)

(6)

Q2:

a) i)  $5x^2 + \frac{1}{4}x^4 + c$

ii)  $x^2 + 3e^x + c$

2x

iii)  $2\ln x + 4x + c$

 $4x^{-3} +$ 

iv)  $\int (x-1)(3x+4)dx = \int 3x^2 + x - 4 dx$   
 $\therefore \cancel{3x^3} + \frac{1}{2}\cancel{x^2} - 4x + c$

v)  $\int_3^6 3x^2 - 2x dx = [x^3 - x^2]_3^6$

$(6^3 - 6^2) - (3^3 - 3^2) = 162$

b)  $\int_0^1 \frac{e^x}{\sqrt{e^x+1}} dx$

$u = e^x + 1$

$\frac{du}{dx} = e^x \therefore dx = \frac{1}{e^x} du$

$\frac{4e}{2} \int_0^1 \frac{e^x}{\sqrt{u}} \frac{1}{e^x} du = \int_2^{e+1} \frac{1}{\sqrt{u}} du$

$= \int_2^{e+1} u^{-\frac{1}{2}} du = 2u^{\frac{1}{2}}$

$[2(e^x+1)^{\frac{1}{2}}]_0^1 = 2\sqrt{e+1} - 2\sqrt{2}$

$\approx 1.028142246$   
 $\approx 1.03$

c) i)  $\int x \cos x dx$

$u = x \quad du = 1$   
 $dv = \cos x \quad v = \sin x$

$uv - \int v du dx \therefore (x)(\sin x) - \int (\sin x)(1) dx$   
 $x \sin x - (-\cos x) + c$   
 $x \sin x + \cos x + c$

ii)  $\int_1^e (\ln x)(1) dx$

$u = \ln x \quad du = \frac{1}{x}$

$dv = 1 \quad v = x + c$

$(\ln x)(x) - \int x(\frac{1}{x}) dx$   
 $x \ln x - x + c$

$[x \ln x - x]_1^e$

$(e \ln e - e) - (1 \ln 1 - 1) =$   
 $e - e - (-1) = 1$

(5)

(3)

(3)

(3)

(3)

(5)

(5)

(4)

Q3: a) i)  $(3+2i)(1-i) = 3+2i-3i-2(-1)$   
 $3-i+2 = 5-i$

ii)  $\frac{3}{(2-i)} \times \frac{2+i}{2+i} = \frac{6+3i}{4+2i-2i-(-1)} = \frac{6+3i}{5}$   
 $= \frac{6}{5} + \frac{3}{5}i$

iii)  $\frac{1}{i} \times \frac{i}{i} = \frac{i}{i^2}$

$= \frac{i}{(-1)} = -i \therefore 0 + (-1)i$

b)  $z_1 = \frac{2-j}{2+j}$

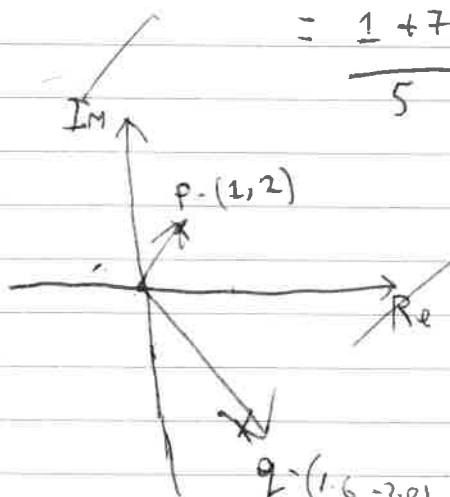
$z_2 = \frac{3+j}{1-2j}$

$\frac{2-j}{2+j} \times \frac{2-j}{2-j} = \frac{4-4j+(-1)}{4-2j+2j-(-1)}$   
 $= \frac{3-4j}{5} \therefore \frac{3}{5} - \frac{4}{5}j = z_1$

$\frac{3+j}{1-2j} \times \frac{1+2j}{1+2j} = \frac{3+6j+j+2(-1)}{1-2j+2j-4(-1)}$   
 $= \frac{1+7j}{5} \therefore \frac{1}{5} + \frac{7}{5}j = z_2$

c) i)

ii)



$(\frac{3}{5} - \frac{4}{5}j) + 2(\frac{1}{5} + \frac{7}{5}j)$   
 $= 1 + 2j$

$3(\frac{3}{5} - \frac{4}{5}j) - (\frac{1}{5} + \frac{7}{5}j)$   
 $2 \cdot (1.6, -3.8) = \frac{8}{5} - \frac{19}{5}i$

d)  $Z = (1+2i)(1+i)$

$$= 1 + i + 2i + 2i^2$$

$$= 1 - 2 + 3i$$

$$= -1 + 3i$$

$$|Z| = \sqrt{(-1)^2 + 3^2}$$

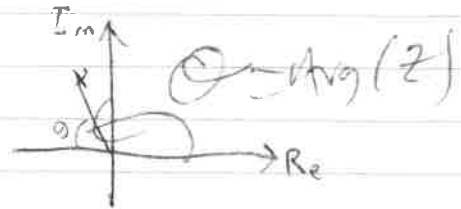
$$= \sqrt{10} = r$$

$$\tan^{-1}\left(\frac{3}{-1}\right) = \tan^{-1}(-3) = -56.3099^\circ \approx -56.3^\circ$$

$$|0| = 56.309^\circ \approx 56.3^\circ$$

$$180^\circ - 0 = 123.69^\circ \approx 124^\circ$$

$$Z = \sqrt{10} (\cos 124^\circ + j \sin 124^\circ)$$



4

e)  $\frac{3+i}{3-i} \times \frac{3+i}{3+i} = \frac{9+6i-1}{9+3i-3i-(-1)}$

$$= \frac{8+6i}{10} = \frac{8}{10} + \frac{6}{10}i$$

$$Z = \frac{4}{5} + \frac{3}{5}i$$

$$|Z| = \sqrt{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = 1$$

$$\tan^{-1}\left(\frac{(\frac{3}{5})}{(\frac{4}{5})}\right) = \tan^{-1}\left(\frac{3}{4}\right) = 36.86989765^\circ \approx 36.9^\circ$$

$$1(\cos 36.9^\circ + j \sin 36.9^\circ)$$

6

Q4:

$$a) i) 4^{2x} = 3^{x+1}$$

$$\log_{17} 4^{2x} = \log_{17} 3^{x+1}$$

$$2x \log_{17} 4 = (x+1) \log_{17} 3$$

$$\frac{2x \log_{17} 4}{(x+1)} = \log_{17} 3$$

$$\frac{2x}{x+1} = \frac{\log_{17} 3}{\log_{17} 4}$$

$$\frac{x}{x+1} = \frac{\log_{17} 3}{2 \log_{17} 4}$$

$$x = \frac{\log_{17} 3}{2 \log_{17} 4} (x+1)$$

$$x = x \frac{\log_{17} 3}{2 \log_{17} 4} + \frac{\log_{17} 3}{2 \log_{17} 4}$$

$$x - x \frac{\log_{17} 3}{2 \log_{17} 4} = \frac{\log_{17} 3}{2 \log_{17} 4}$$

$$x \left( 1 - \frac{\log_{17} 3}{2 \log_{17} 4} \right) = \frac{\log_{17} 3}{2 \log_{17} 4}$$

$$x = \frac{\left( \frac{\log_{17} 3}{2 \log_{17} 4} \right)}{\left( 1 - \frac{\log_{17} 3}{2 \log_{17} 4} \right)}$$

$$\therefore x = 0.6562889815$$

$$\approx 0.66$$

You've made  
Very hard  
Work of  
this.

4

$$\text{ii) } 2 \ln x = \ln(4x - 4)$$

$$\ln x^2 = \ln(4x - 4)$$

$$x^2 = 4x - 4$$

$$x^2 - 4x + 4 = 0$$

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(4)}}{2(1)}$$

$$= \underline{\underline{2}} \text{ (repeated)}$$

2

$$\text{b) i) } \log(x^4 y z) = 4 \log x + \log y + \log z$$

$$\text{ii) } \ln\left(\frac{x^4 y}{z}\right) = \ln(x^4 y z^{-1})$$

$$= \ln x^4 + \ln y + \ln z^{-1}$$

$$= 4 \ln x + \ln y - \ln z$$

2

2

This is a very, very good effort James, very well done!