



AC Response of an RLC Network

BEng Electrical & Electronic Engineering
ACDA5001 – Electrical Engineering

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Abstract

This report investigates the effects of the resonant frequency within a parallel RLC circuit. By using the values of the resistor (R), inductor (L), capacitor (C) and the voltage (V), it is possible to calculate the frequency against the following:

- Circuit current
- The voltage drop across each component.
- The efficiency of how much of the power supply is used as intended, in respect to the change of frequency.
- The effects of the frequencies being out of phase.

These calculations indicate the efficiency of the system and can indicate where improvements can be made. If the system is out of phase, either due to power displacement or power distortion, these can be identified and mitigated through the use of power factor corrections.

The results show that the closer the frequency is to the resonant frequency, the lower the power efficiency will be. As such, upon reaching the resonant frequency, the efficiency will be at zero.

Declaration

‘This report is the result of the author’s original research. It has been composed by the author and has not been previously submitted for examination which has led to the award of a degree.’

Signed: Mr A. Hillam

Date: 08/01/2021

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1.0 Introduction

An RLC circuit includes series or parallel connection of resistors, inductors and capacitors. The circuits are used in a range of applications, where adjustments to frequencies is often required. These include mobile phones, TV sets, oscilloscopes, filtering systems. Anything that receives or transmits a signal uses an RLC. The adjustments are made by applying damping to the circuits to reduce the chance of reaching the resonant frequency. [1] [2]

1.1 Resonance

The resonance occurs once the frequency of at least two types of electrical energy such as magnetic fields for inductors or electrical fields for conductors converge due to filtration to a point of either cancelling each other out also known as a rejecter circuit, thus reaching zero, or merging to create the peak, which is known as an acceptor circuit. [1] [2]

The resonance is created by the collapsing of the magnetic field in the inductor to generate an electrical current in its windings that charge the capacitor. In addition, the capacitor discharges to create an electrical current which charges the magnetic field of the inductor. [2] [1]

Parallel resonance is when the circuit current is in phase with the voltage applied, which zero. As the circuit has a resistive nature. The power factor reaches unity of 1. [1] [2]

1.2 Power Factor Correction

Power factor by definition is the real power delivered to the load, divided by the apparent load, typically expressed as $PF = \frac{P}{S}$. As a result, the closer the V_{rms} is to the I_{rms} , the closer it will be to unity of one. The purpose of power factor correction is to keep the apparant power under the rated requirements. Without power factor correction, the system would be unreliable and potentially exceed above the rated requirements, resulting in system failure. [3] [1]

A good displacement power factor is when both sin waves of the voltage and the current are in phase as close to one as can be. A poor displacement power factor consists of the sin waves being out of phase, the further out of phase they are, the further away from one the system goes. [3] [1]

A good distortion power factor is when the peak of the current changes at the same rate of the voltage peaks. A poor distortion is when the peak / shape of the current is distorted / changes at variable rates compared to the voltage peaks. The rate at which the current is distorted determines how far the power factor is from 1. [3] [1]

1.3 Maximum Power Transfer

To obtain the maximum power from a system, the resistance of the load must be equal to the Thevenin's resistance of the system. The application for this theorem, include: the communication systems, by changing the resistance speaker to be equal to the source resistance of the amplifier, once the resistances are the same, MPT has been reached. In car engines the resistance of the motor and the internal resistance of the car battery need to be equal to reach MPT. [4] [3]

2.0 Method

The initial variables are provided either stated or provided from the student number as shown below.

Initial Variables

Student Number: 1450691

Inductance (L) = 0.145 Henrys

Resistance (R) = 06 = 6 Ohms

Capacitance (C) = 0.000091 Farads

Voltage (V) = 10V

The resonance frequency is determined by applying the values of inductance and capacitance to the following equation.

Resonance Frequency

$$f_{res} = \frac{1}{2\pi\sqrt{LC}} [1]$$

To generate a graph for the circuit current, the equations below are used to extract the range of variables.

Equations to solve for circuit current.

Inductive Reactance

$$X_L = 2\pi fL [1]$$

Capacitive Reactance

$$X_C = \frac{1}{2\pi fC} [1]$$

Impedance of Capacitor and Inductor Reactance

$$Z_{LC} = \frac{X_C * X_L}{X_C + X_L} [1]$$

Impedance of whole system

$$Z = R + j(Z_{LC}) [1]$$

Current

$$I = \frac{V+j}{Z} [1]$$

Absolute Current

$$||I|| = \sqrt{X^2 + Y^2} [5]$$

(please note: (I) values in these curved brackets refer to the separation of complex number, such as X= real number, Y=imaginary number of the impedance. This formula results in the absolute value).

The following equations were used to extract values for VR and absolute VR. The results are used to generate voltage across resistor graph.

Additional Equations to solve Vr

Voltage Across Resistor

$$Vr = R + j(I)$$

Absolute Voltage Across Resistor

$$|Vr| = \sqrt{X^2 + Y^2} [5]$$

The following equations were used to extract values for Vlc and absolute Vlc. The results are used to generate voltage across inductor and capacitor graph.

Additional Equations to solve Vlc

Voltage Across Inductor and Capacitor

$$Vlc = ZLC * (I)$$

Absolute Voltage Across Resistor

$$|VLC| = \sqrt{X^2 + Y^2} [5]$$

Equations to Solve Apparent Power

$$S = V * I$$

Equations to Solve Real Power

$$P = I^2 * R$$

Equations to Solve Reactive Power

As the apparent power (S) and real power (P) have been calculated. The following Pythagoras formula has been used to extract the reactive power (Q).

$$Q = \sqrt{S^2 - P^2}$$

Equations to Solve Power Factor

$$\text{Power Factor} = P/S$$

Additional Equations to Solve Phase Resonance

The following equations have been used to generate to determine the phase angles for I, Vr and Vlc. These are used in the following formula to generate the distance to resonance dependant on the frequency.

$$I \text{ Theta (I)} = \tan^{-1}\left(\frac{Y}{X}\right) [6]$$

$$V_r \text{ Theta (Vr)} = \tan^{-1}\left(\frac{Y}{X}\right) [6]$$

$$V_{lc} \text{ Theta (Vlc)} = \tan^{-1}\left(\frac{Y}{X}\right) [6]$$

$$VLC(x) = |VLC| * \cos(Vlc \text{ Theta} + (I \text{ Theta}) \sqrt{X^2 + Y^2})$$

$$VLC(y) = |VLC| * \sin(Vlc \text{ Theta} + (I \text{ Theta}) \sqrt{X^2 + Y^2})$$

$$V_r(x) = |V_r| * \cos(Vr \text{ Theta} + (I \text{ Theta}) \sqrt{X^2 + Y^2})$$

$$V_r(y) = |V_r| * \sin(Vr \text{ Theta} + (I \text{ Theta}) \sqrt{X^2 + Y^2})$$

$$I(x) = |I| * \cos(I \text{ Theta} + (I \text{ Theta}) \sqrt{X^2 + Y^2})$$

$$I(y) = |I| * \sin(I \text{ Theta} + (I \text{ Theta}) \sqrt{X^2 + Y^2})$$

Equations to Maximum Power Transfer

The following equation will give the total current in the system.

$$I = \frac{V}{Z_{load} + Z_{src}}$$

The following equation gives the real / active power within the system.

$$P = R[I * I * Z_{load}]$$

3.0 Results

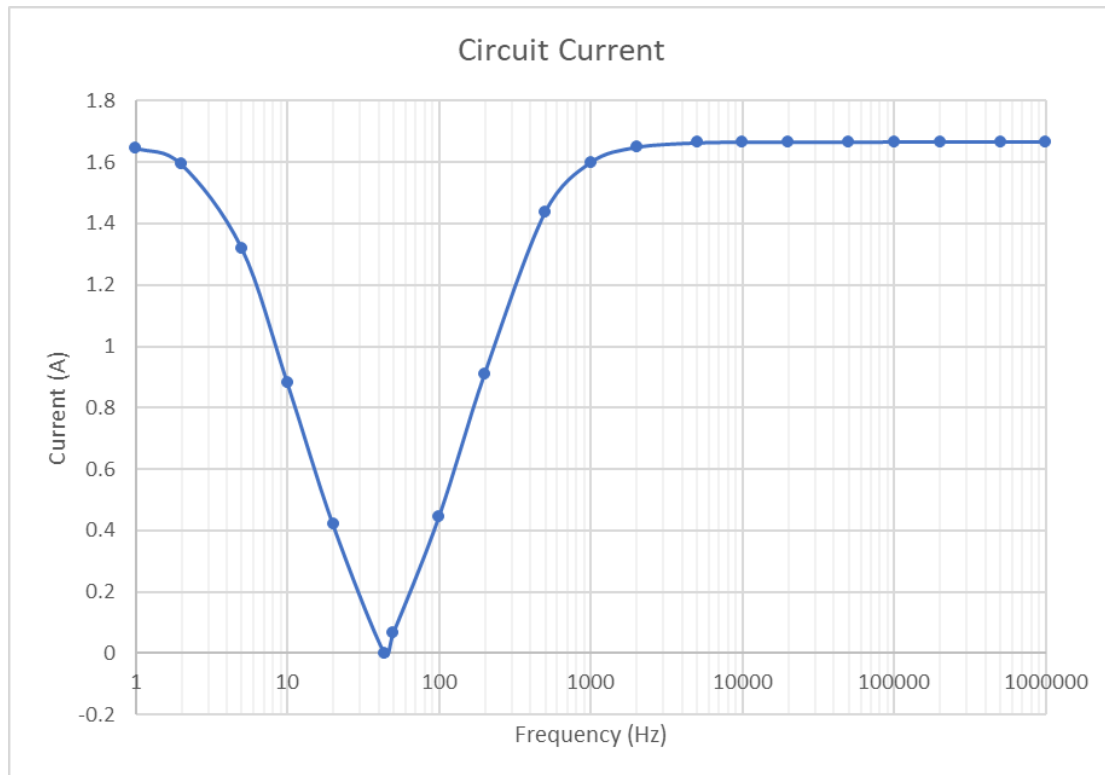


Figure 1.0 Circuit Current: Depicts how the current changes as the frequency changes. Until current reaches the resonant frequency.

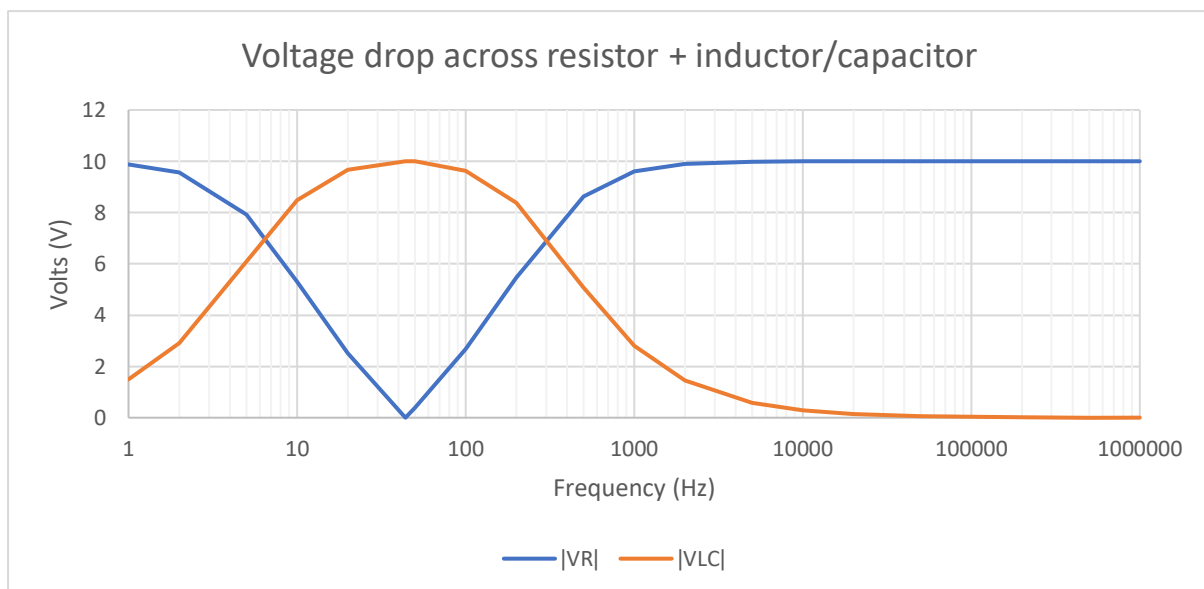


Figure 2.0 Voltage drop across resistance + inductor/capacitor. The blue line depicts how the voltage across the resistor changes as the frequency changes. The orange line depicts how the voltage across the inductor and capacitor changes as the frequency changes.

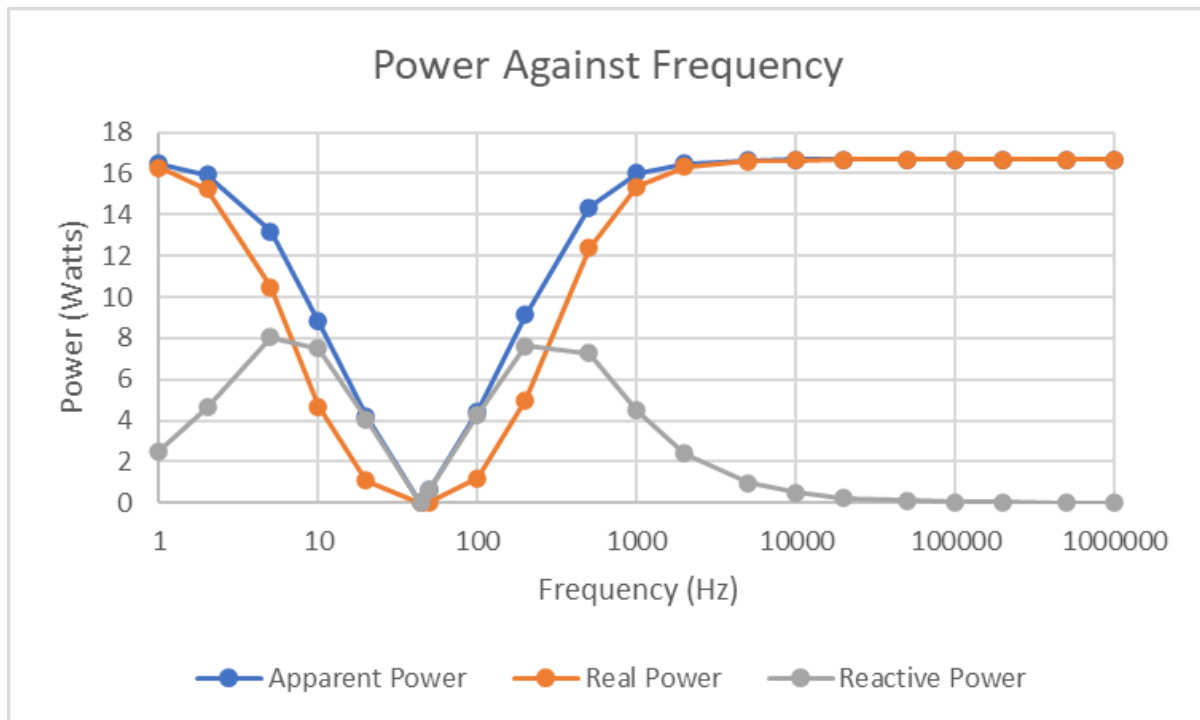


Figure 3.0 Power Against Frequency: The blue line represents the entire power range, as the frequency changes. The orange line represents the actual usable power as frequency changes. The grey line represents the residual power as the frequency changes.

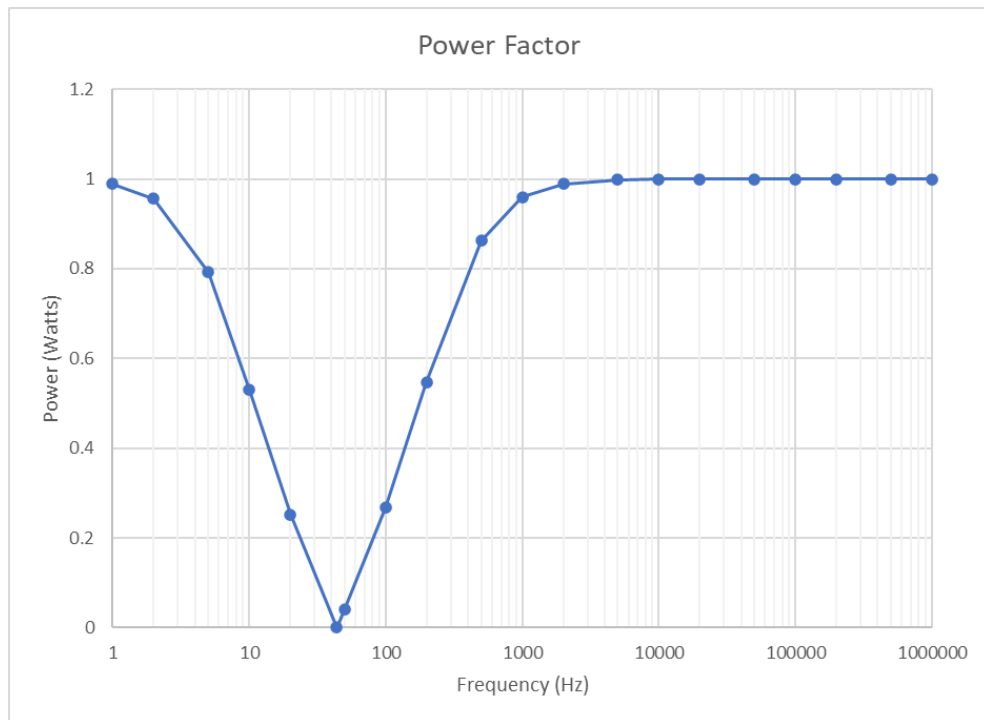


Figure 4.0 Power Factor: The blue line depicts the rate of power change, as the frequency changes.

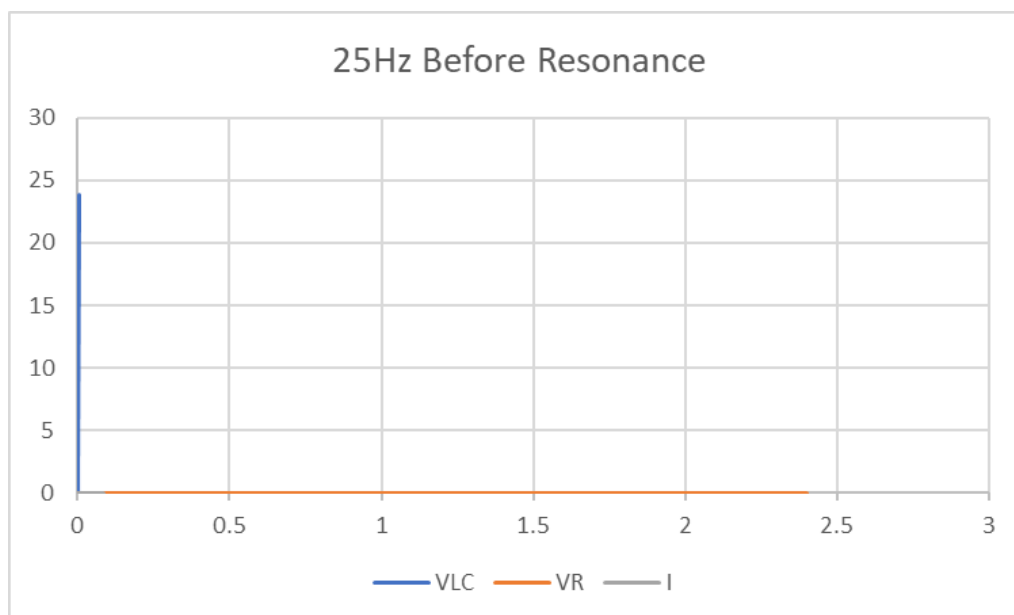


Figure 5.0 25Hz Before Resonance. This represents the phase at 25Hz before the resonance.

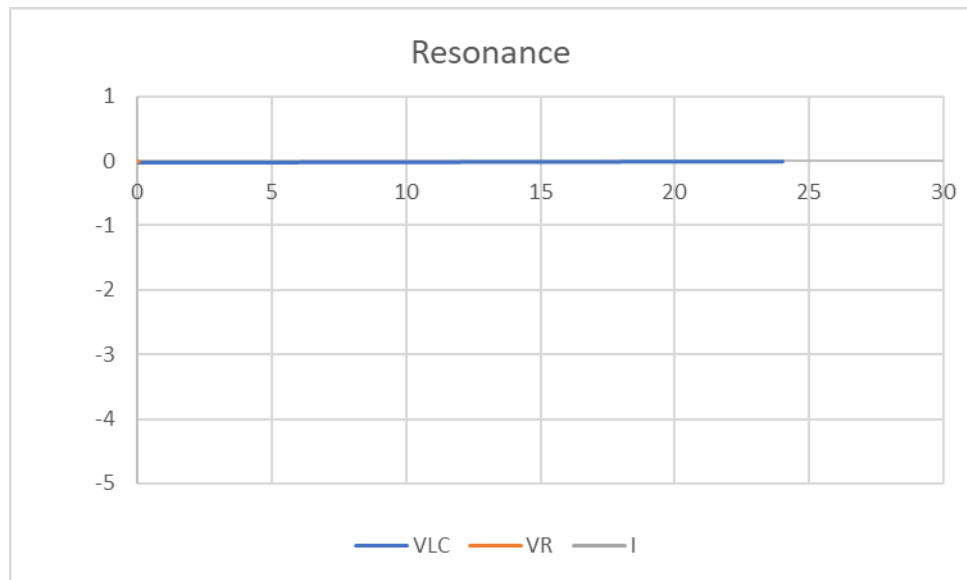


Figure 6.0 Resonance: This represents the phase during resonance.

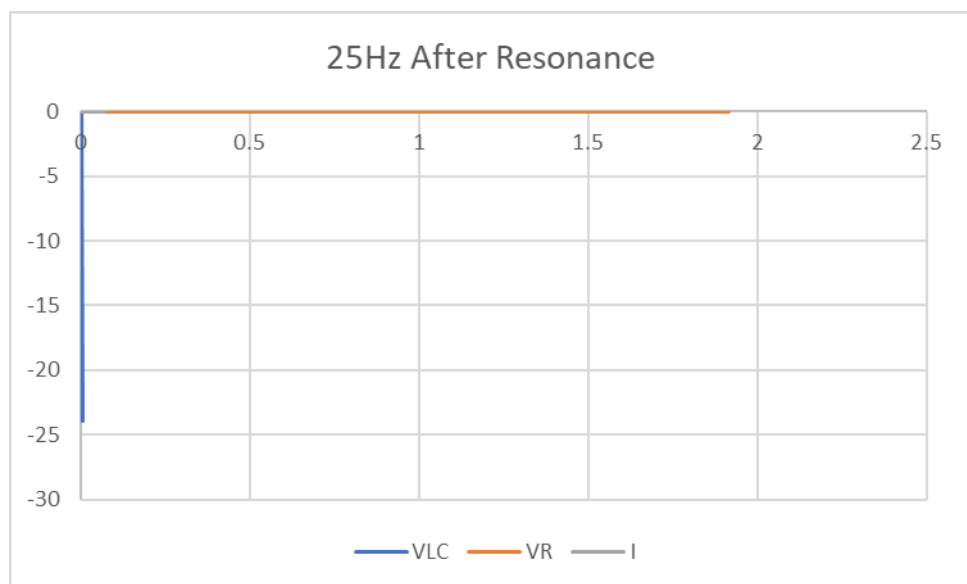


Figure 7.0 25Hz After Resonance: This represents the phase at 25Hz after resonance.

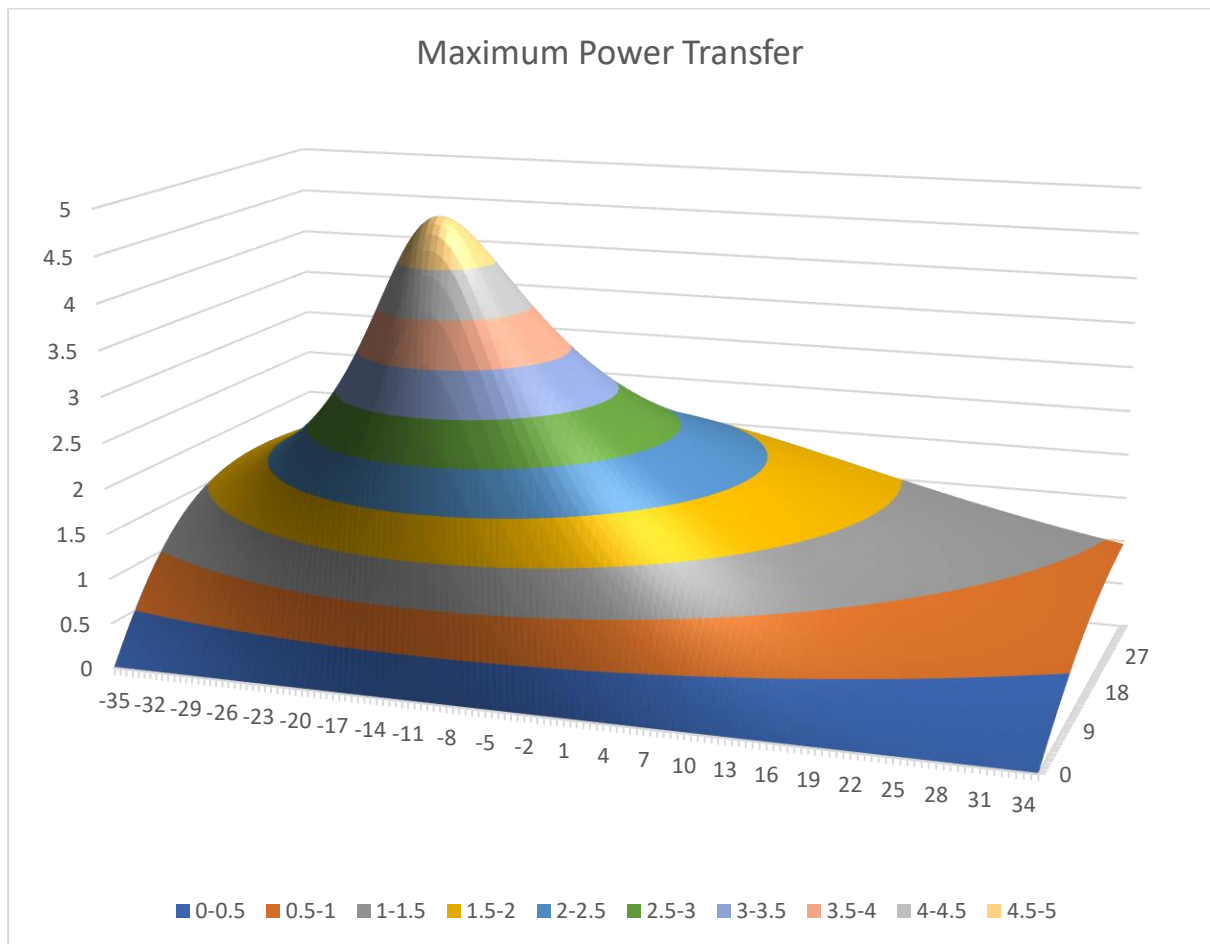


Figure 8.0 Power Transfer: This graph represents efficiency of maximum power transfer.

4.0 Discussion

The circuit current as shown in figure 1.0, shows the rate of current change as the frequency changes. As such, the closer that the current reaches the resonant frequency of 43.814, the lower the current gets. As a result of reaching resonance, the resistance is at maximum, thus placing the current at zero.

Figure 2.0 shows the voltage drops across the components resistor indicated by the blue line and the inductor and capacitor, indicated by the orange line. As such, as the frequency increases, the voltage across the resistor decreases until the resistance impedes all of the voltage. The voltage across the inductor and capacitor increases as the frequencies increases, until the full voltage is used. As a result, to the resistance and the loads, there is a resonant balance at the frequency of 43.814.

Figure 3.0 is used to identify rate of power used, as indicated in orange; and the power wasted, as indicated in grey. The graph demonstrates that, although the full power, indicated in blue; is not the full power used in the load. Again, as the resonant frequency is reached, the wattage is reduced to zero. As the frequency moves away from the resonant point the reactive power is higher than the real power used. However, as the frequency moves further away from the resonant point, the real power used increases greatly over the reactive power. As such, the reactive power can be reduced by remaining away from the resonant frequency. Thus, improving efficiency.

Figure 4.0 shows the efficiency of the power factor. Raising to one being the ideal per efficiency. However, as the frequency moves towards the resonant frequency, the efficiency starts reduce from 1 (100%), to below 1(100%).

Figure 5.0 when compared to figure 6.0 drops the frequency by 25Hz from the resonant frequency. Figure 5.0 shows how the VLC moves to the Y axis, thus showing a steady rate of changes.

Figure 7.0 when compared to figure 6.0 increases the frequency by 25Hz from the resonant frequency. Figure 7.0 shows how the VLC moves to the -Y axis, thus showing a steady rate of changes.

Figure 6.0 shows that at resonance the VLC, VR and I, are all in phase at zero.

At the point resonance, the current is at zero, thus there is no electrical flow. The energy stored in the capacitor and inductor alternate a stored charge as an electrostatic charge in the capacitor and a magnetic force field surrounding the inductor.

Figure 8 illustrates the maximum power transfer approximately at $Z_{sc}=4.2+j14\text{ohms}$.

5.0 Conclusion

Through the range of datasheets and graphs created, the following conclusions can be made:

- Resonance occurs when the voltage across all the RLC components is in a state of balance. Thus, zero power is being used, due to cancellation.
- The efficiency of the power factor can be improved by reducing the reactive power by moving the frequency away from the resonant frequency.
- During the point of resonance, there will be a stored charge within the capacitor and surrounding the inductor.
- The wider the phase angle is, the more reactive power occurs, thus reducing efficiency.
- The narrower the phase angle is, the less reactive power occurs, thus increasing power efficiency.
- If the supply has an overall capacitive internal impedance, then the current will lead the voltage at an angle.

6.0 References

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