

Exam 11/05/20 13:00 - 15:00
Advanced Stress Analysis + FEA
(M6X00420)

Sion Evans (P150600) 2/20

Two

b i) $30 \times 10^3 \times 20 \times 10^3 \text{ m}$ Thickness $2.5 \times 10^{-3} \text{ m}$
 $L = 2.4 \text{ m}$ $\sigma = 480 \times 10^6 \text{ Pa}$ $E = 190 \times 10^9$

$$u = \frac{P^2 L}{2AE} = \frac{\sigma^2 AL}{2E}$$

$$A = (25 \times 10^{-3})(15 \times 10^{-3})$$
$$= 3.75 \times 10^{-4} \text{ m}^2$$

$$A = (30 \times 10^{-3})(20 \times 10^{-3}) = 3.75 \times 10^{-4}$$

$$u = \frac{(480 \times 10^6)^2 (2.25 \times 10^{-4})(2.4)}{(2)(190 \times 10^9)}$$

$$= \underline{\underline{2.25 \times 10^{-4} \text{ m}^2}}$$

$$= \underline{\underline{327.41 \text{ J}}}$$

ii) $P.E = mgh$ $u = \frac{P^2 L}{2AE}$ $P.E = u$

$$Mgh = \frac{P^2 L}{2AE}$$

$$P = \sigma A$$

$$Mg = W$$

$$Wh = \frac{\sigma A^2 L}{2AE} = \frac{\sigma AL}{2E}$$

$$AL = V$$

$$h = \frac{\sigma V}{2WE}$$

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b iii) $\sigma = \frac{h z w E}{\sqrt{}}$ $M = 20 \text{ kg}$ $h = 450 \times 10^{-3} \text{ m}$

$$\sigma = \frac{(450 \times 10^{-3}) (2) (20) (9.81) (190 \times 10^9)}{(2.25 \times 10^{-4}) (2.4)}$$

$$\sigma = 6.21 \times 10^{16} \text{ Pa}$$

iv) $\sigma_{\max} = 6.21 \times 10^{16}$ $d = 20 \times 10^{-3}$ $L = 2.4$

$$V = \frac{h z w E}{\sigma} = \frac{(450 \times 10^{-3}) (2) (20) (9.81) (190 \times 10^9)}{6.21 \times 10^{16}}$$

$$5.4 \times 10^{-4} = \frac{\pi d e^2}{4} - \frac{\pi d i^2}{4} \cdot L$$

$$\frac{5.4 \times 10^{-4}}{2.4} = \frac{\pi (20 \times 10^{-3})^2}{4} - \frac{\pi d i^2}{4}$$

$$\cancel{2.25 \times 10^{-4}} \frac{\pi d i^2}{4} = \frac{\pi (20 \times 10^{-3})^2}{4} - 2.25 \times 10^{-4}$$

$$d i = \sqrt{\frac{(8.916 \times 10^{-5}) (4)}{\pi}} = 10.65 \times 10^{-3} \text{ m}$$

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$$\frac{4}{20}$$

iv) continued. . . .

$$\begin{aligned}\text{Wall thickness} &= D_o - D_i \\ &= 20 \times 10^{-3} - 10.65 \times 10^{-3} \\ &= \frac{9.35 \times 10^{-3} \text{ m}}{2} \\ &= \underline{\underline{4.68 \times 10^{-3} \text{ m}}}\end{aligned}$$

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Four

a) Plates 300 x 12 mm C-channel ^{wide} 100 x ^{ball} 200 mm

C-channel $I_{xx} = 26.2 \times 10^6 \text{ mm}^4$ $I_{yy} = 1.57 \times 10^6 \text{ mm}^4$
 $A = 4400 \text{ mm}^2$

8m long

xx C-channel =

$$(2)(26.2 \times 10^6) = \underline{\underline{52.4 \times 10^6 \text{ mm}^4}}$$

$$\text{plate} = (2)(1 + Ah^2)$$

$$= (2) \left(\frac{(300)(12)^3}{12} + (300)(12)(106)^2 \right)$$

$$= 80.986 \times 10^6 \text{ mm}^4$$

$$I_{xx} = \underline{\underline{132.986 \times 10^6 \text{ mm}^4}} \quad 132.986 \times 10^6 \text{ mm}^4$$

yy channel = $(2)(1.57 \times 10^6 + (4400)(79^2))$

$$= \underline{\underline{58.06 \times 10^6 \text{ mm}^4}}$$

$$\text{plate} = (2) \left[\frac{(12)(300)^3}{12} \right] = \underline{\underline{54 \times 10^6 \text{ mm}^4}}$$

$$I_{yy} = 112.06 \times 10^6 \text{ mm}^4$$

$I_{xx} > I_{yy}$, will buckle about yy

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Four b) $k = \sqrt{\frac{I}{A_T}}$

$$\begin{aligned} A_T &= (2)(4400) + (2)(300)(12) \\ &= 16,000 \text{ mm}^2 \\ &= \underline{\underline{16 \times 10^3 \text{ mm}^2}} \end{aligned}$$

$$k = \sqrt{\frac{112.06 \times 10^6}{16 \times 10^3} \frac{\text{mm}^4}{\text{mm}^2}}$$

$$I_{yy} = 112.06 \times 10^6 \text{ mm}^4$$

$$= \underline{\underline{83.69 \times 10^3 \text{ mm}^2}}$$

$$\text{c) Slender ratio} = \frac{L}{k} = \frac{8}{83.69 \times 10^3} = \underline{\underline{95.54}}$$

$$\begin{aligned} \text{d) } P_e &= \frac{\pi^2 EI}{L^2} = \frac{(\pi)^2 (200 \times 10^9) (112.06 \times 10^6)}{8^2} \\ &= \underline{\underline{3.46 \times 10^6 \text{ N}}} \end{aligned}$$

$$\begin{aligned} \text{e) } P_R &= \frac{\sigma_y A}{1 + \alpha \left(\frac{L}{k}\right)^2} = \frac{(320 \times 10^6) (16 \times 10^3)}{1 + \frac{1}{2500} (95.54)^2} = 2.31 \times 10^6 \text{ N} \end{aligned}$$

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Four F) 224 mm x 6 mm

$$I_{xx} = \frac{(6)(224^3)}{12} + \cancel{224(6)(2)} = 11.239 \times 10^6 \text{ mm}^4 \\ = 11.239 \times 10^6 \text{ mm}^4$$

$$I_{yy} = (2)(I + Ah^2) = (2) \left(\frac{(224)(6)^3}{12} + (224)(6)(153)^2 \right) \\ = 62.93 \times 10^6 \text{ mm}^4 \\ = \cancel{11.239 \times 10^6} \text{ mm}^4$$

$$I_{xx} = 132.99 \times 10^6 \text{ mm}^4 \quad I_{yy} = 174.99 \times 10^6 \text{ mm}^4$$

$I_{yy} > I_{xx}$, will buckle about I_{xx}

$$P_e = \frac{(\pi^2)(200 \times 10^9)(132.99 \times 10^{-6})}{8^2}$$

$$= 4.1 \times 10^6 \text{ N}$$

$$\text{increase of } 4.1 - 3.46 = \underline{\underline{0.64 \text{ MN}}}$$

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Three a) $d_e = 75 \times 10^{-3} \text{ m}$ thickness $= 5 \times 10^{-3}$ Torsion
no axial loading $\sigma_y = 325 \times 10^6$
 $\nu = 0.3$ $n = 6$

$$\frac{T}{J} = \frac{\tau}{R} \quad \text{so } \tau = \frac{TR}{J} \quad J = \frac{\pi d^4}{32}$$

~~add~~ $d_i = 65 \times 10^{-3}$

$$J = \frac{\pi (75)^4}{32} - \frac{\pi (65 \times 10^{-3})^4}{32} = \underline{1.35 \times 10^{-6} \text{ m}^4}$$

$$\tau = \frac{T(237.5 \times 10^3)}{1.35 \times 10^{-6}} = T 27,227$$

~~Hoop stress~~ $= \frac{Pd}{4t} = \sigma_1$ ~~Radial~~ $= 0 = \sigma_3$

~~Longitudinal~~ $= \frac{Pd}{4t} = \sigma_2$

$$\sigma_1 = (\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\sigma_3 = (\sigma_x + \sigma_y) - \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

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Three a)

$$\sigma_y = 0 \quad \sigma_x = 0$$

$$\text{Tresca} = \sigma_1 - \sigma_3 = \frac{\sigma_{\text{yield}}}{n}$$

$$\text{if } a = \sigma_x + \sigma_y \quad b = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\frac{\sigma_{\text{yield}}}{n} = \left(\frac{1}{2}a + \frac{1}{2}b\right) - \left(\frac{1}{2}a - \frac{1}{2}b\right)$$

$$54.17 \times 10^6 = b = \sqrt{\sigma_x^2 + 4\tau_{xy}^2}$$

$$\frac{\sigma_{\text{yield}}^2}{n^2} = \sigma_x^2 + 4\tau_{xy}^2$$

$$\tau_{xy} = \sqrt{\frac{\sigma_{\text{yield}}^2 - \sigma_x^2}{4n^2}} = \sqrt{\frac{(54.17 \times 10^6)^2 - (4)(6^2)}{(4)(6^2)}}$$

$$= 4.514 \times 10^6 \text{ pa}$$

$$T = \frac{4.514 \times 10^6}{27,777} = 162.5 \text{ Nm}$$

clean

10/20

Three

von-Mises

$$\sigma_1^2 - (-\sigma_3^2) + (\sigma_3 - \sigma_1)^2 = \frac{\sigma_y^2}{n}$$

$$\left(\frac{1}{2}a + \frac{1}{2}b\right)^2 + \left(\frac{1}{2}a - \frac{1}{2}b\right)^2 + \left(\frac{1}{2}a - \frac{1}{2}b - \frac{1}{2}a + \frac{1}{2}b\right)^2$$

$$= \left(\frac{1}{4}a^2 + \frac{1}{4}ab + \frac{1}{4}ab + \frac{1}{2}b^2\right) + \left(\frac{1}{4}a^2 - \frac{1}{2}ab + \frac{1}{4}ab + \frac{1}{4}b^2\right) + b^2$$

$$= \frac{1}{2}a^2 + \frac{1}{2}b^2 = \frac{\sigma_y^2}{n}$$

$$2\sigma_3^2 = \frac{\sigma_y^2}{n}$$

$$(2) \sqrt{4\tau_{xy}^2} = \frac{\sigma_y^2}{n}$$

$$\tau_{xy}^2 = \frac{2\sigma_y^2}{n^2}$$

$$\tau_{xy} = \sqrt{\frac{(2)(325 \times 10^6)^2}{(6)^2}}$$

$$= 76.6 \times 10^6 \text{ N/m}^2$$

$$T = \underline{\underline{2757.79 \text{ Nm}}}$$

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Three C)
~~Van-Mises~~

$$C = \frac{T(25 \times 10^3)}{3.62 \times 10^7}$$

$$S = \frac{(51)(50 \times 10^3)^4}{32} - \frac{(51)(40 \times 10^3)^4}{32}$$
$$= 3.62 \times 10^7 \text{ N}^2$$

$$C = 169,060.77$$

$$C = \frac{(2)(880 \times 10^6)^2}{2.75^2} = 452.54 \times 10^6 \text{ Pa}$$

$$T = \underline{\underline{6552.9 \text{ Nm}}}$$

$$d) \frac{6552.9 - 2757.74}{\frac{6552.9 + 2757.74}{2}} \times 100 = 81\%$$