

1) Two points $A(2, 2, 1)$ and $B(3, -4, 2)$ are given in the Cartesian co-ordinates. Obtain a vector from A to B and unit vector from.

$\vec{A}$ to $\vec{B}$

Soln.

$$\vec{A} = 2\vec{a}_x + 2\vec{a}_y + \vec{a}_z$$

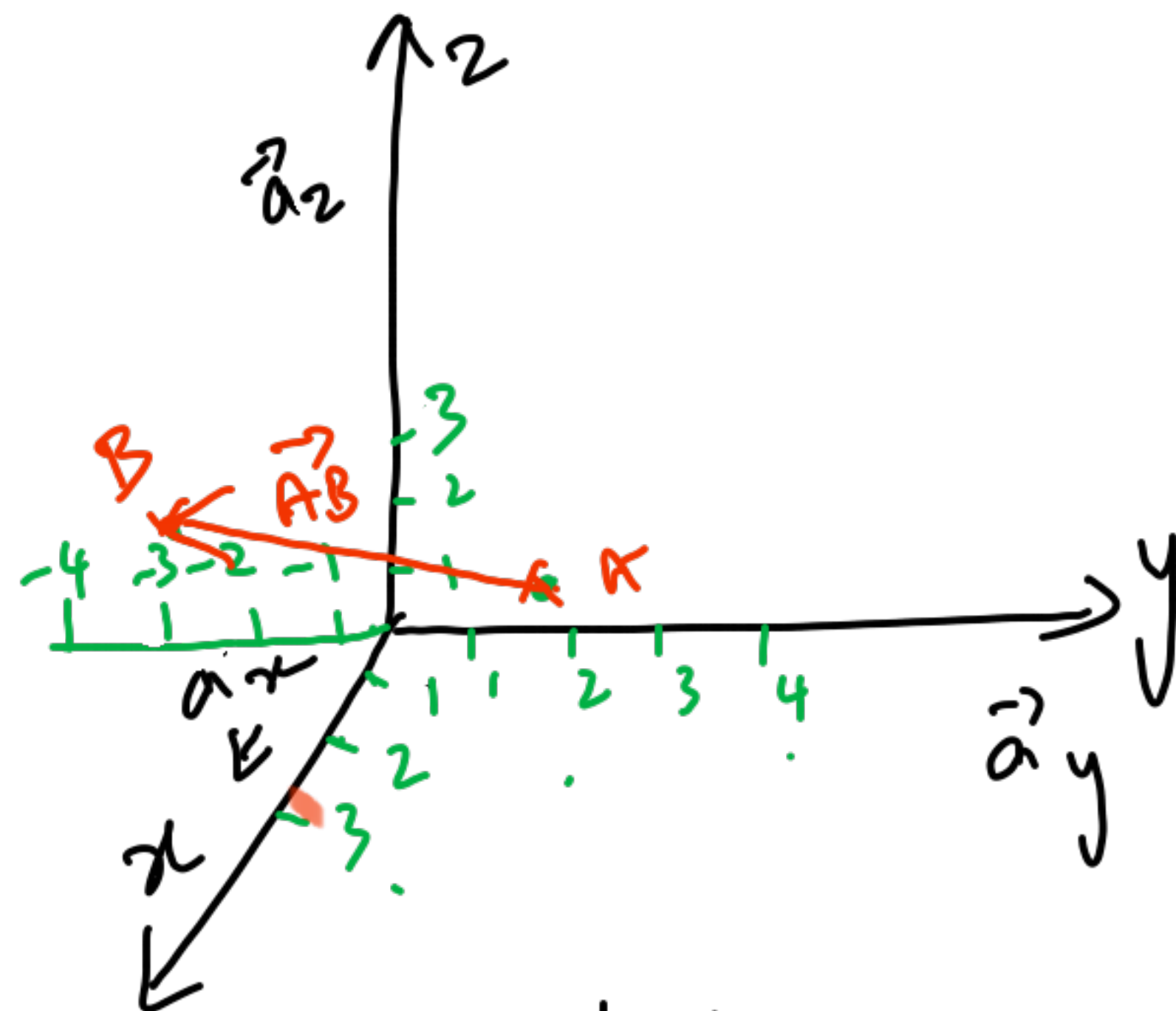
$$\vec{B} = 3\vec{a}_x - 4\vec{a}_y + 2\vec{a}_z$$

Find

$$\vec{AB} = \vec{B} - \vec{A}$$

$$\vec{AB} = (3-2)\vec{a}_x + (-4-2)\vec{a}_y + (2-1)\vec{a}_z$$

$$\vec{AB} = \vec{a}_x - 6\vec{a}_y + \vec{a}_z$$



magnitude.

$$|\vec{AB}| = \sqrt{1^2 + (-6)^2 + 1^2}$$

$$|\vec{AB}| = 6.16$$

Unit vector

$$\vec{a}_{AB} = \frac{\vec{AB}}{|\vec{AB}|}$$

$$\vec{a}_{AB} = \frac{\vec{a}_x - 6\vec{a}_z + \vec{a}_2}{6.16}$$

$$\vec{a}_{AB} = 0.16\vec{a}_x - 0.97\vec{a}_z + 0.16\vec{a}_2$$

$$|\vec{a}_{AB}| = \sqrt{(0.16)^2 + (-0.97)^2 + (0.16)^2}$$

$$|\vec{a}_{AB}| = 0.99 \approx 1$$

2) Show that $\vec{A} = 4\vec{a}_x - 2\vec{a}_y - \vec{a}_z$ and $\vec{B} = \vec{a}_x + 4\vec{a}_y - 4\vec{a}_z$ are perpendicular to each other using dot product.

Soln.

$$\vec{A} \cdot \vec{B} = 0 \quad [\text{Perpendicular}]$$

$$\vec{A} \cdot \vec{B} = (4 \times 1) + (-2 \times 4) + (-1 \times -4)$$

$$\vec{A} \cdot \vec{B} = 4 - 8 + 4 = 0$$

$\vec{A} \cdot \vec{B}$ is perpendicular

3) Given two vectors $\vec{A} = 3\vec{a}_x + 2\vec{a}_y - \vec{a}_z$ and $\vec{B} = -\vec{a}_x + 4\vec{a}_y + 2\vec{a}_z$. Find the vector that is perpendicular to both the vectors and verify that the resulting vector is perpendicular to $\vec{A}$ and $\vec{B}$.

Soln:

$$\vec{A} = 3\vec{a}_x + 2\vec{a}_y - \vec{a}_z$$

$$\vec{B} = -\vec{a}_x + 4\vec{a}_y + 2\vec{a}_z$$

Cross product of two vector is a Perpendicular vector.

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 3 & 2 & -1 \\ -1 & 4 & 2 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \vec{a}_x \begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix} - \vec{a}_y \begin{vmatrix} 3 & -1 \\ -1 & 2 \end{vmatrix} + \vec{a}_z \begin{vmatrix} 3 & 2 \\ -1 & 4 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \vec{a}_x (4 - (-4)) - \vec{a}_y (6 - (-1 \times -1)) + \vec{a}_z (12 - (2 \times -1))$$

$$\vec{A} \times \vec{B} = 8\vec{a}_x - 5\vec{a}_y + 14\vec{a}_z$$

$$\vec{R} = 8\vec{a}_1 - 5\vec{a}_2 + 14\vec{a}_3$$

To verify if $\vec{R}$ is perpendicular to $\vec{A}$ and $\vec{B}$

$$\vec{A} \cdot \vec{R} = 0$$

$$\vec{B} \cdot \vec{R} = 0$$

To verify $\vec{A}$ perpendicular to $\vec{R}$

$$\vec{A} \cdot \vec{R} = (3 \times 8) + (2 \times -5) + (-1 \times 14)$$

$$\vec{A} \cdot \vec{R} = 0$$

As $\vec{A} \cdot \vec{R}$ is zero, both vectors are perpendicular

To verify $\vec{B}$ perpendicular to $\vec{R}$

$$\vec{B} \cdot \vec{R} = (-1 \times 8) + (4 \times -5) + (2 \times 14)$$

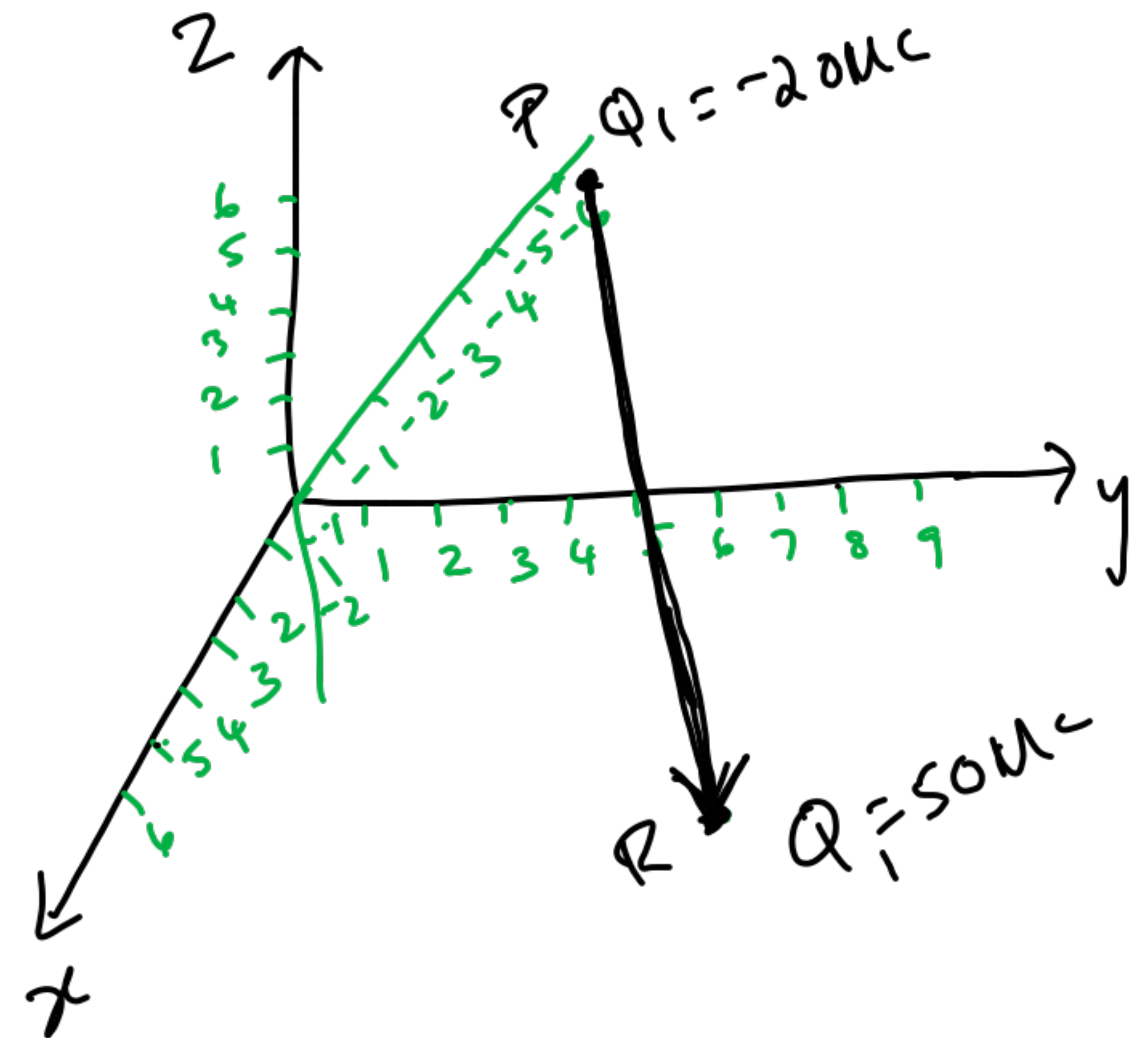
$$\vec{B} \cdot \vec{R} = 0$$

$\vec{B}$ and $\vec{R}$ are perpendicular to each other

4) A charge $Q_1 = -20 \mu\text{C}$ is located at a point $P(-6, 4, 6)$ and a charge $Q_2 = 50 \mu\text{C}$ is located at a point $R(5, 8, -2)$ in free space. Calculate the force exerted on Q_2 by Q_1 also the force vector. The distance given are in meter

Soln.

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 d^2} \hat{a}_{12} \quad \left[\sum v = 1 \right]$$



$$\vec{P} = -6\vec{a}_x + 4\vec{a}_y + 6\vec{a}_z$$

$$\vec{R} = 5\vec{a}_x + 8\vec{a}_y - 2\vec{a}_z$$

$$\vec{d}_{PR} = \vec{R} - \vec{P}$$

$$= (5 - (-6))\vec{a}_x + (8 - 4)\vec{a}_y + (-2 - 6)\vec{a}_z$$

$$\vec{d}_{PR} = 11\vec{a}_x + 4\vec{a}_y - 8\vec{a}_z$$

$$|\vec{d}_{PR}| = \sqrt{11^2 + 4^2 + (-8)^2}$$

$$|\vec{d}_{PR}| = 14.2$$

$$\vec{a}_{PQ} = \frac{\vec{d}_{PR}}{|\vec{d}_{PR}|}$$

$$= \frac{11\vec{a}_x + 4\vec{a}_y - 8\vec{a}_z}{14.2}$$

$$\vec{a}_{PQ} = 0.78\vec{a}_x + 0.28\vec{a}_y - 0.56\vec{a}_z$$

$$\vec{F} = \frac{-20 \times 10^{-6} \times 50 \times 10^{-6}}{4\pi \times 8.854 \times 10^{-12} \times (14.2)^2} \vec{a}_{PQ}$$

$$\vec{F} = \underline{-0.045} \vec{a}_{PQ}$$

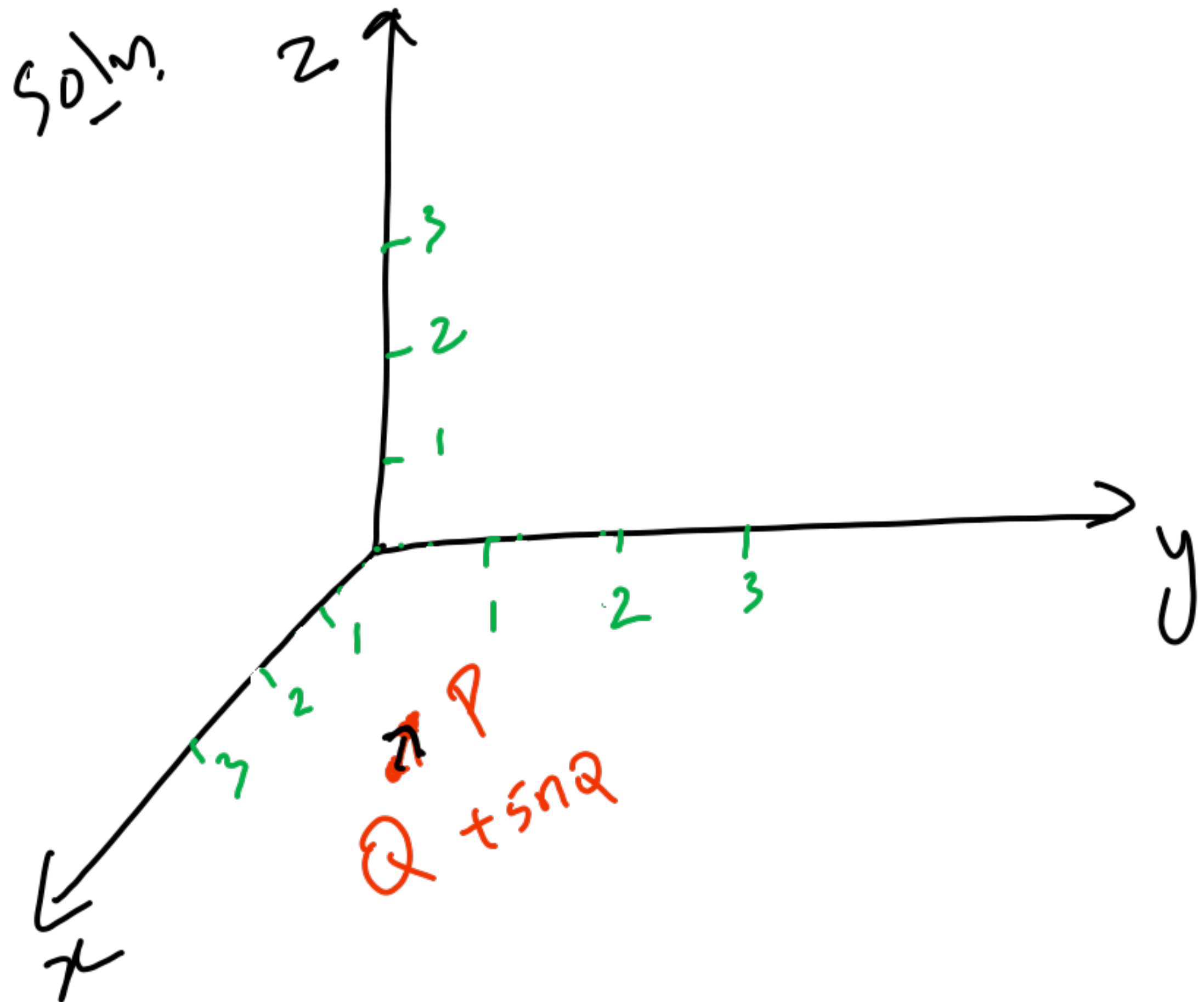
$$\vec{F} = -0.045(0.78 \vec{a}_x + 0.28 \vec{a}_y - 0.56 \vec{a}_z)$$

$$\vec{F} = -0.035 \vec{a}_x - 0.013 \vec{a}_y + 0.025 \vec{a}_z$$

$$|\vec{F}| = 0.045 \text{ N}$$

$$|\vec{F}| = \underline{45 \text{ mN}}$$

5) Determine the electric field intensity vector at $P(-0.2, 0, -2.3) \text{ m}$ due to a point charge $+5 \text{ nC}$ at $Q(0.2, 0.1, -2.5) \text{ m}$ in free space



$$\vec{E} = \frac{Q}{4\pi\epsilon d^2} \vec{a}$$

$$\vec{P} = -0.2\vec{a}_x - 2.3\vec{a}_z$$

$$\vec{Q} = 0.2\vec{a}_x + 0.1\vec{a}_y - 2.5\vec{a}_z$$

$$\vec{d}_{QP} = \vec{P} - \vec{Q}$$

$$\vec{d}_{QP} = (-0.2 - 0.2)\vec{a}_x + (0 - 0.1)\vec{a}_y + (-2.3 - (-2.5))\vec{a}_z$$

$$\vec{d}_{QP} = -0.4\vec{a}_x - 0.1\vec{a}_y + 0.2\vec{a}_z$$

$$|\vec{d}_{QP}| = \sqrt{(-0.4)^2 + (-0.1)^2 + (0.2)^2} = 0.46$$

$$\vec{a}_{QP} = \frac{\vec{d}_{QP}}{|\vec{d}_{QP}|}$$

$$= \frac{-0.4\vec{a}_x - 0.1\vec{a}_y + 0.2\vec{a}_z}{0.46}$$

$$\vec{a}_{QP} = -0.88\vec{a}_x - 0.22\vec{a}_y + 0.43\vec{a}_z$$

$$\vec{E} = \frac{5 \times 10^9}{4\pi \times 8.854 \times 10^{-12} \times (0.46)^2} \vec{a}_{QP}$$

$$\vec{E} = \underline{212.4} \vec{a}_{QP}$$

$$\vec{E} = 212.4(-0.88\vec{a}_x - 0.22\vec{a}_y + 0.43\vec{a}_z)$$

$$\vec{E} = -184\vec{a}_x - 46.17\vec{a}_y + 92.34\vec{a}_z.$$

$$|\vec{E}| = \sqrt{(-184)^2 + (-46.17)^2 + (92.34)^2}$$

$$|\vec{E}| = 212.4 \text{ N/C}$$

b) At a point in free space, the electric and magnetic field vectors are

$$\vec{E} = 4\vec{a}_x - 3\vec{a}_y + 2\vec{a}_z \quad \text{V/m}$$

$$\vec{H} = 1\vec{a}_x + 2\vec{a}_y - \vec{a}_z \quad \text{A/m}$$

a) Calculate the Poynting vector

b) Calculate the magnitude of the Poynting vector

c) Determine the unit vector in the direction of power flow.

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\vec{S} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 4 & -3 & 2 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= \vec{a}_x [(-3 \times -1) - (2 \times 2)] \\ - \vec{a}_y [(4 \times -1) - (2 \times 1)] \\ + \vec{a}_z [4 \times 2 - (1 \times -3)]$$

$$\vec{S} = -1\vec{a}_x + 6\vec{a}_y + 11\vec{a}_z \text{ W/m}^2$$

b) Magnitude $|\vec{S}|$

$$|\vec{S}| = \sqrt{(-1)^2 + 6^2 + 11^2}$$

$$|\vec{S}| = 12.57 \text{ W/m}^2$$

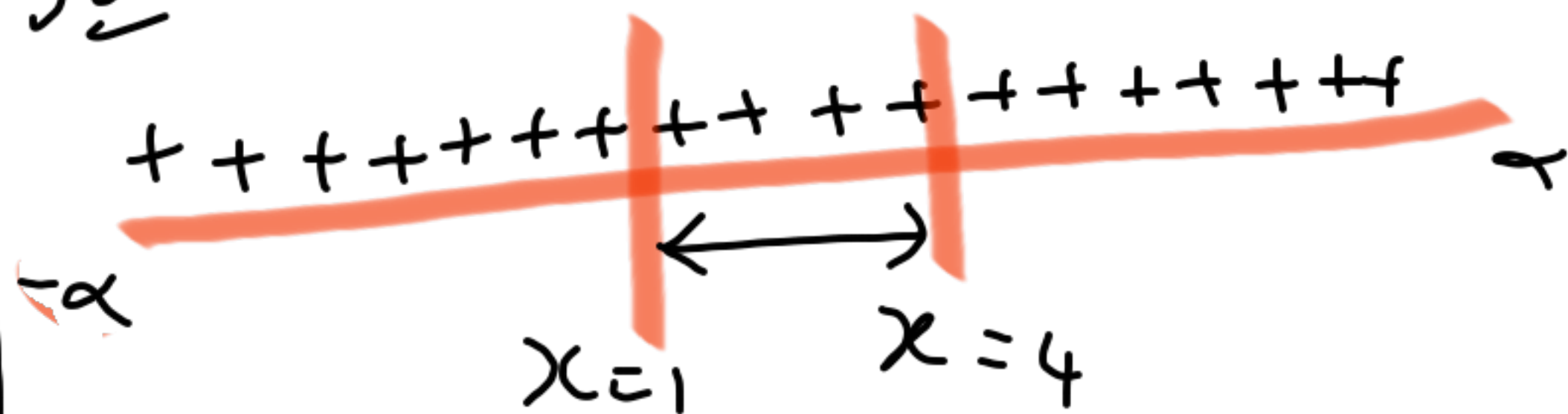
c) unit vector

$$\vec{a}_s = \frac{\vec{S}}{|\vec{S}|} = \frac{-1\vec{a}_x + 6\vec{a}_y + 11\vec{a}_z}{12.57}$$

$$\vec{a}_s = -0.08\vec{a}_x + 0.48\vec{a}_y + 0.88\vec{a}_z$$

7) An infinitely long charge is distributed along the x -axis with a linear charge density $\rho_L = 2x^2 \text{ C/m}$. Calculate the total charge Q distributed along the segment of the line from $x = 1\text{m}$ to $x = 4\text{m}$.

Soln.



$$Q = \int_L p_L \cdot dl$$

given. $p_L = 2x^2$

$$Q = \int_1^4 2x^2 dx$$

$$Q = 2 \int_1^4 x^2 dx$$

$$Q = 2 \left[\frac{x^3}{3} \right]_1^4$$

$$\left[\int x^n dx = \frac{x^{n+1}}{n+1} \right]$$

$$Q = 2 \left[\frac{4^3}{3} - \frac{1^3}{3} \right]$$

$$= 2 \left[\frac{64}{3} - \frac{1}{3} \right]$$

$$= 2 \left[\frac{64-1}{3} \right]$$

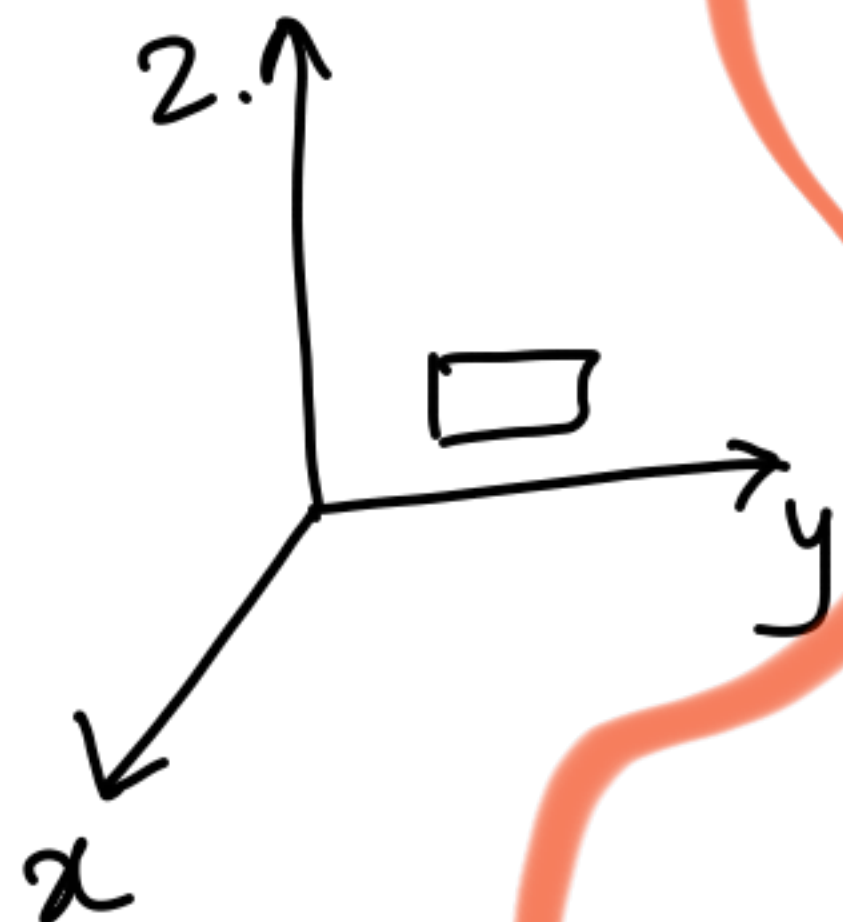
$$Q = 42$$

8) A volume charge density is expressed as $\rho_v = 10z^2 x \sin \pi y$. Find the total charge inside the volume $(-1 \leq x \leq 2)$ $(0 \leq y \leq 1)$ $(3 \leq z \leq 3.6)$

Sol. Given $\rho_v = 10z^2 x \sin \pi y$

$$Q = \int_V \rho_v \cdot dv$$

$$Q = \int_{-1}^2 \int_0^1 \int_3^{3.6} 10z^2 x \sin \pi y \, dx \, dy \, dz$$



$$Q = 10 \int_{-1}^2 x \, dx \int_0^1 \sin \pi y \, dy \int_3^{3.6} z^2 \, dz$$

$$Q = 10 \left[\frac{x^2}{2} \right]_{-1}^2 \left[-\frac{\cos \pi y}{\pi} \right]_0^1 \left[\frac{z^3}{3} \right]_3^{3.6}$$

$$Q = 10 \left[\left[\frac{2^2}{2} - \frac{(-1)^2}{2} \right] \left[-\frac{\cos \pi}{\pi} + \frac{\cos 0}{\pi} \right] \left[\frac{3.6^3}{3} - \frac{3^3}{3} \right] \right]$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$\int \sin ny dy = -\frac{\cos ny}{n}$$

$$Q = 10 \left[\left[\frac{4}{2} - \frac{1}{2} \right] \left[-\frac{(-1)}{\pi} + \frac{1}{\pi} \right] \left[\frac{46.6}{3} - \frac{27}{3} \right] \right]$$

$$Q = 10 \left[\left[\frac{3}{2} \right] \left[\frac{2}{\pi} \right] \left[\frac{19.6}{3} \right] \right]$$

$$Q = 62.4 \text{ C}$$