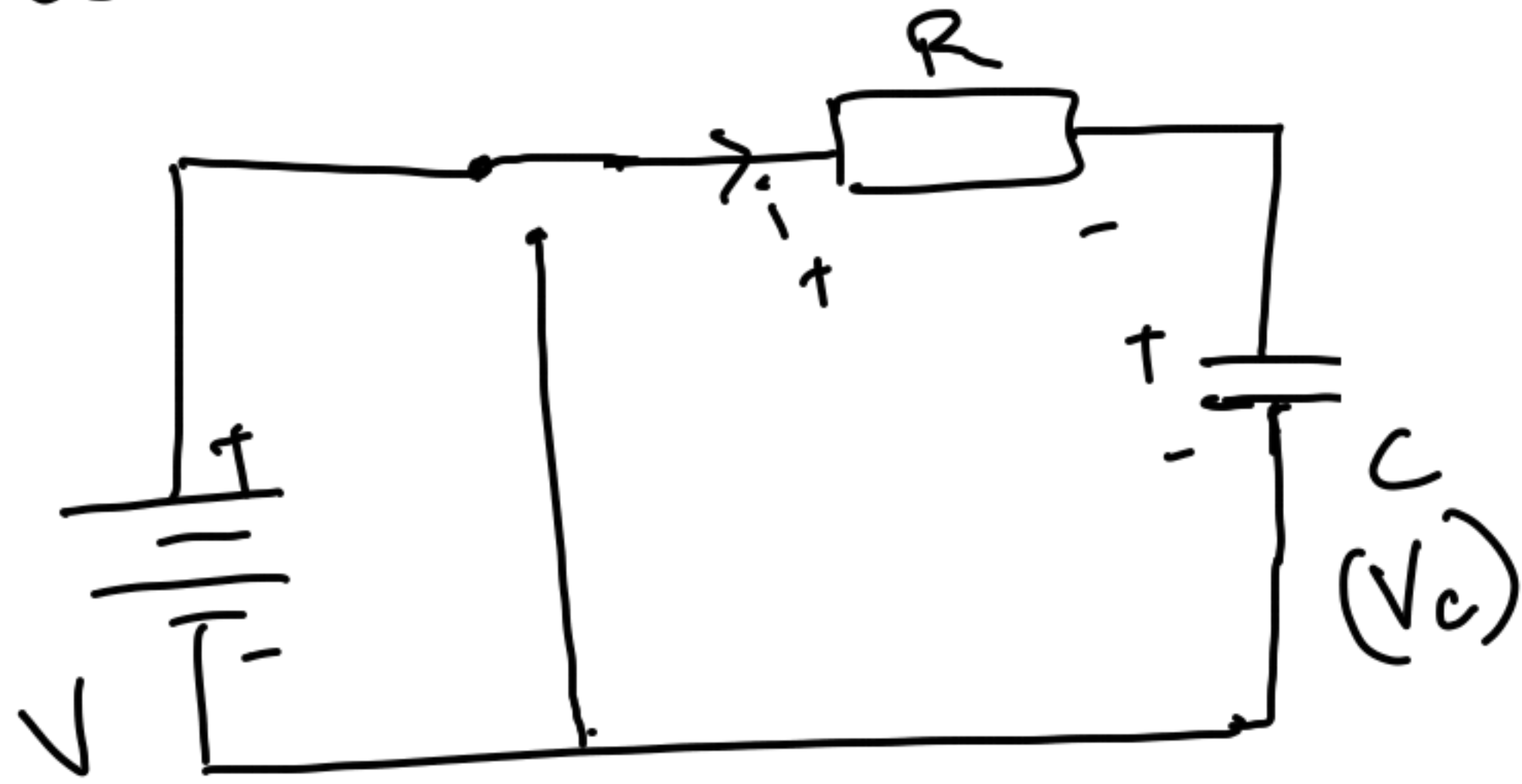


RC transient circuit.



Apply KVL,

$$-iR - V_C + V = 0 \quad \text{--- (1)}$$

Voltage across the capacitor.

$$V_C = \frac{q}{C}$$

$$q = C V_C$$

Diff. with respect 't'.

$$\frac{dq}{dt} = C \frac{dV_C}{dt}$$

$$i = C \frac{dV_C}{dt}$$

Sub. 'i' in eq (1)

$$\text{(1)} \Rightarrow -C \frac{dV_C}{dt} \cdot R - \underline{V_C} + V = 0$$

$$V - V_c = CR \frac{dV_c}{dt}$$

$$\frac{dt}{CR} = \frac{dV_c}{V - V_c}$$

Sub, $dV_c = -du$, $u = V - V_c$

$$\frac{dt}{CR} = \frac{-du}{u}$$

integrating on both the side.

$$\int \frac{dt}{CR} = - \int \frac{du}{u}$$

$$u = V - V_c$$

Diff. both the side

$$du = -dV_c$$

$$dV_c = -du$$

⊗ by ⊖ both the side

$$-\frac{1}{CR} \int dt = \int \frac{du}{u}$$

for t , $t = 0$, to $t = t$

for V_c , $V_c = 0$, to $V_c = V_c$

$$-\frac{1}{CR} \int_0^t dt = \int_0^{V_c} \frac{du}{u}$$

$$-\frac{1}{CR} [t]_0^t = \left[\ln(u) \right]_0^{V_c}$$

Sub. $u = V - V_c$

$$\frac{-1}{CR} [t - 0] = \left[\ln(V - V_c) \right]_0^{V_c}$$

$$\frac{-t}{CR} = \ln(V - V_c) - \ln(V)$$

$$\frac{-t}{CR} = \ln\left(\frac{V - V_c}{V}\right)$$

taking exponential on both the side

$$e^{-t/CR} = \frac{V - V_c}{V}$$

$$V \cdot e^{-t/RC} = V - V_c$$

$$V_c = V - V e^{-t/RC}$$

$$V_c = V(1 - e^{-t/RC})$$

$$\text{time constant} = \tau = RC$$

$$V_c = V(1 - e^{-t/\tau}) \rightarrow$$

Substitute V_c in eq ①

$$\begin{aligned} \text{①} \Rightarrow -iR - V_c + V &= 0 \\ -iR - V(1 - e^{-t/\tau}) + V &= 0 \end{aligned}$$

$$iR = V - V(1 - e^{-t/\tau})$$

$$iR = \cancel{V} - \cancel{V} + V e^{-t/\tau}$$

$$i = \frac{V}{R} e^{-t/\tau}$$

$$i = I_0 e^{-t/\tau}$$

$$V_C = V(1 - e^{-t/\tau})$$

$$w = t = \tau$$

$$V_C = V(1 - e^{-1})$$

$$V_C = V(1 - 0.368)$$

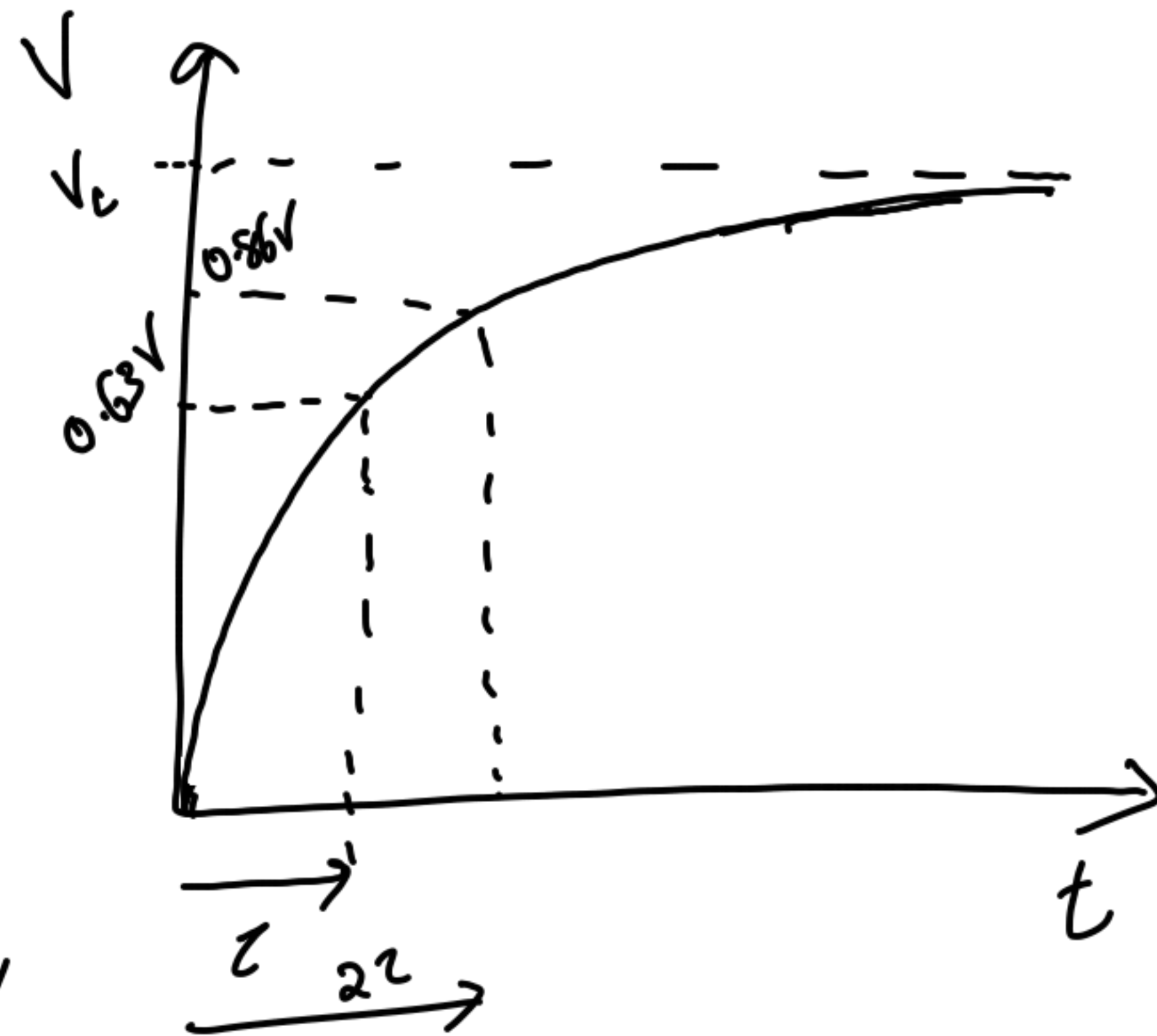
$$V_C = 0.632 V$$

$$\text{When } t = 2\tau : 0.865 V$$

$$t = 3\tau : 0.95 V$$

$$t = 4\tau : 0.98 V$$

$$t = 5\tau : 0.993 V$$



$$i = \hat{I}_0 e^{-t/\tau}$$

when $t = \tau = 0.368 \hat{I}_0$

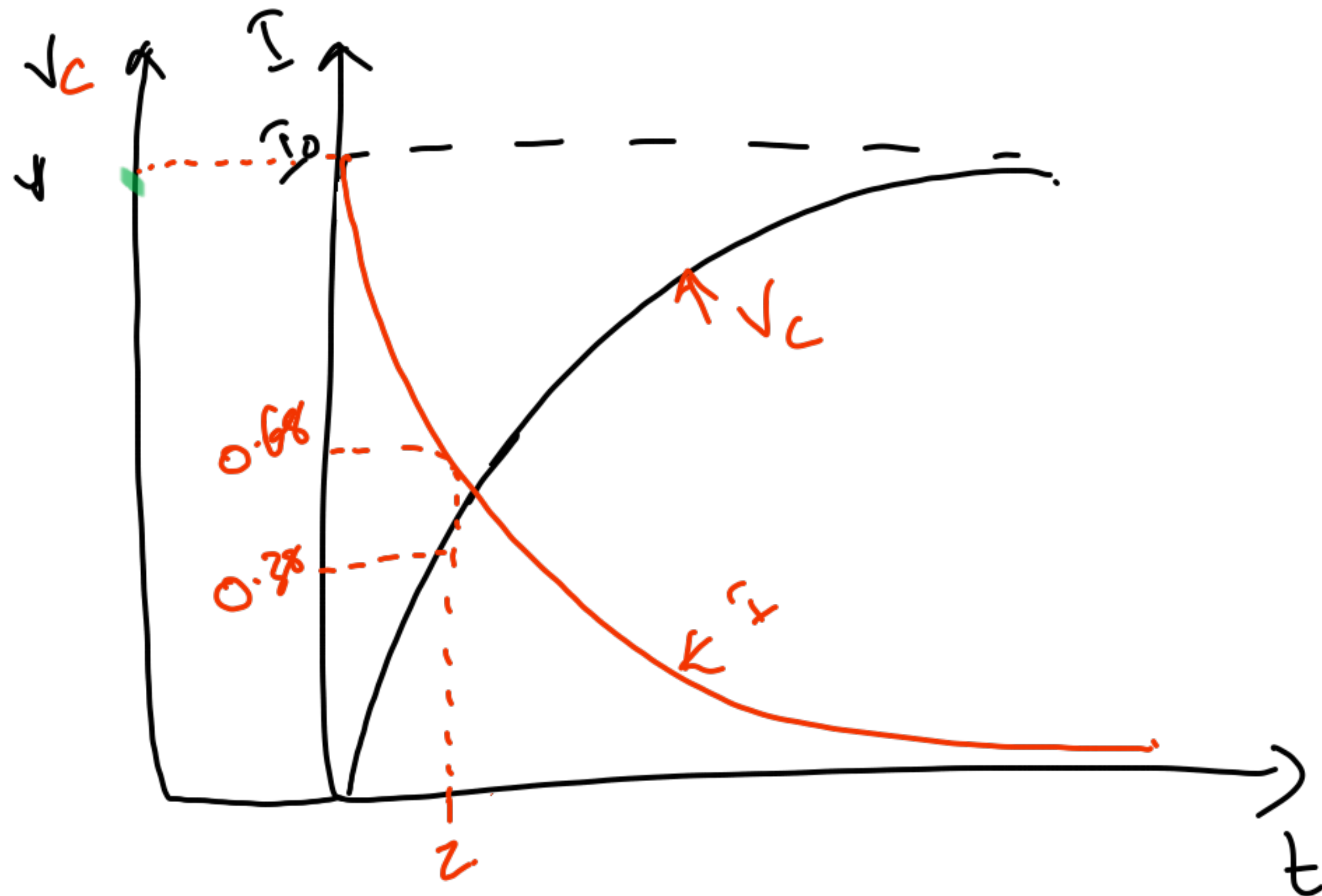
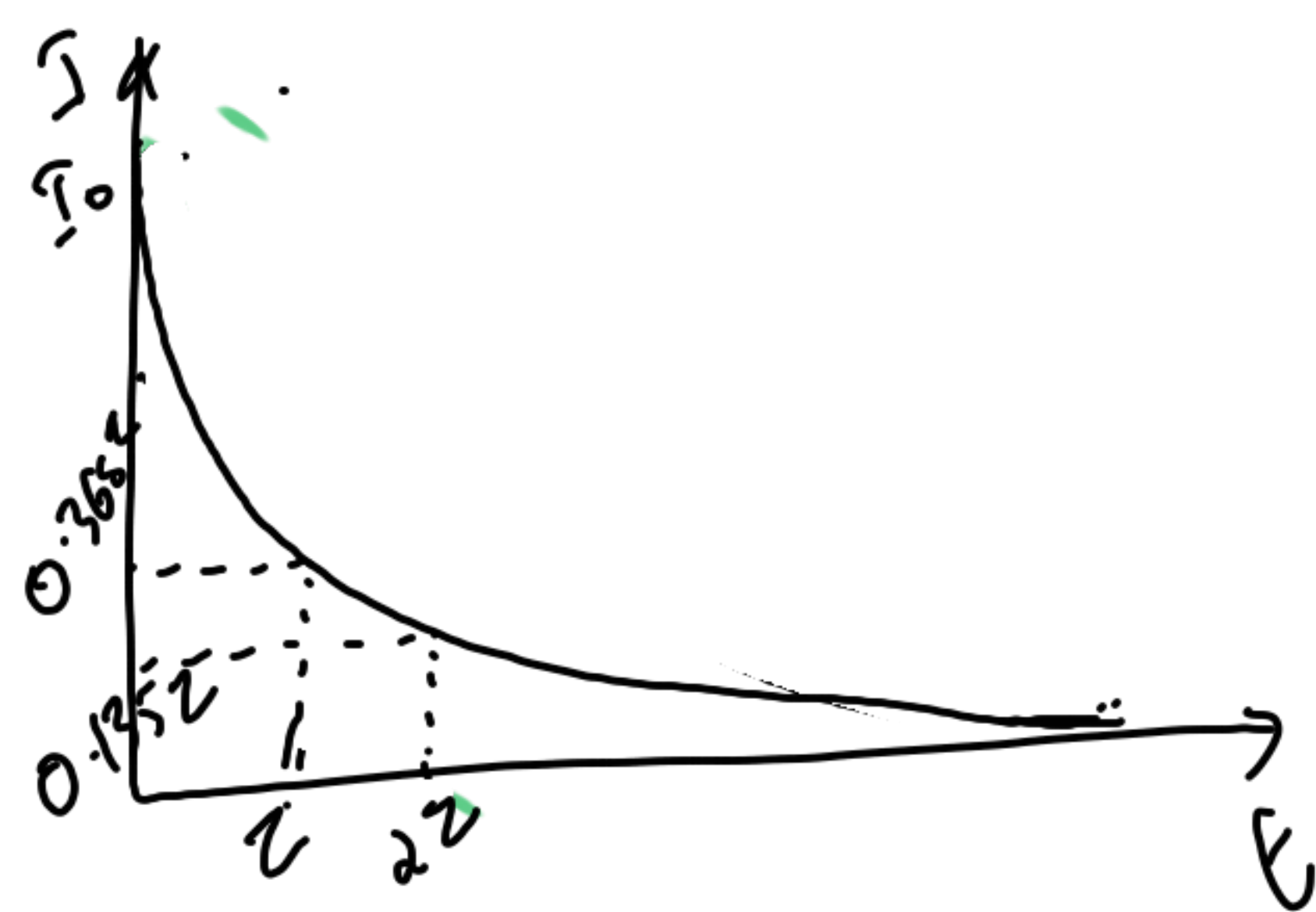
$t = 2\tau = 0.135 \hat{I}_0$

$t = 3\tau = 0.05 \hat{I}_0$

$t = 4\tau = 0.018 \hat{I}_0$

$t = 5\tau = 0.0067 \hat{I}_0$

$t = 6\tau = 0.0025 \hat{I}_0$



i) A $20\mu\text{F}$ Capacitor is connected in series with a $50\text{k}\Omega$ resistor and the circuit is connected to a 20V DC supply. Determine.

a) initial value of current flowing

b) the time constant of the circuit

c) the value of the current one second after connection.

d) Value of the capacitor voltage two seconds after connection

e) time after connection when the resistor voltage is 15V .

$$a) I_0 = \frac{V}{R} = \frac{20}{50\text{k}}$$

$$I_0 = 0.4\text{mA}$$

$$b) = \tau = RC$$
$$= 50 \times 10^3 \times 20 \times 10^{-6}$$

$$\tau = \underline{\underline{1\text{sec}}}$$

c) To find $\hat{I}$, when $t = 1 \text{ sec}$

$$\begin{aligned}\hat{I} &= \hat{I}_0 e^{-t/\tau} \\ &= 0.4 \times 10^{-3} e^{-1/1}\end{aligned}$$

$$\hat{I} = 0.147 \text{ mA}$$

d) To find V_C , when $t = 2 \text{ sec}$

$$V_C = V(1 - e^{-t/\tau})$$

$$V_C = 20(1 - e^{-2/1})$$

$$V_C = 20(1 - e^{-2}) = 17.3 \text{ V}$$

e) Resistor Voltage $V_R = 15 \text{ V}$, $t = ?$

$$V_R = V e^{-t/\tau}$$

$$15 = 20 e^{-t/1}$$

$$\frac{15}{20} = e^{-t}$$

$$0.75 = e^{-t}$$

taking natural log on both the sides

$$\ln(0.75) = -t$$

$$-0.29 = -t$$

$$t = \underline{\underline{0.29 \text{ sec}}}$$

2) A capacitor of value $1\mu\text{F}$ and Resistor of $5.47\text{M}\Omega$ are connected in series across a 220V power supply through a switch. Calculate the time by which the capacitor will be charged to 60% of the supply voltage.

Soln:
 $C = 1\mu\text{F}$, $R = 5.47\text{M}\Omega$.
 $V = 220\text{V}$.

$$\tau = RC = 5.47 \times 10^6 \times 1 \times 10^{-6}$$
$$\tau = 5.47\text{S}$$
$$V_C = V(1 - e^{-t/\tau})$$

60% of supply voltage 220V is 132V .

$$V_C = \frac{220}{100} \times 60 = 132\text{V}$$
$$132 = 220(1 - e^{-t/5.47})$$
$$\frac{132}{220} = 1 - e^{-t/5.47}$$

$$0.6 = 1 - e^{-t/5.47}$$

$$e^{-t/5.47} = 1 - 0.6$$

$$e^{-t/5.47} = 0.4$$

taking natural log on both the Side.

$$\frac{-t}{5.47} = \ln(0.4)$$

$$\frac{-t}{5.47} = -0.91$$

$$t = 0.91 \times 5.47$$

$$t = \underline{\underline{5.03 \text{ s}}}$$

3) A Capacitor of Value $1.5 \mu\text{F}$ is connected to a $2\text{k}\Omega$ resistor in series via a Switch to the 20V power supply. Calculate the time taken by the Capacitor to reach $V_C = 5\text{V}$.

Soln.

$$C = 1.5 \mu\text{F}$$

$$R = 2\text{k}\Omega$$

$$V = 20\text{V}.$$

$$V_C = 5\text{V}.$$

$$t = \underline{\underline{0.86\text{ms}}}$$

$$V_c = V(1 - e^{-t/\tau})$$

$$5 = 20(1 - e^{-t/\tau})$$

$$\tau = 1.5 \times 10^{-6} \times 20000$$

$$\tau = 0.003 \text{ s}$$

$$\frac{5}{20} = 1 - e^{-t/0.003}$$

$$0.25 = 1 - e^{-t/0.003}$$

$$e^{-t/0.003} = 1 - 0.25$$

$$e^{-t/0.003} = 0.75$$

Taking natural log. on both the side

$$\frac{-t}{0.003} = \ln(0.75)$$

$$\frac{-t}{0.003} = -0.288$$

$$t = 0.288 \times 0.003$$

$$\underline{\underline{t = 0.86 \text{ ms}}}$$