

1ai.

$$\frac{x^2}{2x+1} = y.$$

$$u(x) = x^2$$

$$u'(x) = 2x$$

$$v(x) = 2x+1$$

$$v'(x) = 2$$

$$\frac{dy}{dx} = \frac{du \cdot v - u \cdot dv}{v^2}$$

$$\text{So, } \frac{dy}{dx} = \frac{2x(2x+1) - x^2(2)}{(2x+1)^2}$$

$$= \frac{4x^2 + 2x - 2x^2}{(2x+1)^2} = \frac{2x^2 + 2x}{(2x+1)^2}$$

ii

$$y = \sqrt{x^2 + 1}$$
$$y = (x^2 + 1)^{1/2}$$

$$u = x^2 + 1$$

$$\frac{du}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$$

$$y = u^{1/2}$$
$$\frac{dy}{du} = \frac{1}{2} u^{-1/2}$$

$$\text{So, } \frac{dy}{dx} = \frac{1}{2} u^{-1/2} \times 2x$$

$$= \frac{1(x^2+1)^{-1/2}}{2} \times 2x = x(x^2+1)^{-1/2} \text{ or } \frac{x}{\sqrt{x^2+1}}$$

$$\text{iii } y = 3x^{3/2} + \frac{3}{x^2} - 6x$$

$$y = 3x^{3/2} + 3x^{-2} - 6x$$

$$\begin{aligned} \frac{dy}{dx} &= 3 \times \frac{3}{2} x^{1/2} + 3(-2)x^{-3} - 6 \\ &= \frac{9}{2} x^{1/2} - 6x^{-3} - 6 \end{aligned}$$

$$\text{iv. } y = \sec^3 4x$$

$$\text{So, } 3\sec^2(4x) \cdot \frac{d}{dx} \sec(4x)$$

$$\sec(4x) \rightarrow \sec(4x)(\tan(4x))$$

$$3\sec^2(4x) \cdot \sec(4x)\tan(4x) \cdot \frac{d}{dx} 4x$$

So,

$$\text{So, } 3\tan(4x)\sec^3(4x) \times 4 \cdot \frac{d}{dx} x$$

$$12\tan(4x)\sec^3(4x)$$

$$= 12\sec^3(4x)\tan(4x)$$

b.

$$y = \frac{1}{x} + \cos 2x \quad \text{find } \frac{d^5 y}{dx^5}$$

$$\frac{dy}{dx} = \frac{-1}{x^2} + 2\sin 2x$$

$$\frac{1}{x} = -\frac{1}{x^2}$$

$$\cos \rightarrow -\sin$$

$$\sin \rightarrow \cos$$

$$\frac{d^2 y}{dx^2} = \frac{2}{x^3} + 4\cos 2x$$

$$\frac{-1}{x^2} = -x^{-2} \rightarrow +2x^{-3}$$

$$\frac{d^3 y}{dx^3} = -6x^{-4} - 8\sin 2x$$

$$\frac{d^4 y}{dx^4} = +24x^{-5} - 16\cos 2x$$

$$\frac{d^5 y}{dx^5} = -120x^{-6} + 32\sin 2x$$

$$\text{Simplify} = -15x^{-6} + 4\sin 2x$$

c.

$$f(x) = \ln(\operatorname{cosec} x - \cot x) \quad \text{Show}$$

$$f'(x) = \operatorname{cosec} x$$

$$\frac{d}{dx} \frac{1}{\operatorname{cosec} x - \cot x} \cdot \frac{d}{dx} (\operatorname{cosec} x - \cot x)$$

$$= \frac{\frac{d}{dx} \operatorname{cosec} x - \frac{d}{dx} \cot x}{\operatorname{cosec} x - \cot x}$$

$$= \frac{(-\cot x) \operatorname{cosec} x - (-\operatorname{cosec}^2 x)}{\operatorname{cosec} x - \cot x}$$

$$= \frac{\operatorname{cosec}^2 x - \cot x \operatorname{cosec} x}{\operatorname{cosec} x - \cot x} = \operatorname{cosec} x$$

v. $y = x + \sin 2x$

$$\frac{dy}{dx} = 2\cos 2x + 1$$

$$g(x) = x$$

$$g'(x) = 1$$

$$h(x) = \sin 2x$$

$$h'(x) = 2\cos 2x$$

$$y = \frac{1-10x}{(2x-1)^2}$$

$$u = 1-10x$$

$$\frac{du}{dx} = \frac{d}{dx} 1-10x$$

$$\frac{u}{v} = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$v = (2x-1)^2$$

$$\frac{dv}{dx} = \frac{d}{dx} (2x-1)^2$$

so,

$$\frac{\frac{d}{dx}(1-10x) \cdot (2x-1)^2 - (1-10x) \cdot \frac{d}{dx}(2x-1)^2}{((2x-1)^2)^2}$$

$$\text{So, } \frac{-10 \cdot (2x-1)^2 - (1-10x) \cdot 4(2x-1)}{(2x-1)^4}$$

$$y = u^2$$

$$\frac{dy}{du} = 2u$$

$$u = 2x-1$$

$$\frac{du}{dx} = 2$$

$$\frac{dy}{dx} = 4u$$

$$= 4(2x-1)$$

$$= \frac{-10(2x-1)^2 - 4(1-10x)(2x-1)}{(2x-1)^4}$$

Simplify

$$\frac{-10}{(2x-1)^2} - \frac{4(1-10x)}{(2x-1)^3}$$

$$\frac{-10(2x-1) - 4(1-10x)}{(2x-1)^3} = \frac{-20x+10-4+40x}{(2x-1)^3}$$

$$\rightarrow \frac{20x+6}{(2x-1)^3} = \frac{2(10x+3)}{(2x-1)^3}$$

Q2ai

$(+C)$ ← don't forget!!

$$\int \frac{2x}{4+3x^2} dx$$

$$u = 4+3x^2$$
$$\frac{du}{dx} = 6x$$

$$dx = \frac{1}{6x} du$$

$$= \frac{1}{3} \int \frac{1}{u} du$$

$$= \frac{1}{3} \ln u = \frac{1}{3} \ln(4+3x^2) + C$$

$$\int_0^4 e^{1/2x} dx$$

$$= 2 \int_0^4 e^u du$$

$$u = \frac{1}{2}x$$

$$\int_0^4 2e^{1/2x} + C \quad \text{so, } \left[2e^{1/2(4)} \right] - \left[2e^{1/2(0)} \right] = 12.78$$

2aiii $\int_0^4 \frac{5x+13}{(2x+1)(x+4)} dx$

So, $\int \left(\frac{3}{(2x+1)} + \frac{1}{(x+4)} \right) dx \Rightarrow 3 \int \frac{1}{(2x+1)} + \frac{1}{(x+4)} dx.$

For

$$\int \frac{1}{2x+1}$$

don't forget x3

$$u = 2x+1$$

$$\frac{du}{dx} = 2$$

$$\frac{1}{2} \int \frac{1}{u} du = \frac{\ln u}{2}$$

$$dx = \frac{1}{2} du$$

$$\frac{\ln(2x+1)}{2}$$

$$\int \frac{1}{x+4} = \ln u$$

$$u = x+4$$

$$\frac{du}{dx} = 1$$

$$= \ln(x+4)$$

$$dx = \frac{du}{1} = du$$

$$3 \times \frac{\ln(2x+1)}{2} + \ln(x+4)$$

$$\text{So, } \frac{3\ln(2x+1)}{2} + \ln(x+4) + C$$

Now limits. $4 \neq 0$

$$\frac{3\ln(2x+1)}{2} + \ln(x+4)$$

$$\text{So, } \left(\frac{3\ln(2(4)+1)}{2} + \ln(4+4) \right) - \frac{3\ln(2(0)+1)}{2} + \ln(0+4)$$

$$= \frac{3\ln 9}{2} + \ln 8 - \ln 4 = 3.99$$

2a) $\int_0^4 \frac{5x+13}{(2x+1)(x+4)} dx$

So, $\int \left(\frac{3}{(2x+1)} + \frac{1}{(x+4)} \right) dx \Rightarrow 3 \int \frac{1}{(2x+1)} + \frac{1}{(x+4)} dx.$

For

$$\int \frac{1}{2x+1}$$

don't forget $\times 3$

$$u = 2x+1$$

$$\frac{du}{dx} = 2$$

$$dx = \frac{1}{2} du$$

$$\frac{1}{2} \int \frac{1}{u} du = \frac{\ln u}{2}$$

$$\frac{\ln(2x+1)}{2}$$

$$\int \frac{1}{x+4} = \ln u$$

$$u = x+4$$

$$\frac{du}{dx} = 1$$

$$= \ln(x+4)$$

$$dx = \frac{du}{1} = du$$

$$3 \times \frac{\ln(2x+1)}{2} + \ln(x+4)$$

$$\text{So, } \frac{3\ln(2x+1)}{2} + \ln(x+4) + C$$

b.

$$f(x) = \frac{4}{x(1+4\ln x)^2} dx.$$

$$u = 4\ln x + 1$$

$$\frac{du}{dx} = \frac{4}{x}$$

$$dx = \frac{x du}{4}$$

$$= \int \frac{1}{u^2} \quad \text{So} = -\frac{1}{u}$$

$$= -\frac{1}{4\ln x + 1} + C.$$

$$y = \frac{1}{u^2} \text{ or } u^{-2}$$

$$\frac{dy}{du} = -u^{-1}$$

$$= -u^{-1} \text{ or }$$

$$-\frac{1}{u}$$

$$\text{Ci} \int_{\ln 2}^{\ln 5}$$

$$\frac{3e^{2x}}{\sqrt{e^x - 1}} dx$$

$$= 3 \int_{\ln 2}^{\ln 5} \frac{e^{2x}}{\sqrt{e^x - 1}}$$

$$u = e^x - 1$$

$$\frac{du}{dx} = e^x$$

$$dx = e^{-x} du.$$

$$= 3 \int \frac{u+1}{\sqrt{u}} du. = \int \frac{e^x + 1}{\sqrt{e^x - 1}} du.$$

$$\int \sqrt{u} du = \frac{u^{3/2}}{3/2} = \frac{2u^{3/2}}{3}$$

$$e^{2x} = (u+1)^2$$

3aii

$$\frac{3}{(1+i)(2+i)} = \frac{3}{2+i+2i+i^2} = \frac{3}{1+3i}$$

$$\frac{3}{1+3i} \times \frac{1-3i}{1-3i} = \frac{3-9i}{1-3i+3i-9i^2} = \frac{3-9i}{10}$$

$$= \frac{3}{10} - \frac{9i}{10}$$

3aii

$$\left(\frac{1+i}{1-i} \right)^3$$

$$\left(\frac{1+i}{1-i} \right)^3 = \frac{(1+i)^3}{(1-i)^3} = \frac{1+i+i^2+i^3}{1-i-i^2-i^3} = \frac{1+2i-1-1}{1-i+1-i} = \frac{-1+2i}{2-2i} = \frac{-1+2i}{2(1-i)}$$

So, $i + 0$.

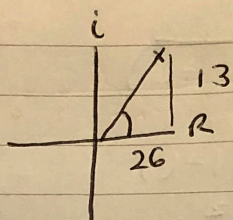
$$\left(\frac{1+i}{1-i} \right) \left(\frac{1+i}{1-i} \right) = \frac{1+i+i^2+i^2}{1-i-i+i^2} = \frac{1+2i-1-1}{1-2i-1} = \frac{-1+2i}{-2i} = \frac{1-2i}{2i}$$

$$\frac{-1}{1} \times \left(\frac{1+i}{1-i} \right) = \frac{-1-i}{1-i} = -i$$

So, $-i + 0$.

SO, $26 + 13i$

$$R = \sqrt{26^2 + 13^2} \quad R = \sqrt{676 + 169} = 29.1$$



$$\tan \theta = \frac{0}{a} \quad \tan \theta = \frac{13}{26}$$

$$\theta = 26.6^\circ$$

SO $z = 29 \angle 27^\circ$

rounded.

C.

$1 + 3i$ and $1 + i$

$$\frac{1+3i}{1+i} \times \frac{1-i}{1-i} = \frac{1-i+3i-3i^2}{1+i-i-i^2} = \frac{4+2i}{2} = 2+i$$

$$b. z = (4+7i)(3-2i)$$

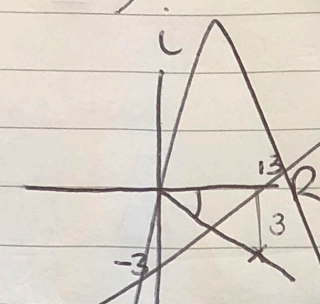
polar form.

$$z = R \angle \theta$$

$$\begin{aligned} z &= 12 - 8i + 2i - 14i^2 \\ &= 12 - 6i + 14 \\ &= 26 - 6i \end{aligned}$$

$$= 26 - 6i$$

not quite!



$$R = \sqrt{(13)^2 + (-3)^2}$$

$$R = 12.65$$

$$\tan \theta = \frac{b}{a}$$

$$\tan \theta = \frac{3}{13}$$

$$\theta = 12.99^\circ$$

$$360^\circ - 13 = 347^\circ$$

So, $z = 13 \angle 347^\circ$

$$z = (4+7i)(3-2i)$$

$$\begin{aligned} &= 12 - 8i + 2i - 14i^2 = 12 - 6i + 14(-1) \\ &= 12 + 13i - 14(-1) \\ &= 26 + 13i \end{aligned}$$

So, $(i) - (-2+6i) + (1-2i) =$
 $i + 2 - 6i + 1 - 2i = -7i + 3.$

iv $2i(i-1) + (\sqrt{3}-i) + (1+i)(1+i)$

$$= (2i^2 - 2i + \sqrt{3} - i) + (1+i)(1+i)$$

$$= -2 - 2i + \sqrt{3} - i + 1 - \cancel{i^2}^{(+1)}$$

$$= \sqrt{3} - 3i.$$

b $(4+7i)(3-2i)$

polar form

3ai

$$\left(\frac{1+i}{1-i}\right)^3$$

$$\left(\frac{1+i}{1-i}\right) \times \left(\frac{1+i}{1-i}\right) = \frac{1+i+i+i^2}{1-i-i+i^2} = \frac{1+2i+i^2}{1-2i+i^2} = \frac{+2i}{-2i} = \frac{+i}{-i} = -1$$

$$\left(\frac{1+i}{1-i}\right) \times \left(\frac{1+i}{1-i}\right) = \frac{i+i^2}{-i+i^2} = \frac{i+(-1)}{-i+(-1)} = \frac{i}{-i} = -1$$

$$-1 \times \left(\frac{1+i}{1-i}\right) = -\frac{(1+i)}{(1-i)}$$

+1+1+1
0 + -i
-1
-1

3i

$$\frac{3}{(1+i)(2+i)} = \frac{3}{2+i+2i+i^2} = \frac{3}{1+3i}$$

$$\frac{3}{1+3i} \times \frac{3}{1-3i} = \frac{9}{1-3i+3i-9i^2} = \frac{9}{10} = \frac{9}{10} + 0i$$

iii

$$\left(\frac{1+i}{1-i}\right) - (1+2i)(2+2i) + \left(\frac{3-i}{1+i}\right)$$

$$\frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{1+2i+i^2}{1-i^2} = \frac{2i}{2} = i$$

$$(1+2i)(2+2i) = 2+2i+4i+4i^2 = 2+6i+4(-1) = -2+6i$$

$$\frac{3-i}{1+i} \times \frac{1-i}{1-i} = \frac{3-3i-i+i^2}{1-i+i-i^2} = \frac{2-4i}{2} = 1-2i$$

$$\int \frac{1}{\sqrt{u}} du = 2\sqrt{u}.$$

$$= \frac{2u^{3/2}}{3} + \underline{2\sqrt{u}}.$$

$$u^{-1/2}$$

$$\frac{du^{1/2}}{1/2} = 2u^{1/2}.$$

$$= 2\sqrt{u}.$$

$$e^{x+1} \Rightarrow \frac{2(e^{x+1})^{3/2}}{3} + 2\sqrt{e^{x+1}} + C \cdot x 3.$$

$$= 2(e^{x+1})^{3/2} + 6\sqrt{e^{x+1}} + C.$$

then plug in limits. $\ln 5 \ln 2$

$$(2(e^{\ln 5+1})^{3/2} + 6\sqrt{e^{\ln 5+1}}) - (2(e^{\ln 2+1})^{3/2} + 6\sqrt{e^{\ln 2+1}}) = 20.$$

$$\text{ii } \int_1^e \ln x dx =$$

$$\int f g' = f g - \int f' g.$$

$$f(x) \ln x \rightarrow \frac{1}{x}.$$

$$g'(x) = 1$$

$$\text{so } g(x) = x.$$

$$= x \ln(x) - \int 1 dx.$$

$$\Rightarrow x \ln x - x + C.$$

$$x(\ln x - 1) + C.$$

plug in e & 1

$$(e \ln e - e) - (1 \ln 1 - 1) = 1.$$