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Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can check the script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

1	Student Number	2418593/1
2	Title of Examination Paper	MATHEMATICS Paper 2
3	Module Code	ACDA 4008(2)
4	Date and Time of Examination	7.1.25 9:00 am
5	Examination Venue	1Q310
6	Seat Number	64
7	Number of Answer Books handed in	

List the numbers of the questions you have attempted	Office Use
Q1	30
Q2	26
Q3	26
Q4	10
Total	92

92%

✓✓✓✓✓ Very good

✓ CAT Feb 25

i) $y = x + 3$
 $\frac{dy}{dx} = 1$ ✓

ii) $y = 4x^3 + 10x^{\frac{1}{2}}$
 $\frac{dy}{dx} = 12x^2 + 5x^{-\frac{1}{2}}$ ✓

iii) $y = \frac{1}{x^5} + x^3$
 $= x^{-5} + x^3$
 $\frac{dy}{dx} = -5x^{-6} + 3x^2$ ✓

iv) $y = \sin(8x)$ ✓ $8x = u$ $v = \sin(x)$ $\frac{du}{dx} = 8$ $\frac{dv}{dx} = \cos(x)$
 $\frac{dy}{dx} = 8\cos(8x)$ ✓ $du \cdot \frac{dv}{du} = 8\cos(x)$

vi) $y = (2x-3)^{10}$ ✓ $2x-3 = u$ $u' = 2$ $v = ()^{10}$ $v' = 10()^9$
 $\frac{dy}{dx} = 20(2x-3)^9$ ✓ $u' \cdot v' = 20(2x-3)^9$

v) $y = x \cos(x)$ ✓ $x = u$ $v = \cos(x)$ $u' = 1$ $v' = -\sin(x)$
 $\frac{dy}{dx} = \cos(x) - x \sin(x)$ ✓ $-x \sin(x) + \cos(x)$

vii) $y = 3x + \tan(x)$ ✓
 $\frac{dy}{dx} = 3 + \sec^2 x$ ✓

b) $y = \frac{1}{x} + \cos(2x)$
 $y = x^{-1} + \cos(2x)$
 $\frac{dy}{dx} = -x^{-2} - 2\sin(2x)$ ✓

$\frac{d^2y}{dx^2} = 2x^{-3} + 4\sin(2x)$ ✓

$\frac{d^3y}{dx^3} = -6x^{-4} + 8\cos(2x)$ ✓

2

3

3

2

3

5

2

20

4

24

$$c) \frac{(x-1)}{(4+x^2)}$$

$$x-1 = f \\ f' = 1$$

$$g = 4+x^2 \\ g' = 2x$$

$$\frac{dy}{dx} = \frac{(4+x^2)(1) - (x-1)(2x)}{(4+x^2)^2}$$

$$= \frac{4+x^2-2x^2+2x}{(4+x^2)^2} = \frac{-x^2+2x+4}{(4+x^2)^2}$$

when $x=0$

$$\frac{dy}{dx} = \frac{-(0)^2+2(0)+4}{(4+0^2)^2} = \frac{1}{4}$$

$$2i) \int (10x + x^3) dx \\ y = 5x^2 + \frac{1}{4}x^4 + c$$

$$ii) \int (x - 3e^x) dx \\ y = \frac{1}{2}x^2 - 3e^x + c$$

$$iii) \int \left(\frac{1}{x} + 4\right) dx \\ y = \ln x + 4x + c$$

$$iv) \int 2x(3x+4) dx \\ 6x^2 + 8x dx \\ y = 2x^3 + 4x^2 + c$$

$$v) \int_0^2 (3x^2 + 2x) dx \\ y = x^3 + x^2 \\ \text{Sub } x=2 \quad \text{Sub } x=0 \\ 12 - 0 = 12 \\ = 12$$

6

3

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2

3

b) $\int_0^1 \frac{x}{\sqrt{x^2+1}} dx$

let $x^2+1 = u$
 $\frac{du}{dx} = 2x$

Sub $x=1$ into u
 $1^2+1 = 2 = u$
 Sub $x=0$
 $0^2+1 = 1 = u$

$\int_1^2 \frac{x}{\sqrt{u}} \times \frac{1}{2x} du$
 $= \int_1^2 \frac{1}{2} u^{-\frac{1}{2}} du = \frac{1}{2} \int_1^2 u^{-\frac{1}{2}} du$
 $= \left(2u^{\frac{1}{2}} \right) \times \frac{1}{2} \Big|_1^2$
 $= (2)^{\frac{1}{2}} - (1)^{\frac{1}{2}} = 0.414$

Sub $u=2$ and 1
 $= 0.609$

$\frac{1}{2} \left(\frac{2}{3} u^{\frac{3}{2}} \right) \Big|_1^2 = \frac{1}{3} (2^{\frac{3}{2}} - 1)$

$= (2) - (1) = 0.414$

ci) $\int x \sin(2x) dx$

let $x = u$ $\frac{du}{dx} = \sin(2x)$
 $\frac{du}{dx} = \sin(2x)$
 $v = -\frac{1}{2} \cos(2x)$

$-x \frac{1}{2} \cos(2x) - \int -\frac{1}{2} \cos(2x) \times 1 dx$
 $= -x \frac{1}{2} \cos(2x) + \frac{1}{4} \sin(2x) + C$

ii) $\int_1^e x \ln x dx$

$\ln x = u$ $\frac{du}{dx} = \frac{1}{x}$
 $x = \frac{du}{dx}$ $v = \frac{1}{2} x^2$

$= \frac{1}{2} x^2 \ln x - \int \frac{1}{x} \times \frac{1}{2} x^2 dx \rightarrow \frac{1}{2} \int \frac{1}{x} x^2 = \int x$
 $= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$
 $= \frac{1}{2} x^2 \left(\ln x - \frac{1}{2} \right) + C$
 $\frac{1}{2} x^2 (\ln x - \frac{1}{2}) + C$
 $\frac{1}{2} x^2 (\ln x - \frac{1}{2}) + C$

3ai $(3+2i)(1-i)$

let $i^2 = -1$

$3 - 3i + 2i - 2i^2$
 $3 - i - 2(-1)$
 $= 5 - i$

ii $\frac{3}{(2+i)(2-i)} = \frac{6-3i}{4-i^2} = \frac{6-3i}{5} = \frac{6}{5} - \frac{3}{5}i$

$$\text{ii)} \frac{4}{i^5} \rightarrow \frac{4}{(i^4)i} = \frac{4 \times i}{i \times (i)} \quad \text{let } i^2 = -1 \quad \frac{4i}{i^2(-1)} = -4i + 0$$

$$\text{b)} \quad z_1 = \frac{2-i}{2+i} \quad z_2 = \frac{3+i}{3-i}$$

$$\frac{2-i}{2+i} \times \frac{(2-i)}{(2-i)}$$

$$= \frac{4-4i+i^2}{4-i^2} = \frac{3-4i}{5} = z_1 = \frac{3}{5} - \frac{4}{5}i$$

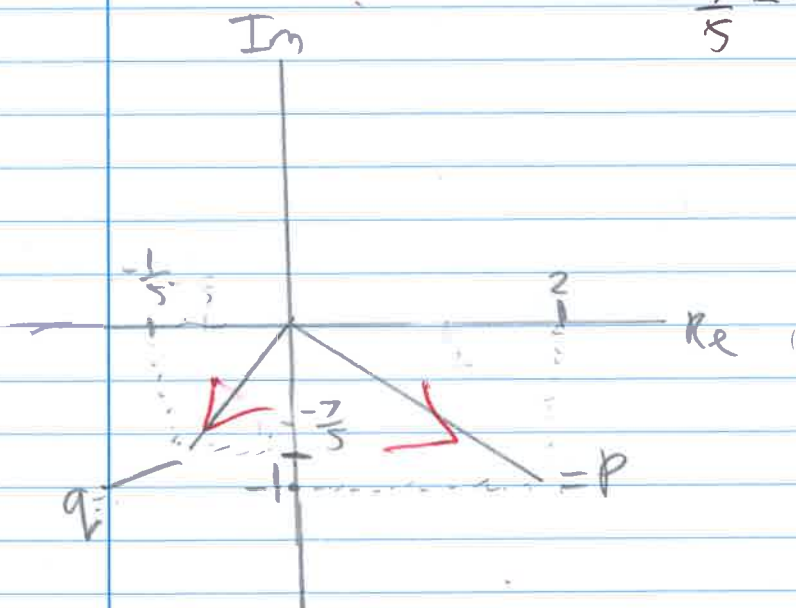
$$z_2 = \frac{3+i}{3-i} \times \frac{(3+i)}{(3+i)} = \frac{9+6i+i^2}{9-i^2} = \frac{8+6i}{10} = z_2 = \frac{8}{10} + \frac{6}{10}i$$

$$z_2 = \frac{4}{5} + \frac{3}{5}i$$

$$1 = 2z_1 + z_2 = \frac{4}{5} + \frac{3}{5} = \frac{7}{5}$$

$$\frac{3}{5}i + -\frac{4}{5}i = -\frac{1}{5}i$$

$$= \frac{7}{5} - \frac{1}{5}i$$



$$p = 2\left(\frac{3}{5} - \frac{4}{5}i\right) + \frac{4}{5} + \frac{3}{5}i$$

$$\frac{6}{5} - \frac{8}{5}i + \frac{4}{5} + \frac{3}{5}i$$

$$p = 2 - i$$

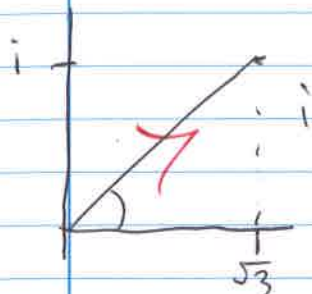
$$q = \left(\frac{3}{5} - \frac{4}{5}i\right) - \left(\frac{4}{5} + \frac{3}{5}i\right)$$

$$q = -\frac{1}{5} - \frac{7}{5}i$$

$$dz \quad z = \sqrt{3} + i$$

$$r = \sqrt{(\sqrt{3})^2 + (1)^2} = 2$$

$$r(\cos \theta + i \sin \theta)$$



$$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{6}\pi = \frac{\pi}{6}$$

$$p.d.r. = 2\left(\cos\left(\frac{1}{6}\pi\right) + i \sin\left(\frac{1}{6}\pi\right)\right)$$

$$\frac{u}{v} = \frac{d}{dz} = \frac{v \frac{du}{dz} - u \frac{dv}{dz}}{v^2}$$

$$e) \quad \frac{1+i}{1-i}$$

$$\frac{1+i}{1-i} \cdot \frac{1+i}{1+i} = \frac{(1+i)^2}{1-i^2} = \frac{2i}{2} = i$$

$$r=1, \theta=90^\circ$$

$$g' = 1$$

$$s' = -1$$

$$\frac{d}{dz} \left(\frac{1}{1-i} - \frac{1}{1+i} \right) = \frac{(1-i) - (-1-i)}{(1-i)^2} = \frac{2}{(1-i)^2}$$

$$1+i \text{ in polar } = \sqrt{2} \left(\cos \frac{1}{4}\pi + i \sin \frac{1}{4}\pi \right)$$

$$1-i = \sqrt{2} \left(\cos \frac{1}{4}\pi + i \sin \left(-\frac{1}{4}\pi\right) \right)$$

$$\frac{\sqrt{2} \left(\cos \frac{1}{4}\pi + i \sin \frac{1}{4}\pi \right)}{\sqrt{2} \left(\cos \frac{1}{4}\pi + i \sin \left(-\frac{1}{4}\pi\right) \right)}$$

$$4i) \quad 3^{2x} = 2^{x+1}$$

$$\log_3 3^{2x} = \log_2 2^{x+1}$$

$$2x \log_3 3 = (x+1) \log_2 2$$

$$2x \log_3 3 = x \log_2 2 + \log_2 2$$

$$x(2 \log_3 3 - \log_2 2) = \log_2 2$$

$$x = \frac{\log_2 2}{2 \log_3 3 - \log_2 2} = 0.461$$

$$ii) \quad 2 \ln x = \ln(5x-6)$$

$$\ln x^2 = \ln(5x-6)$$

$$x^2 = 5x-6$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2)$$

$$x = 3, 2$$

$$\text{bi } \log(x^6 y z) = \log x^6 + \log y + \log z$$

$$= 6 \log x + \log y + \log z$$

$$\text{ii) } \ln\left(\frac{x^3 y}{z}\right)$$

$$= \ln x^3 + \ln y - \ln z$$

$$= 3 \ln x + \ln y - \ln z$$

This is a very good effort
Well Done!

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