

Suspension Behaviour

Ride characteristics

Frequency and compromises

Pitch control

Damping

Ride Comfort

Summary of comfort data

The following results are summarised from Car Suspension and Handling - Bastow and indicate the vibration levels acceptable to drivers and passengers. Generally the aim is to keep suspension frequencies below 2.0 Hz for ride comfort and above 1.0 Hz. for physical well being.

0.5 to 0.75 Hz	Dizziness and sickness produced
5.0 to 7.0 Hz	Internal organs resonant frequency
18 to 20 Hz	Head and neck sensitivity
1.5 Hz	Longitudinal and lateral acceleration sensitivity

Types of vehicle vibration

"Body Bounce" body bouncing on springs up to	c.2.5 Hz
"Wheel hop" wheel hops on road above	c. 6 Hz
"Boulevard Jerk" body and springs effectively solid and tyres act as the sole springing medium which is resonant at above	c. 12 Hz.

Sprung Mass Behaviour

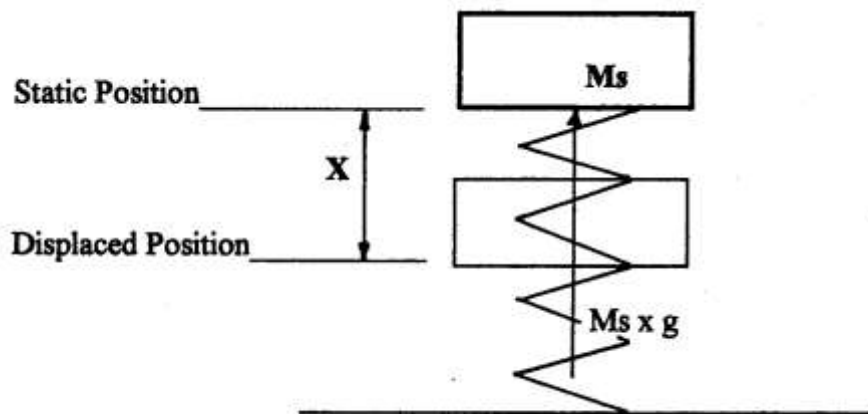


Fig 1

With reference to the simple spring and mass system in Fig 1. In the static case the force exerted by the weight of the car ($M_s \times g$) is balanced by the force exerted by the spring. If the mass is displaced by X the spring force will increase to:

$$(M_s \times g) + KX$$

where K = **spring stiffness**.

The motion of displaced mass on being released is Simple Harmonic.

Review of Simple Harmonic Motion

Fig. 2 shows point P moving at constant angular velocity about point O. The projection of this point to a linear representation of time T gives simple Harmonic Motion.

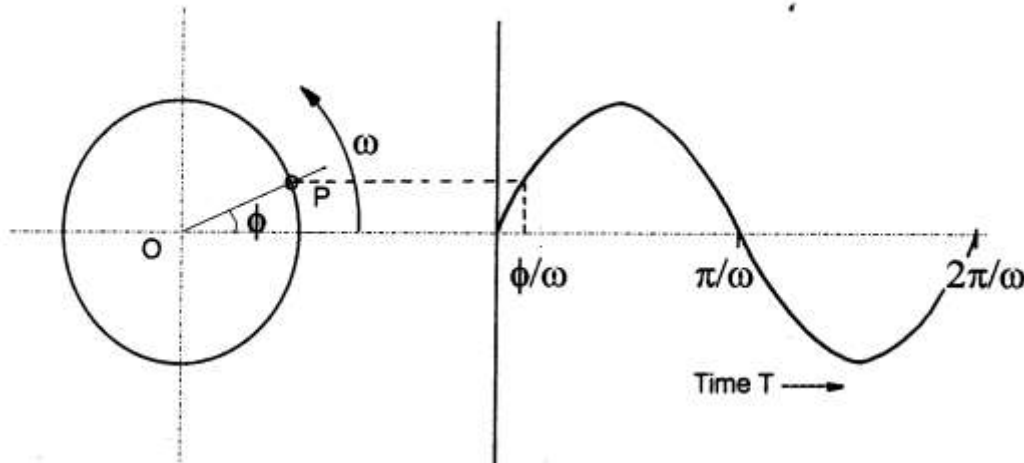


Fig.2

SHM with Springs

Equation 1
$$f = \frac{\omega}{2\pi}$$

Equation 2
$$f = \frac{1}{2\pi} \sqrt{K / M_s}$$

Equation 3
$$f = \frac{1}{2\pi} \sqrt{g / d}$$

Where

- M_s = mass acting on spring (sprung mass) [kg]
- f = natural frequency [Hz]
- d = deflection [m]
- g = acceleration due to gravity [9.81m/s]

Acceleration transmitted to the sprung mass by a step input to the spring

Take the example of a wheel meeting a step. By re-arranging Equation 3 the acceleration transmitted to the body is given by :-

Equation 4
$$a = 4 f^2 \pi^2 d$$

Example:

In the case of a suspension with a natural frequency of 1.25 Hz meeting a step of height 0.05 m the acceleration transmitted to the sprung mass would be 2.46 m/s^2

If the spring rate were increased to give a natural frequency of 2.5 Hz the acceleration would be 12.34 m/s^2 ie four times greater.

It can be seen that the acceleration transmitted increases with the square of the suspension frequency.

Wheel rate (Kw)

Wheel rate can be defined as the effective spring rate seen at the wheel. For example, if the spring acted vertically over the centre of the wheel, the wheel rate would equal the spring rate (K_s). When analysing the behaviour of suspension systems it is convenient to deal with the wheel rate which then takes into account any suspension leverage or spring inclination that is present.

When dealing with automotive suspension systems all the previous equations should be used with K_w as the spring rate to take account of the suspension linkages employed.

Equation 2 then becomes

$$f = \frac{1}{2\pi} \sqrt{\frac{K_w}{M_s}}$$

Where f is the natural frequency of the suspension

This is an approximation of the true frequency which is modified by the damping in the system but the error is relatively small where the damping is close to critical (this will be explained later)

SUSPENSION FREQUENCY .

Obviously, by transposing this equation, K_w can be found from :

Equation 5
$$K_w = (2\pi f)^2 \times M_s$$

Maintenance of wheel contact

Suspension	Wheel Rate	Static deflection under 300kg
Frequency [Hz]	[N/mm]	[mm]
1.00	11.8	248
1.25	18.5	159
1.5	26.7	110
1.75	36.3	81
2.00	47.3	62
2.5	74	40

Table 1

Table 1 demonstrates the effect of suspension frequency on static deflection and spring rate.

For example, a car of sprung mass 1200 kg equally distributed front and rear has a front suspension frequency of 1.5 Hz. and so a static deflection - 110 mm

If a dip in the road has a depth of 150mm then, as this exceeds the static deflection the wheel, it loses contact with the road (the body is assumed to be maintaining a relatively constant height due to its inertia. A suspension with this level of static deflection would require around 300mm of travel which may cause problems with camber change etc.

By softening the spring and so reducing the natural frequency to 1.25Hz the deflection of 159mm will allow the ripple to be followed by the tyre.

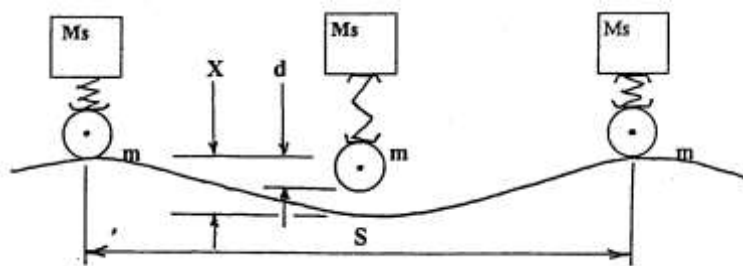


Fig 3 Showing limitation of road contact in a ripple by initial static deflection of the suspension

In practice the suspension travel is often (but not always) constrained by the damper stroke (or when independent springs and dampers are used by straps). This has the effect of using up the initial part of the travel and so the movement of the wheel from full droop to static ride height is not as great as the simple compression of the spring would suggest. The depth of ripple that can be followed would also be limited in the same way. An alternative is to use a short, low rate helper spring in series with the main spring. This becomes coil bound when the car is sitting on

the road and so plays no part in determining the effective wheel rate at the static ride height. It does, however, allow the wheel to travel further on droop than the static ride wheel rate would suggest and so maintain wheel contact into dips and on the inner wheel of a corner. This is especially useful on rally cars where long suspension travel is required but the low wheel rates required to achieve this would make the car unstable. Open wheel racing cars and sports racing cars often have very high wheel rates and so very limited wheel travel and in these situations the behaviour of the tyre as the primary spring mechanism becomes increasingly significant.

Maintaining contact with the road at speed

The natural frequency (f_{us}) of the UNSPRUNG mass (M_{us}) under the action of the suspension spring on one side and the tyre (which acts as a spring) on the other is given by :-

$$f_{us} = \frac{1}{2\pi} \sqrt{\frac{K_w + K_t}{M_{us}}}$$

Where K_t is the tyre rate

Using this equation Table 2 shows the relatively small influence that suspension frequency actually has on the natural frequency of the unsprung mass for a car with the following data:-

$M_s = 400\text{kg}$ $M_{us} = 40\text{ kg}$ $K_t = 256\,000\text{ N/m}$

Natural frequency of sprung mass f [hz]	K_w [N/m]	Natural frequency of unsprung mass f^* [hz]
1.0	15796	13.1
1.5	35540	13.6
2.0	63182	14.2
2.5	98723	15.0

Table 2

Now take the example of the same car with a natural suspension frequency of 1.5 hz. travelling at 100kph Table 3 shows the influence of unsprung mass on the minimum spacing of ripples for matching of road input frequency and natural frequency of the unsprung mass

Unsprung mass Mus [kg]	Natural frequency of unsprung mass fus[hz]	Separation of ripples for exact matching of wheel input frequency and natural frequency of unsprung mass [m]
20	19.2	1.5
30	15.7	1.8
40	13.6	2
50	12.2	2.3

Table 3

It can be seen that by reducing the unsprung mass (and so raising its natural frequency) the ripple separation that can be accommodated will be made less. In practice this means that the wheel can maintain contact with the road surface, even if it is not smooth, at higher speeds. This will translate into better road holding and is the reason so much effort is made to reduce unsprung mass in suspension systems despite the added cost this usually incurs.

Special case of ‘Washboard’ Surface



Showing ‘washboard’ surface which has developed on an unmade road in Southern Spain.

Regular ripples form on unmade roads with heavy vehicular traffic. Drivers find that they can drive with more comfort at certain speeds. The ripple spacing can be anything from 0.2 m to 0.7 m . In the case of this particular section of road, cars were observed to be travelling at around 50kph or 13.9 m/s. The ripple separation was approximately 0.5m . If the wheel mass is taken to be 40kg and the unsprung mass 400kg then Table 3 shows that the natural frequency of the unsprung mass will be approximately 13.6 Hz.

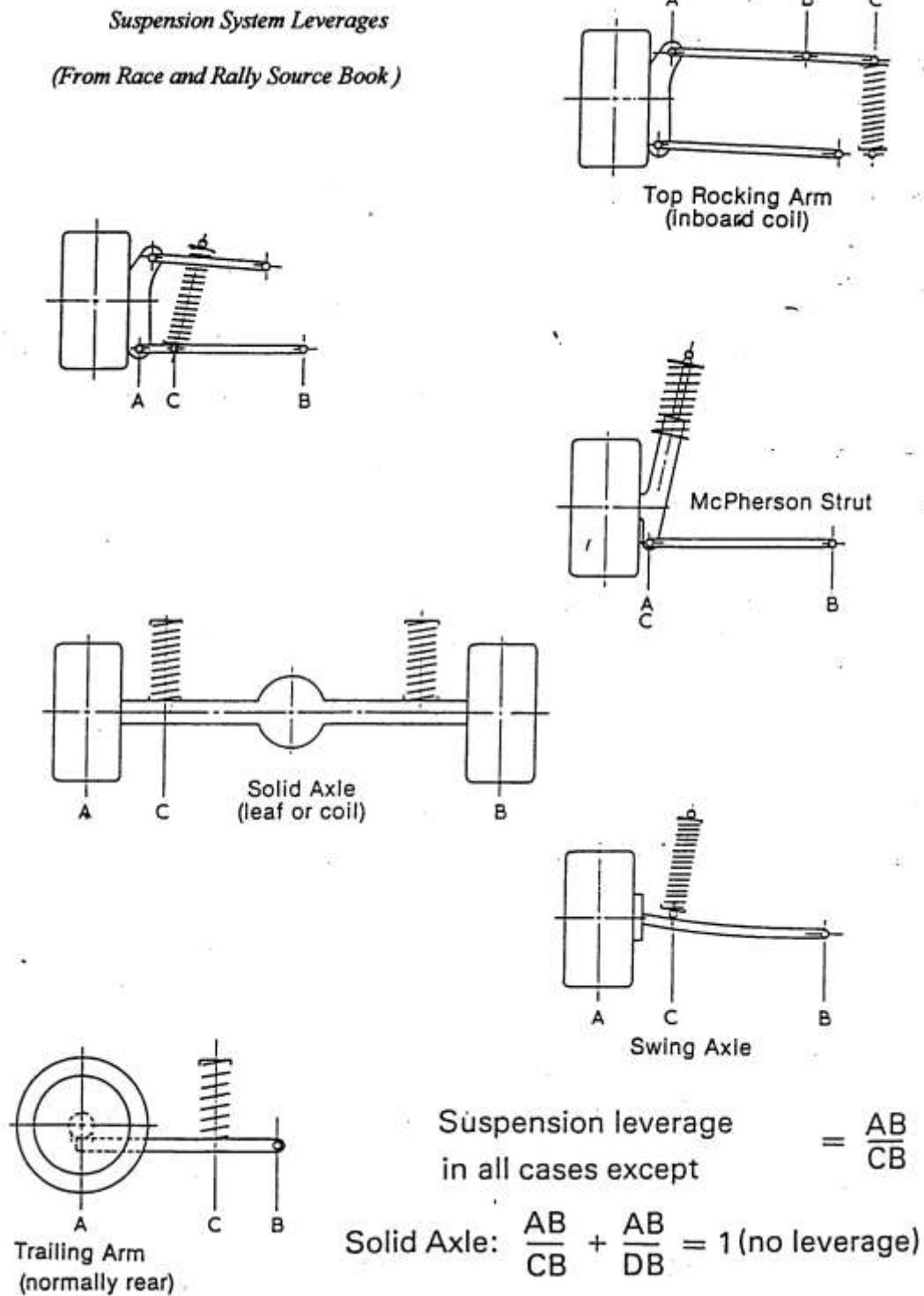
The speed for matching the frequency of the ripples in the road with the natural frequency of the unsprung mass is found from

$$\text{Speed} = \text{Spacing} \times \text{Frequency}$$

So in this example this would be 6.8 m/s ie 25 kph.

This is twice the typical road speed observed. The multiple of unsprung mass frequency that provides greatest comfort will depend on a number of factors such as the natural frequency of the suspension, damping, polar moment of inertia of the sprung mass etc. The limiting factor is that if too many ripples are 'jumped' at a time the tyre will spend less time in contact with the road and so directional stability will suffer.

Fig 4



Method used to determine spring specification

If the spring is mounted perpendicular to the suspension arm the **Suspension Leverage** (this is termed Velocity Ratio in WinGeo3) can be determined using Fig 7 as a guide for various suspension systems and is given by:

$$SL = \frac{AB}{CB}$$

Where:

AB = Distance from suspension arm pivot to outer ball joint or wheel centre line

CB = Distance from suspension arm pivot to spring centre line

If the spring is inclined relative to the perpendicular to the suspension arm then the value of the suspension leverage will increase to take account of the reduced travel of the spring and is then given by :

$$SL = \frac{AB}{CB} \times \frac{1}{\cos\theta}$$

Where:

θ = Inclination of spring to the perpendicular to the suspension arm

If the spring rate is known then the wheel rate can be determined from:

$$K_w = \frac{K_s}{SL^2}$$

Where: K_w = Wheel Rate K_s = Spring Rate

The suspension frequency can now be found using Equation 2

It can be seen that if the spring is mounted vertically at static ride height then as the suspension is deflected on bump the spring will be inclined away from the perpendicular to the suspension arm and the resultant wheel rate will reduce with suspension travel leading to a falling wheel rate. This is in contrast to the ideal requirement for a suspension to have a rising or at least a constant rate

Having obtained the spring rate, the free length of the spring is now required to enable the correct spring to be installed. As a spring of the correct rate could come in any length a method to determine how far the spring is compressed on static load is required. Knowing the dimension between spring pans in this static load position and then adding the static load compression will give the free length.

The static compression is given by:

$$SC = \frac{W_s \times SL}{K_s}$$

Where:

SC = Static Spring Compression

W_s = Sprung Weight = $M_s \times g$

The spring free length is given by:

$$FL = SC + PS$$

Where:

FL = Spring Free Length

PS = Spring Pan Separation at static load condition.

Clarification of the relationship between spring rate and wheel rate.

(Refer to any diagram in Fig 4)

Vertical displacement at A = X

Vertical displacement at C = Y

Load acting on spring = F

$$\frac{X}{AB} = \frac{Y}{CB}$$

$$\text{displacement at A} = X = \frac{AB \times Y}{CB}$$

$$= SL \times Y$$

$$\text{Force at A} = Kw \times X$$

$$= Kw \times SL \times Y$$

Taking moments about B

$$F \times CB = Kw \times SL \times Y \times AB$$

$$F = Kw \times SL^2 \times Y$$

$$Ks = \frac{F}{Y}$$

$$Ks = \frac{Kw \times SL^2 \times Y}{Y}$$

$$\text{Wheel Rate Kw} = \frac{Ks}{SL^2}$$

Relationship between Spring Compression and Sprung Weight

$$\text{Wheel deflection} = \frac{Ws}{Kw}$$

Spring compression

$$SC = \frac{\text{Wheel deflection}}{SL}$$

$$SC = \frac{W_s}{K_w \times SL}$$

But

$$K_w = \frac{K_s}{SL^2}$$

$$SC = \frac{W_s \times SL^2}{K_s \times SL}$$

$$\text{Spring Compression } SC = \frac{W_s \times SL}{K_s}$$

Interaction between front and rear suspensions

As the front suspension meets a road bump first, the car will always be made to pitch.

The aim of suspension and vehicle design is to limit the level and duration of that pitch as this was found (by M. Olley in the 30s) to be the factor most likely to produce discomfort.

The curves of spring movement shown in Fig 5 show the relationship between front and rear suspension frequency and pitch behaviour and are drawn from the relationship:

$$X = Ae^{-nt} \cos(\omega t)$$

Where:

- X = Displacement of the sprung mass after time t [m]
- A = Initial displacement [m]
- e = Euler's constant = 2.718
- n = Damping factor (defined by Bastow as weight / twice damping force) for the purpose of this exercise a value of 0.8 gives satisfactory results
- t = time after initial displacement [s]
- C = Natural frequency of the suspension [rad/s] (i.e. $2\pi \times f$)

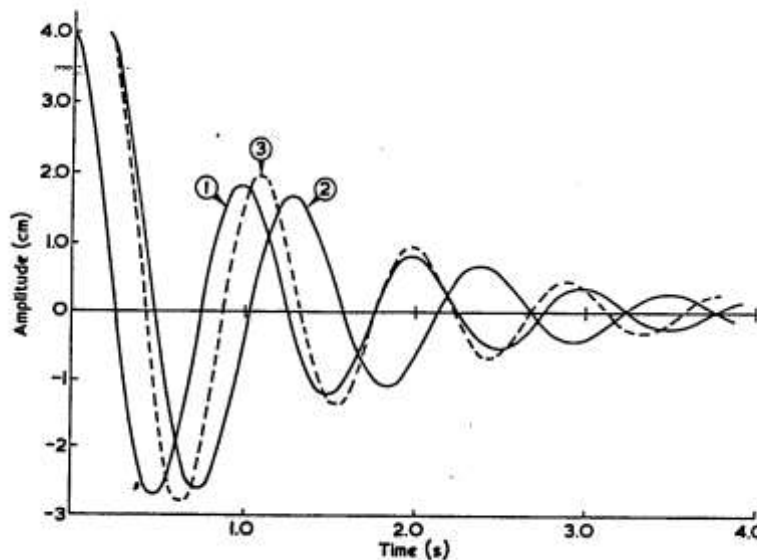


Fig 5 Amplitudes for damped movement of front and rear of sprung mass from single disturbance: Curve 1, front, frequency 1 Hz; Curve 2, rear, frequency 0.909 Hz; Curve 3, rear, frequency 1.11 Hz. Car speed 50 kph.

The curves in Figs. 6 and 7 represent the resulting pitch movement produced by different combinations of front and rear spring frequency at 100 kph (Fig 6) and 50 kph (Fig 7). They are produced by subtracting the displacement of the rear sprung mass from that of the front after a time interval, dependent on wheelbase and road speed (0.1 seconds in Fig 6 and 0.2 seconds in Fig 7). Downward displacements on these graphs correspond to the front end below the rear.

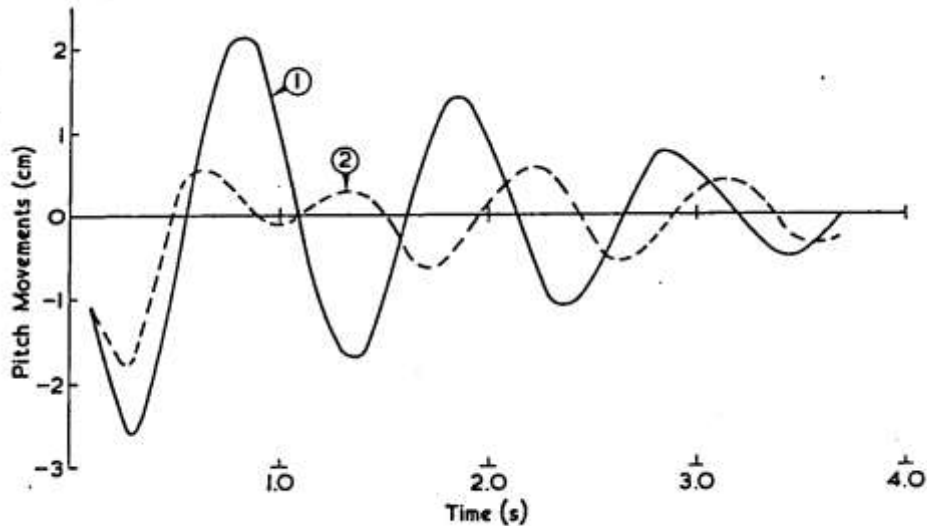


Fig 6 Resulting pitch movements. Car speed 100kph: Curve 1, lower rear frequency; Curve 2, higher rear frequency.

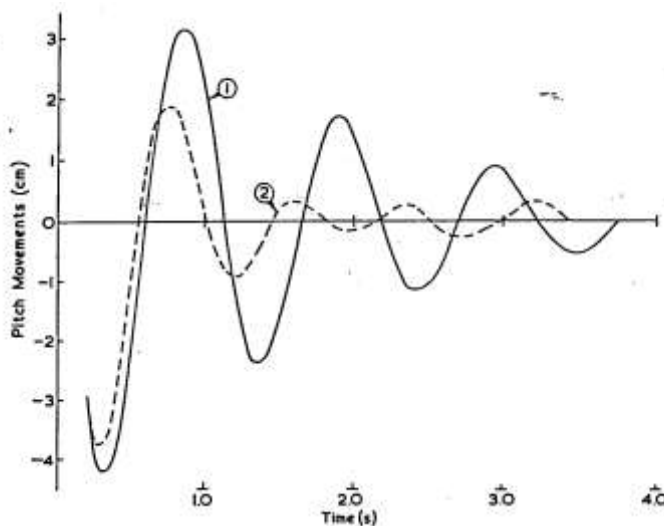


Fig 7 Resulting pitch movements. Car speed 50 kph: Curve 1, lower rear frequency; Curve 2, higher rear frequency.

The following points should be noted:

- (1) With the lower frequency rear suspension there is a strongly persisting pitch movement.
- (2) With the higher frequency rear suspension the initial pitch movement is very quickly reduced.
- (3) With the higher frequency rear suspension the initial pitch movement is never so severe as with the lower frequency.
- (4) The initial pitch movement is less severe at the higher than at the lower speed, because the time interval is less. This time interval is at both speeds less than half a cycle of the front suspension.

It can be seen from the curves that for minimum pitch the rear frequency should be higher (i.e. the spring is stiffer) than the front frequency. A problem with this is that to maintain roll stiffness at the front an anti roll is often needed.

An example the ratio of front to rear frequencies used on the old Jaguar XJ6 was 1:1.6.

Early Ford Fiesta suspension frequencies:

-with driver only	Front Freq. = 1.55
	Rear Freq. = 2.17
	Pitch Freq. = 1.96
-with full load	Front Freq. = 1.45
	Rear Freq. = 1.69
	Pitch Freq. = 1.56

To minimise pitch amplitude and duration the rear suspension frequency needs to be higher than that of the front.

Mass Distribution

The distribution of the mass of the body will affect the **pitch frequency**

If the masses are set far away from the centroid of the vehicle there will be greater resistance to pitching due to inertia of those masses than if they were close to the centroid.

If the moment of inertia of each small mass δm about the CG is taken to be :-

$$r^2 \delta m$$

then the total moment $I = \sum r^2 \delta m$

$$I = MK^2$$

where:

K = Radius of gyration (i.e. radius at which all small masses can be considered to be concentrated relative to the axis of the centroid.)

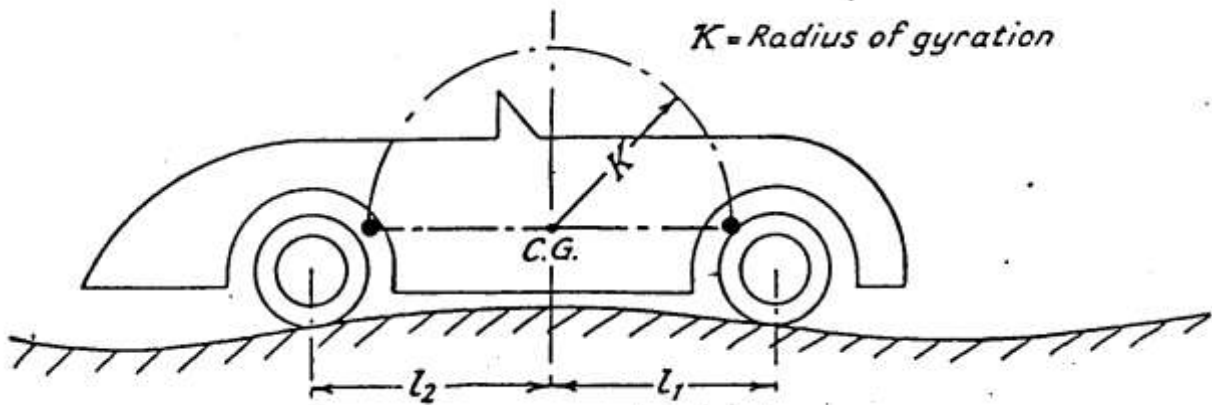


Fig 8 Radius of gyration of a vehicle

The period of the pitch frequency is given by:-

$$T' = 2\pi \sqrt{\frac{Ms}{Kw} \times \frac{K^2}{I_1 \times I_2}}$$

Where:

T' = Pitch frequency

The aim of suspension design is to achieve bounce frequency = pitch frequency.

i.e.
$$\mathbf{K}^2 = \mathbf{I}_1 \times \mathbf{I}_2$$

or
$$\frac{\mathbf{K}^2}{\mathbf{I}_1 \times \mathbf{I}_2} = 1$$

This is required for the following reasons:-

1. If $\frac{\mathbf{K}^2}{\mathbf{I}_1 \times \mathbf{I}_2}$ is less than 1 pitch frequency increases.

Longitudinal acceleration of the head and neck will increase with pitch frequency

Aim of suspension design is to have low pitch frequency, so $\frac{\mathbf{K}^2}{\mathbf{I}_1 \times \mathbf{I}_2}$ is made close to 1

2. The higher the polar moment of inertia, **I**, the greater will be the resistance to initial pitch of the sprung mass on its suspension. As $\mathbf{I} = \mathbf{MK}^2$ then for a given vehicle mass this resistance is dependent on K which should be large enough to ensure $\frac{\mathbf{K}^2}{\mathbf{I}_1 \times \mathbf{I}_2}$ is close to 1.
3. Physical limitations exist but if **K** is too large and $\frac{\mathbf{K}^2}{\mathbf{I}_1 \times \mathbf{I}_2} > 1$ then yaw behaviour on road-responsiveness and stability will be affected.

N.B.

Limitations on how close pitch and bounce frequencies can be made is the changing load conditions that will be experienced and the need for different front and rear frequencies to minimise amplitude and duration of pitching motion.

Damping

The natural frequency of the suspension is designed to be as low as possible within the constraints imposed by the need to maintain road/wheel contact and passenger comfort. Any vibration above that frequency produced by road ripples, bumps etc. will naturally be attenuated. At the natural frequency of the suspension (under the sprung mass of the body) road induced vibration will cause the body to resonate.

Suspension damping is therefore introduced to provide a force opposing the road shock (bump) and a force opposing the extension of the spring after deflection(rebound). However, any such force will tend to increase harshness of ride.

Damping Constant B

This is a measure of the degree of damping produced and is usually taken as the slope of the force against speed curve which for a road car will typically be in the region of 2kN s/m.

Critical Damping Constant B_c

This is the damping necessary to prevent the initial displacement of the sprung mass causing an oscillation that crosses the original equilibrium line.

B_c for a simple sprung damped mass is given by:

$$B_c = 2 \times \sqrt{Ms \times K} \quad [\text{Ns /m}]$$

Where :

M = Sprung mass

K = Spring rate

Critical damping force

It can be seen that, by multiplying B_c by damper piston velocity **v** in m/s the critical damping force at a given wheel speed can be found i.e.

$$\mathbf{Fdc} = \mathbf{B} \times \mathbf{v} \quad [\text{N}]$$

The force at the damper is related to the force at the wheel by a factor of SL² (in the same way that wheel rates and spring rates are related)

Typically the ratio of damping constants Rebound / Bump will be in the region of 70/30

Damping Factor ζ

This is a measure of the degree of damping being used and is effectively the ratio of the damping coefficient actually used to that required for critical damping i.e. ζ = B/B_c

Critical damping is regarded as too severe for a practical passenger car suspension but does give a yardstick by which the damping given to a practical system can be expressed.

For competition cars critical damping (i.e.when $\zeta = 1$) is regarded as being close to the level of damping required . The actual ratio used in passenger vehicles for road use is generally between 0.2 and 0.6.

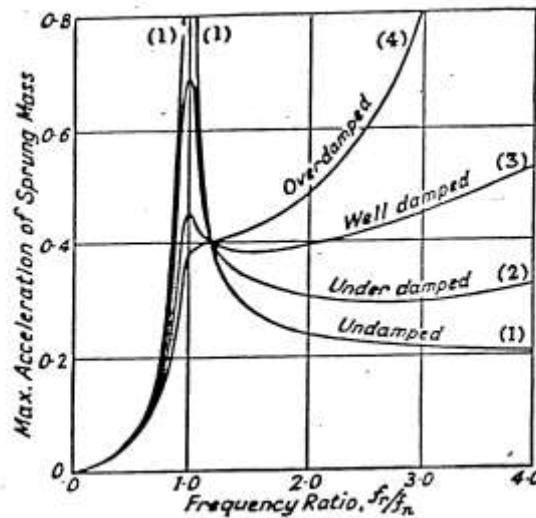


Fig. 9 Damping characteristics of a sprung damped mass

CURVE 1 Undamped

CURVE 2 $B/B_c = 0.15$ giving maximum acceleration at resonance of 0.7g

CURVE 3 $B/B_c = 0.25$ giving maximum acceleration at resonance of 0.45g

CURVE 4 $B/B_c = 0.65$ giving maximum acceleration at resonance of 0.4g

Note that although curves 3 and 4 reduce resonant frequency response the accelerations experienced at higher frequencies increase (curve 4 with $B/B_c = 0.65$ especially so). B/B_c of 0.2 to 0.6 is generally regarded as the optimum range for road use.

Hydraulic Dampers

There are 3 mechanisms by which suspension oscillations are damped out.

1. Friction F is constant
2. Air Resistance $F \propto V^2$
3. Viscous $F \propto V$ and then V^2 as piston speed increases

Where:

V = Velocity of displacement

F = Damping force

Air resistance is small enough to be ignored.

If friction exists in a suspension system then no movement of the suspension can take place until that friction has been overcome which will make the ride harsher. It should therefore be eliminated as far as practicable. The majority of damping in a suspension system must come from viscous damping from hydraulic dampers. The graph in Fig 10 shows typical damping characteristics for an aftermarket one way adjustable twin tube damper manufactured by AVO.

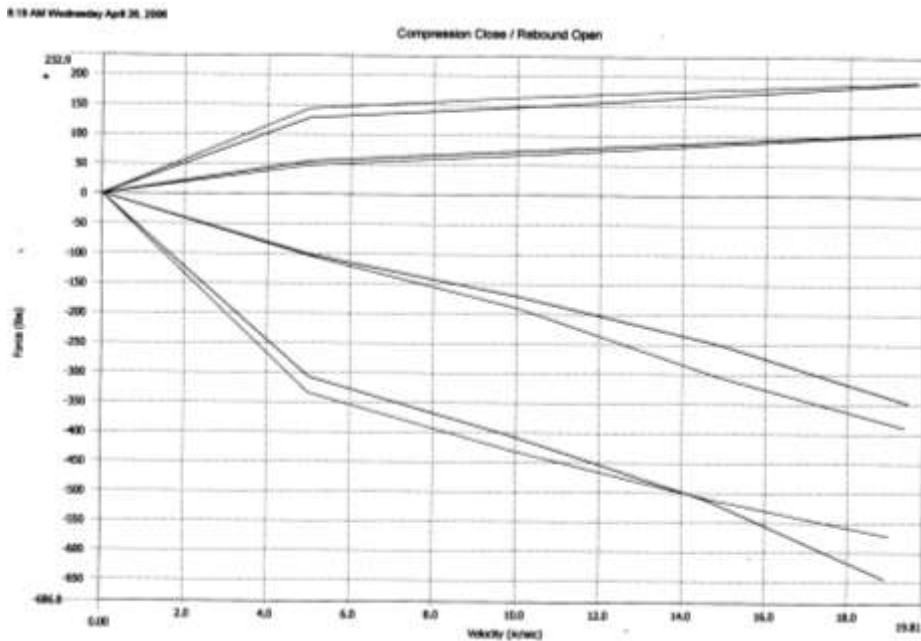


Fig 10 Characteristics for AVO aftermarket damper

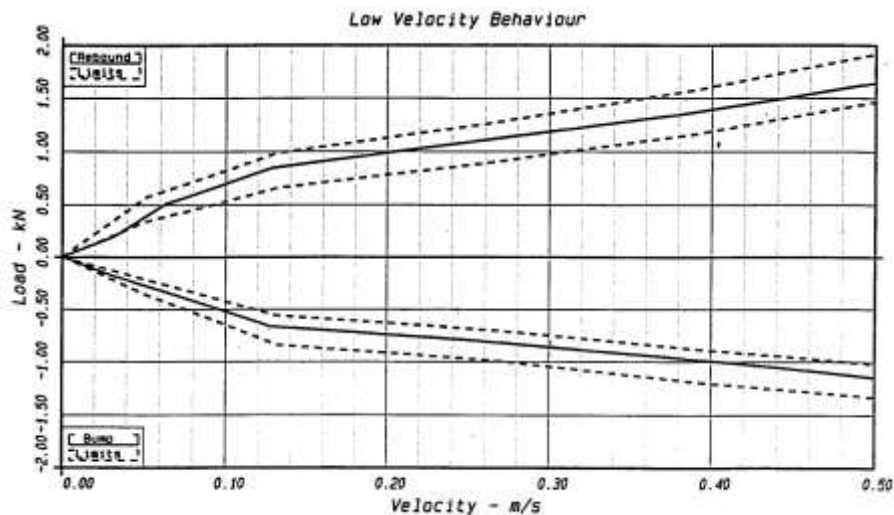


Fig 11 Low speed characteristics for a hydraulic damper for a Range Rover on bump and rebound

SUSPENSION AND STEERING SYSTEMS DESIGN AND ANALYSIS

In both the previous cases the initial setting or 'bleed', is to control slow suspension movement such as found in roll, gentle ripples and braking. If this was not available and the main piston valve alone was used the effect would be the same as with friction i.e. until the valve spring pressure was overcome the suspension would be solid. Because the 'bleed' flow involves turbulent flow damping force $F \propto V^2$ the differentiation between bleed and blow-off settings is normally rounded off to make a smooth transition.

During the 'blow-off' phase the damping force $F \propto V$

At higher velocities again the flow once more becomes turbulent and $F \propto V^2$.

The valving used comprises the following:-

REBOUND, BLOW OFF Valve controlled activated above c.0.1 m/s

REBOUND BLEED Orifice & piston leakage controlled below c.0.1 m/s

BUMP BLOW OFF Valve controlled activated above c.0.1 m/s

BUMP BLEED Orifice and piston leakage controlled below c.0.1 m/s

The energy extracted from the motion by the damper is in the form of work done on the fluid:-

$$\text{Pressure drop} \times \text{fluid displacement} = \text{Work Done}$$

(The required energy extracted is also described as piston force $F \times$ piston travel)

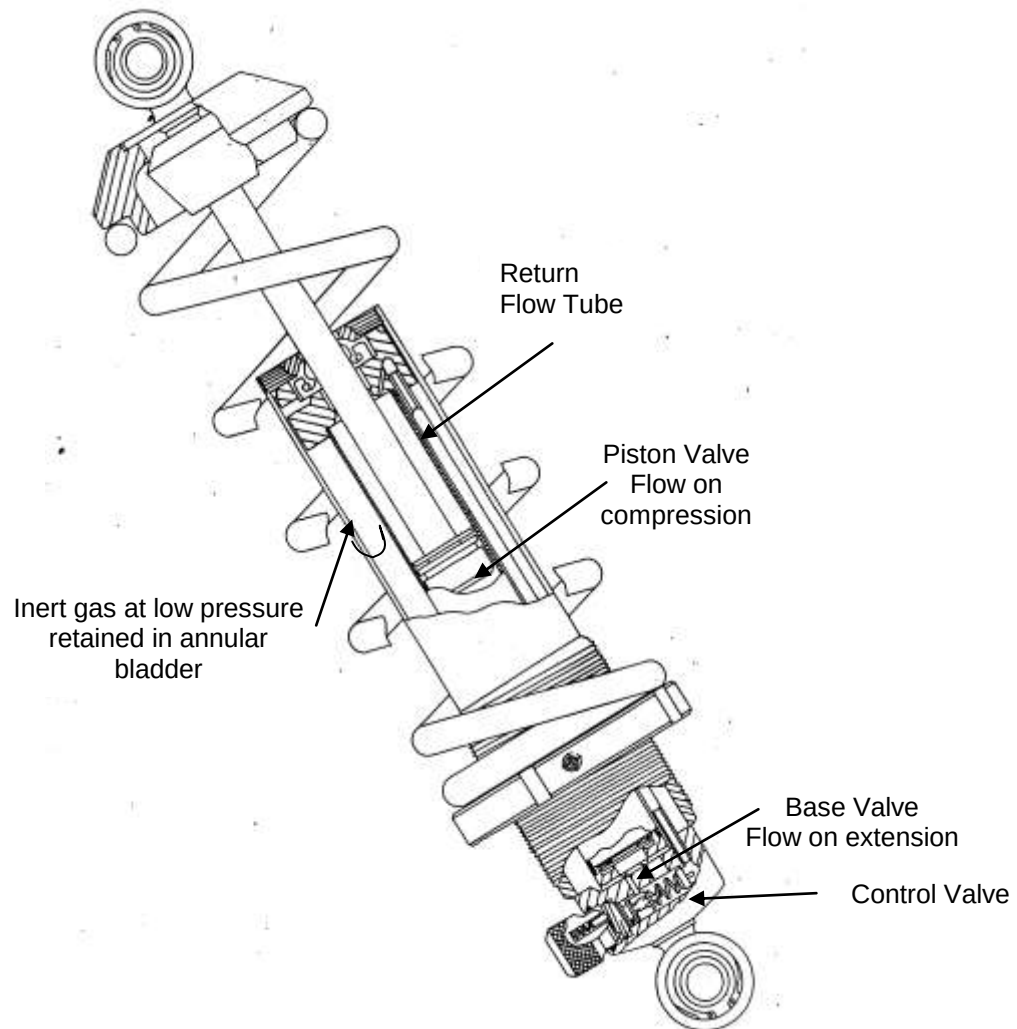


Fig 12 AVO Twin tube damper

The AVO twin tube damper uses a single adjustable control valve to control damping. On compression the base valve is closed and the flow is through the piston one way valve. Fluid is displaced by the piston rod and its flow is controlled by the control valve in the return flow tube. On extension, the piston valve closes, oil is forced through the return flow tube and is again controlled by the same control valve.