

Improving Primary Mathematics Teaching and Learning



Edited by
Mary McAteer

9 Talk

The key to mathematical understanding?

Mike Askew

Introduction

This chapter looks at the importance of talk in mathematics teaching and learning. I argue that effective teaching – that is teaching that helps learners construct mathematical meaning rather than simply learn procedures and routines – rests on a ‘teaching tripod’ of tasks, tools and talk. Although all three of these are essential elements in mathematics lessons, talk, and productive talk in particular, is the primary means through which learners construct mathematical meaning.

Chapter aims

The aims of this chapter are to:

- Examine why talk is so important in learning mathematics and how it helps learners’ construct mathematical meaning
- Provide examples of productive talk
- Present ways to promote and support productive talk in classrooms
- Think about how lessons can best be planned and enacted to bring about productive talk.

The mathematics curriculum and children’s learning

Talk and children’s development of mathematics knowledge and understanding

As a teacher in training I was brought up on this motto:

I hear and I forget
I see and I remember
I do and I understand.

(Nuffield Mathematics Project/British Council 1978)

Complete tosh. How many things have I seen that I no longer remember? When I am awake at three in morning because something someone said to me is playing on a continuous loop in my head, where is that stuff about hearing and forgetting? What is perhaps most striking, though, is that talk is conspicuously missing from this prescription.

While I have no grandiose expectations that what I write will seep into the collective consciousness, I am aware of the possibility of some future historian, looking through quaint artefacts called 'books' ('Curious aren't they, however did they manage before Kindle implants?'), might chuckle at the naivety of my ideas. But anyway I am claiming that

Productive talk is the key to mathematical understanding.

I am not arguing that we throw away the base-ten blocks, burn the exercise books and sit round and chat. Learners of all ages need to be doing things in mathematics lessons, manipulating objects both physical and virtual. They need to be recording in images, symbols and words, on paper and on screens. I'm taking these as necessary givens. The position that I am strongly taking is that talk is what helps learners construct meaning from their actions and records. Doing and recording are the bones and muscles of learning mathematics: talk is the lifeblood that moves them.

One of the difficulties in writing about talk (or dialogue) in mathematics classrooms is that it is not monolithic; dialogue varies across three dimensions – purposes, content and context (Boerst et al. 2011). The purposes of classroom dialogue can be as diverse as examining different representations of a mathematical concept, exploring connections between seemingly different ideas, or sharing different solutions to a problem. The content of the dialogue can range through recall of facts and definitions, describing procedures, sharing different strategies. The context can be in small groups or whole class, with or without designated roles for group members, highly structured or more freewheeling.

There is not enough space here to cover all these possibilities so I restrict the focus to dialogue with the purpose of supporting growth in mathematical meaning through the content of problem solving. Within this purpose and content I look at two contexts in which learners can engage in productive talk: the private talk that takes place in pairs or small groups and the public conversations in whole class settings (Askew

2012). In private talk, learners make sense of problems, try out ideas and risk getting things wrong. In public conversations, learners collectively share and consider different solutions, examine a range of representations, strategies and methods, and connect different ideas, all arising out of their problem-solving activity. A key idea is that the private talks give rise to public conversations that move everyone's mathematical knowledge and understanding forward, rather than simply share different solutions to the problem.

I have chosen this purpose (meaning making), content (problem solving) and these contexts (private talk, public conversations) for several reasons. First, the role of the teacher is subtly different in the two contexts; supporting pairs or small groups of children as they solve problems makes different demands from orchestrating whole class dialogue arising out of these solutions. Second, problem solving is a challenging part of the curriculum, often mistakenly thought of as something children can do only after they have learnt the 'content'. Research demonstrates the power of learning content through problem solving – problem solving as a means to the end of developing mathematical meaning (Fosnot and Dolk 2001). This is difficult to enact in classrooms. Third, if children are really engaged in solving problems – that is, figuring out the mathematics that might be helpful and choosing methods or strategies, as opposed to applying a procedure to a piece of mathematics 'wrapped up in words' – then the ensuing dialogue is contingent upon the strategies and solutions that children produce. The content of such dialogue cannot be completely predicted or controlled and places demands on the teacher in the moment-to-moment unfolding of the lesson. In summary, dialogue around problem solving provides a 'critical case' that can be used to reflect upon dialogue in other parts of the curriculum.

Conceptual and theoretical perspectives

The importance of discussions in mathematics teaching is rooted in the nature of mathematics as a discipline, the ways in which students learn, and the context of teaching' (Boerst et al. 2011: 2). Boerst and colleagues neatly encapsulate three reasons for the importance of dialogue in mathematics lessons. Let us look briefly at each.

Dialogue and the nature of mathematics

It is tempting to think of the 'canon' of mathematical knowledge existing in books and journals as being different from the mathematics that

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children have to learn. But it is possible to extend the idea of learning, taking it out of the heads of individuals and thinking of mathematics, as a discipline, as 'learning' (Davis et al. 2000). For example, the history of the development of an idea like zero challenges the popular notion of the isolated mathematician suddenly discovering or inventing an idea that was unequivocally accepted by the community of mathematicians. Mathematical ideas have grown through mathematicians coming together and debating them, following false leads, rejecting and only then subsequently accepting ideas. The idea of zero in western mathematics was not widely accepted before the twelfth century and it took hundreds of years for mathematicians to become fully comfortable with the idea. From this perspective, the growth of knowledge within the community of mathematicians is analogous to the growth of mathematical knowledge within classroom communities. A main difference, however, is the time-scale: what might have taken hundreds of years in the history of mathematics might have the luxury of only a few weeks in school. I am not arguing that children have to reinvent the whole of mathematics – to go through the history of the discipline in weeks rather than centuries. I am pointing to the parallels in the importance of dialogue in each domain – mathematics as a discipline and school mathematics.

Dialogue and how children learn

The 'hear, see, do' slogan was developed at a time when Piaget's theories were influential (although whether they actually changed teachers' practices or were used to justify what was already going on in classrooms is debatable: Valerie Walkerdine (1984) presents a powerful argument in support of the latter position).

The Piagetian notion of 'stages of thinking' was powerful, especially the idea that primary-aged children were mostly in a stage of 'concrete operations' requiring 'hands on' experiences through which they would 'discover' mathematics. Although not so explicitly argued now, the view of children's thinking being qualitatively different from adults' thinking still echoes through much of primary education.

Recent research, however, challenges the 'stages' theory of thinking (Goswami and Bryant 2007), suggesting that the forms of thinking that we use as adults are pretty much in place from birth. Of course children's thinking is different from that of adults, but that is down to lack of experience rather than their thinking itself being essentially different. Young children can, and do, reason about abstract relationships (just as adults can, and do, need concrete materials to help them think). While children's thinking will grow from their direct experiences of the

world, talk is a primary means of extending and reflecting on our direct experiences.

Dialogue and teaching

The research into children's thinking suggests that Vygotsky's (1986) theories about learning and development were closer to the mark. Piagetian theories were used to argue that children had to have reached a particular developmental level before certain learning was possible; for example, children could not be introduced to algebra before reaching the formal operations 'level' of development. In contrast Vygotsky (1986) argued that learning leads development; it is through working on algebraic ideas that one becomes able to think about formal relationships. The Piagetian view of the learner was one of inside-out – the child develops (inside) and only then can engage with ideas 'out there'. The Vygotskian view is one of outside-in – through interactions with others, in contexts that the child may not initially fully comprehend, the child grows and develops into that understanding. Again, talk is central to those interactions with others.

But not any old talk will do: it has to be productive in that it builds on and develops the collective understanding. Aspects of being productive can be thought of as similar to the notion of 'accountable talk' that Lauren Resnick (2010) and colleagues introduced to highlight that classroom talk must be judged against something. *Productive classroom talk* can be accountable to three things: building the community, reasoning and knowledge (Michaels et al. 2008).

Resnick's (2010) research shows that developing accountable talk that is directed to building the community is probably the easiest to implement in classrooms. By asking, 'Who agrees with what Lynne has just said?', 'Jennie, you had a different idea, how does that fit?' or 'Who can re-explain in their own words what Russell has just said?' teachers can change the dynamic of classroom dialogue from one of 'show-and-tell' to one of collective engagement with the mathematics.

The talk that then arises also has to be accountable to reasoning: the arguments and ideas children produce must be commensurate with the logic of mathematical argument. And the talk must also be accountable to knowledge: the mathematics that emerges must eventually be correct. Resnick (2010) suggests, perhaps surprisingly, that it is easier to encourage talk that is accountable to reasoning than it is to produce talk that is accountable to knowledge. She bases this claim on the observation that children can produce well-reasoned arguments grounded in mathematical ideas that are incorrect. An example illustrates this.

reflecting on our direct

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A boy I once met visiting a lesson was listing even numbers and confident that 2, 4, 6, 8 were correct. He was equally confident that 9 was also even.

MA: How do you know if a number is even?

Jo: It's when you can split it into even towers. Look, six can be split into two even towers (*demonstrating this with interlocking cubes and showing how two towers of three were 'even' by putting his hand flat across the top of them*).

MA: So what about nine?

Jo: You can split this into three even towers (*shows three towers of three with the cubes and their 'evenness' with his hand*).

Jo's logic was not flawed: this is the reasoning behind defining composite numbers (that is, non-prime) numbers, of which the even numbers are a particular subset (those composite numbers with two as a factor). At the same time it is incorrect to say that nine is even.

As this short example shows dialogue that is accountable to the mathematical knowledge makes demands on the teacher that may be more difficult to manage than the accountability to the community or reasoning. Helping Jo make sense of his answer, to value what he had done (rather than simply 'correct' him) and to have a conversation about composite numbers depended on my mathematical understanding. Talk that is accountable to the knowledge is particularly challenging as not only does it have to steer learners' thinking in the right direction, but also at its best it has to do this in ways that still value the learner's thinking even when that thinking is incorrect.

Case studies

So what might all this look like in practice? This section presents two case studies from primary school classrooms. In each case, commentary woven into the account explores and unpicks the nature and purpose of the talk that has taken place, and its role in meaning-making.

Reflection

I invite you to look for similar examples in your own teaching and think about the role of talk.

Case Study 9.1: Division

This case study comes from a lesson with a Year 5 class (9 and 10 year olds) about division problems where the answer has to be rounded down. We (the lesson was team taught by the teacher and me) had previously introduced the children to the T-table to help them keep track of their calculations in multiplicative problems involving simple ratios. For example, putting 7 postcards to a page in an album and figuring out how many postcards would be needed to fill an album of 18 pages might involve recording this as:

Pages	Postcards
1	7
2	14
4	28
8	56
16	112
2	14
18	126

The children were not skilled in dealing with written word problems. In particular they would read the problem but not appreciate the need to go back and reread parts to help them understand it. More often than not, having read the question at the end of the problem they would answer it on the basis of what they thought might be the problem, rather than going back and rereading to check that their interpretation was correct.

We set up problems in a 'deconstructed' way to encourage the children to talk about what the problem meant rather than rush to find an answer. In pairs the children shared a paper with the opening sentence of a problem, the closing question and space for working out. Each child was given a slip with a 'clue' on it. The rule was that they could read out their clue but not show it to their partner (to discourage anyone from putting their paper on the table and opting out of working on the problem). In this particular lesson the children were working on this problem:

Anansi makes shoes for spiders.

How many boxes of spider shoes is Anansi able to fill?

Their two clues were:

Clue 1. Anansi makes 52 spider shoes.

Clue 2. Spider shoes come in boxes of 8.

Commentary

While the pairs were starting on the problem, the teacher and I made two 'tours' around the class. The first tour was to check that everyone had understood the problem correctly; children often claim to be stuck, not because they lack the resources to solve the problem, but because they haven't fully understood what they have been asked to do. If that's the case with several children, then the best thing may be to momentarily bring the class back together to clarify things with everyone.

The second tour round was to monitor everyone's progress on the task, where the teacher's role then includes giving feedback when a line of inquiry looks productive, asking questions to prompt further thinking, and suggesting possible approaches to take.

Private talk

All the children seemed to have understood the problem and were able to start, so I chose to listen in closely to one pair, Dee and Jay. Dee had only recently joined the class and I was interested to learn more about her mathematical thinking.

Dee, on hearing Jay's clue (1), immediately jotted down '52 spider shoes' and drew eight boxes (gleaned (incorrectly) from her Clue 1).

MA: (to Jay) Do you agree with what Dee has recorded?

Jay: (hesitatingly) Yes.

MA: (pointing to the eight boxes) So tell me what these represent?

Dee: The eight boxes.

MA: Oh, OK. Jay, can you remember what Dee's clue was?

Jay: I think it was that there are eight spider shoes in a box.

Dee: (reading) 'Spider shoes come in boxes of 8'.

Jay: We don't need eight boxes. We have to find out how many boxes.

Dee: Yeah, boxes of eight (which she records as $B \ O \ 8$). It's a divide.

Jay: Yes, divide (writing down $52/8$).

Dee immediately wrote down this in 'bus stop' notation, but Jay wanted to use a T-table.

MA: Why not do it both ways and see if it works out the same?

Jay set up a table:

Shoes	Boxes
8	1

...class (9 and 10 year olds) ...rounded down. We (the ...had previously introduced ...ack of their calculations ...for example, putting 7 ...ut how many postcards ...might involve recording

...en word problems. In ...ciate the need to go ...More often than not, ...they would answer it ...em, rather than going ...was correct.

...encourage the children ...to find an answer. In ...entence of a problem, ...child was given a slip ...ut their clue but not ...utting their paper on ...this particular lesson

Reflection

It would be more conventional to have this in the order:

Boxes	Shoes
1	8

I chose not to suggest that they change the order: does that matter?

Together they filled it in, doubling to get two and four boxes and adding one more to find the total for five.

Shoes	Boxes
8	1
16	2
32	4
40	5

Dee: Another box will be 48 shoes, so 7 boxes is 56.

Recording:

Shoes	Boxes
8	1
16	2
32	4
40	5
56	7

MA: So what's the answer?

Both: Seven boxes.

MA: OK, and that fits with your clues?

Dee: (pointing to the top row of the table). Yes, eight shoes in a box

Jay: And 52 shoes.

Dee: So 52 shoes, that needs seven boxes.

MA: (to Jay) You agree?

Jay: Yes.

Dee: (reading, as if to make the point clear to me!) 'How many boxes of spider shoes is Anansi able to fill?'

MA: OK, so he fills seven boxes. Lovely.

Jay: (*pausing*) No, he fills six boxes. The last box isn't full.

MA: What do you think, Dee?

Dee added a final row to the T-table:

Shoes	Boxes
8	1
16	2
32	4
40	5
56	7
48	6

Dee: There are four left over. It's six full boxes and four shoes over.

Commentary

Teacher inputs in private talk depend on careful listening in on what the children are saying to decide whether or not to join in the conversation. If the children are working productively in a direction that looks likely to yield a solution, then any intervention is likely to be minimal, although some feedback needs to be given on the approach taken and why it seems to be working. This is important not only for the particular learners getting the feedback, but also in terms of the 'cocktail party effect' – that good ideas get shared around the class. Other children will pick up and use ideas that they've heard are successful (and no, it's not cheating, it's effective collaboration).

If a pair are working well but on listening in the teacher decides their approach may be leading them away from a productive solution, then there is no easy answer as to the type of intervention to make; this will depend on many factors including judging how these particular children will cope with possibly spending time on a trail not leading anywhere. Some children are fine with this; they have the resilience to deal with temporary setbacks. If they are confident that they are on the right tracks, then trying to change their approach may not be successful anyway. Others, however, may become frustrated. The danger with stepping in too soon is that the children will stop thinking for themselves and learn to wait for the teacher to bail them out. The danger with waiting too long is that they become discouraged and see themselves as failures and not

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able to do mathematics. Careful listening is the key – getting the children to explain through in detail what they are doing and thinking often leads to them reappraising their approach for themselves.

In this particular case I held off from indicating whether or not their answer was correct, choosing instead to keep putting it onto the children to check. This was something the class was used to (and Jay in this pair). ‘Are you sure?’ was a question we regularly asked the children whether or not their answer was correct (often a correct answer is met with ‘Fine’, so children learn that being asked ‘Are you sure?’ is code for ‘You’ve got it wrong’!).

In this case the children did self-correct. What if they hadn’t? Then I might have asked pairs to join up and share each other’s methods and solutions – careful matching up of pairs can lead to groups becoming aware of different answers. Sometimes differences will get resolved in the small groups, sometimes they need to be brought up in the whole class discussion.

Dee was then keen to do the standard algorithm that she had been taught at her previous school. She was able to do this using her fluent number fact knowledge and was pleased to get the same answer. We talked about where the numbers in the standard division were represented in the algorithm and how these related to the figures in the T-table.

I told Dee and Jay that they would be coming to the front to share this second method with the class.

Public conversation

As many of the children had used a T-table to find the answer, we had one pair share this method first. Dee and Jay then came and talked through how to do the standard algorithm method.

Reflection

Some would argue that it is more efficient simply to demonstrate upfront how to do the standard division algorithm and then get the children all using this.

What is gained from working with different representations like the T-table?

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and thinking often leads
to...

...whether or not their
...it onto the children
... (and Jay in this pair).
...the children whether or
...is met with 'Fine', so
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...demonstrate upfront
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Commentary

The majority of children in class were using the T-table as a model for finding the solution to the problem. The conversation with Dee and Jay made clear that they were beginning to use this as a tool for thinking with (Gravemeijer et al. 2000). The layout of the ratios helped them realize that this was a division calculation where the answer needed to be rounded down. Although Dee was confident in her use of the division algorithm, the compact nature of the algorithm means that it can be used in a mechanical way without reading meaning into the numerals. Having the dialogue with Dee about the links with the T-table not only strengthened and confirmed her understanding, but also provided a quick assessment that she could use the algorithm appropriately and not simply as a mindless procedure. It then seemed appropriate to share this with the class.

Conclusion

The private talk in this lesson helped the children to sort out the meaning of a division problem and agree on an appropriate strategy for finding the answer. It was productive talk that was accountable both to reasoning and mathematical knowledge (of division). While the public conversation confirmed these answers it was used to take the lesson beyond simply finding the answer to a particular problem and to use the solutions to that problem as a springboard for beginning to move to using a more formal approach for division calculations. This again was productive through being accountable to the community as well as reasoning and knowledge.

Case Study 9.2: Chopsticks

This lesson was carried out with a Year 2 (6 and 7 year olds) class towards the end of the first term so the majority of children were still aged 6. As in the previous example, the teacher and I team-taught the lesson as part of a project working with a school to raise learners' attainment. The children were a diverse group, including many immigrants, so part of what we were doing was working to build up their ability to talk mathematics (for details, see Askew 2012).

We orally posed a simple problem:

Some friends go out for a Chinese meal.
How many chopsticks do they use?

We spent time at the beginning of the lesson playing with pencils to make sure everyone, even if they'd never been to a Chinese restaurant, had a sense of chopsticks coming in twos. Working in pairs, children were given a different number of people in their 'party' so that pairs might share methods but not be able to simply copy from each other.

We were interested in whether the children approached the problem by counting in twos or whether anyone realized that the answer could be found by doubling the number of people. For example, if there were 15 people having the meal, did the children add 15 and 15 or did they count up 15 twos? It turned out the class was fairly evenly split in the two approaches.

Private talk

As the children were working, the teacher and I did our 'tours' of the class. The children were confident in their problem solving and the task was one that they easily understood. So most of our attention was given to discussing which solutions we wanted shared with the whole class.

Commentary

This was the second year of working with this class and the children were confident that they could work in whatever ways they liked and could represent their working in a variety of ways. They were also used to cooperating effectively in pairs. In short, the history of the class contributed to their success. It takes time for good dialogue habits to be built up and we need to take that into account when choosing tasks and expecting children to engage with them productively.

The pairs were fairly evenly split between those doubling the number of eaters and those counting in twos. Children doubling the number of people, in the main, used a number line to find the answer, so we chose a pair whose working was typical. One pair had, however, set out the addition in a traditional column way, so we took this to be an opportunity to explore this with the class.

Several pairs finding the solution by counting in twos represented this in a fashion reminiscent of a double number line (a horizontal T-table). For example, in Figure 9.1 we have the count of the stick figures and the pairs, while Figure 9.2 shows a child's even more iconic version of this. In anticipation of drawing on this in the public conversation, we showed the pair working on Figure 9.2 how to record this as a double number line with the number of children on the top and the number of chopsticks below the line.

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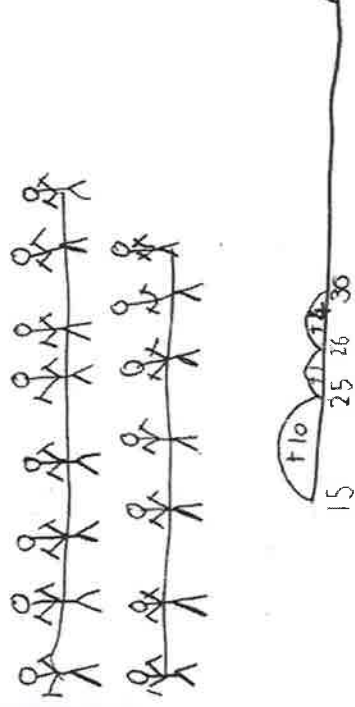


Figure 9.1 Finding the number of chopsticks for 15 people

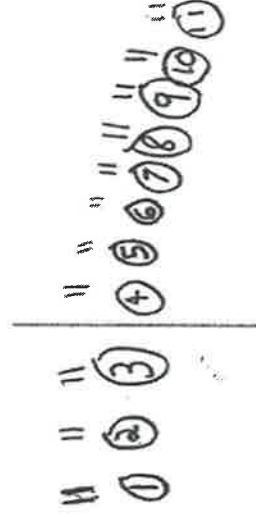


Figure 9.2 Coordinating chopsticks with people

We told all the children who we had selected to share methods to be prepared to come and address the whole class.

Commentary

Selecting which solutions methods means judging which are most likely to be beneficial to the whole class. The public conversation is not simply a time for 'show-and-tell'; it needs to be organized around mathematical ideas.

Public conversation

The first part of the public conversation was based around a pair of children coming up and sharing their number line solution and then the pair who had used column arithmetic coming and explaining that. With both of these representations on the board, we discussed the similarities and differences.

The second half of the conversation had a different focus. Those children we had primed came and recreated the double number line on the interactive white board (Figure 9.3). The class then engaged in a dialogue about how this double number line could be used to answer a whole range of chopsticks problems, such as:

$$\begin{array}{l} \text{Double } 50 \\ = 100 \end{array} \quad \begin{array}{l} 100 = 50 \times 2 \\ 100 = 2 \times 50 \end{array}$$

Bradley

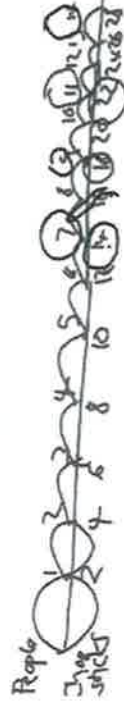


Figure 9.3 Working with a double number line

- If there were 12 people how many chopsticks would they use?
- If 18 chopsticks had been used, how many people was that?

In their pairs the children posed similar problems for each other to share.

In the light of the discussion we were able to 'push' the children's thinking further. Suppose, we said, we could extend the double number line further:

'There's not enough room on the board, but imagine we continued until there were 50 people on the top. How many chopsticks would that be and what mathematical sentence could we use to record this?'

Back to a private talk in pairs, the children quickly figured out that the answer would be 100. Together again as a whole class, we recorded the different ways children thought this could be expressed:

$$\begin{array}{l} 50 + 50 \\ 2 \times 50 \\ \text{Double } 50 \\ 50 \times 50 \\ 50 \times 2 \end{array}$$

different focus. Those children who were on the interactive board in a dialogue about how many chopsticks

$$50 \times 50 = 2500$$

$$50 \times 2 = 100$$

$$50 \times 50 = 2500$$



chopsticks would they use?
any people was that?

each other to share.
the children's thinking
able number line further:

Imagine we continue
how many chopsticks
could we use to

and out that the answer
recorded the different

Again we gave them a few minutes of private talk to decide which of these would be acceptable. Everyone agreed that they would all be fine except 50×50 which was going to be much too big. As we were clearing up, a boy came up and quietly said that 50×50 would be 'two thousand five hundred'. Of course we stopped the clearing up so that he could share this answer with the class and how he'd worked it out:

Well, ten times fifty is five hundred, so twenty times is one thousand, another ten is one thousand five hundred, another ten two thousand so fifty fifties is two thousand five hundred.'

Commentary

The role of the teacher in public conversation is complex. It involves attending to the solution being offered by the children who have come to the front but doing this in such a way as to draw in the rest of the class to be thinking about and engaging with the solution being offered, not simply listening politely in the hope (or fear) that they may be called upon next. The public conversation needs to be crafted around carefully chosen examples, which means not seeking volunteers. The examples may be chosen for a variety of reasons:

- Different representations that can be used to help children make links between
- Different solutions that need reconciling
- Different children having the opportunity to speak and share with the whole class.

Managing this whole class discussion in part involves putting yourself, as the teacher, in the position of being one of the other children and trying to imagine what it must be like to be hearing the solution for the first time; acting as an advocate for these listeners in prompting the children explaining to give an account in such a way that the rest of the class can engage with their solution. This is easier said than done. As teachers, having listened to and talked with the children prior to their presenting makes it easier, to be able to make sense of their explanation through 'filling in the gaps'. But other children in the class may be struggling to make sense of the explanation because they neither have the knowledge of the mathematics nor the benefit of prior conversations.

Public conversation can be less than productive when it turns into a private talk between the teacher and the children presenting. Simple things like moving to the back of the class so that the children have to

face the class rather than talk to you at the side of the board can make a big difference.

Finally, in this example, the direction that the public conversation took, together with the support of the introduction of the double number line, enabled one child to demonstrate a sophisticated level of mathematical reasoning that might not otherwise have been revealed.

Conclusion

The whole class discussion plays a key role in crafting and bringing out the methods and making the mathematics collectively 'owned'. The focus on the whole class discussion needs to be on the mathematics, not on particular children. The spirit needs to move from one of this is 'their' solution, to 'how does this help us all with our mathematics?'

Summary

It is tempting to try to 'script' dialogue in advance, particularly by pre-determining the questions that one is going to ask. However, if teaching is going to be truly interactive, then responses (not just from the teacher but from the other children) depend upon what emerges in the lesson and so are not completely pre-scriptable (pre-dictable, literal origin of 'say before'). However, this observation is not a recommendation for 'anything goes'. The lessons from which these cases are drawn were planned with clear intentions; the first in the connection between multiplication and division in the context of division problems that involved rounding down, while in the second to explore the connection between counting in twos and doubling.

As part of this planning the problems that the children were going to engage in were carefully chosen as ones that had 'low thresholds' (McClure 2011) in the sense that we expected that all the children would be able to make sense of the problems and find some way of solving them, even if that was mathematically 'naive'. But the problems also had reasonably 'high ceilings' in that they could lead to some quite sophisticated mathematics. 'Anansi' for some children simply helped their understanding of the nature of division problems, for others it introduced them to a more efficient calculation strategy. For some children 'Chopsticks' would (and indeed did) only provide some consolidation in counting in twos, while others began to make some connections between doubling and counting in twos and so some understanding of the connection between multiplication and division. We could not know in advance, however,

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ence, particularly by pre-ask. However, if teaching not just from the teacher at emerges in the lesson attable, literal origin of a recommendation for cases are drawn were connection between multi-problems that involved the connection between

the children were going at had 'low thresholds' at all the children would one way of solving them, problems also had real-ome quite sophisticated helped their understand-it introduced them to a when 'Chopsticks' would on in counting in twos, between doubling and the connection between in advance, however,

which children would learn what. We were, however, confident that everyone would learn something.

Such a view of learning outcomes is different to some popularly held advice that learning outcomes can be closely pre-specified at the beginning of a lesson. This may be possible if children are being trained to do something. I can be reasonably confident that over the course of a lesson I can get most children to learn to measure angles using a protractor. But developing understanding or making meaning of mathematics is not within the control of the teacher and unfolds over time. Good lessons built around problem solving and dialogue works with the fact that in any class there will be a range of understandings (even in the most carefully 'settled' group) and that the rate at which these understandings grow and develop is down to too many factors for a single teacher to control.

In summary, productive classroom talk has to embody at least three things:

- It has to have a clear focus: for example, a solution to a problem, particular representations, and connections between ideas. This focus has to be sufficiently rich to sustain dialogue and to promote collective mathematical activity.
- Everyone has to be involved in speaking and listening – engaging in collective mathematical activity – in pairs, small groups and whole class settings.
- The teacher has a dual role here. The first is to be an active member of the collective – listening, prompting, questioning and manager of the discussion. The second is to provide 'space' for everyone to contribute, judging how 'on board' everyone is, encouraging a productive ethos.

Further reading

- Fosnot, C.T. and Dolk, M. (2001) *Young Mathematicians at Work: Constructing Multiplication and Division*. Portsmouth, NH: Heinemann.
- Fosnot, C.T. and Dolk, M. (2001) *Young Mathematicians at Work: Constructing Number Sense, Addition and Subtraction*. Portsmouth, NH: Heinemann.
- Fosnot, C.T. and Dolk, M. (2002) *Young Mathematicians at Work: Constructing Fractions, Decimals and Percents*. Portsmouth, NH: Heinemann.

There are three titles in the *Young Mathematicians at Work* series. Each book has some detailed case studies of children talking about their

solutions to problems, with commentary on the importance of talk in promoting learning and the role of the teacher.

Askew, M. (2012) *Transforming Primary Mathematics*. Abingdon: Routledge. In this book I elaborate on the ideas introduced here. As well as providing further theoretical background, I look at how talk needs to be coordinated with tasks and tools.

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