
QUADRATIC EQUATIONS

A quadratic equation is one in which the highest power of the unknown quantity is 2.

There are multiple methods for solving quadratic equations. Here we will look at the 2 main ones; Factorisation and by using the quadratic formula.

Factorising

A quadratic equation can sometimes be factorised into two brackets in the form of $(x + a)(x + b)$ where a and b can be any term, positive, negative or zero.

Examples:

1. Factorise $x^2 + 7x + 10 = 0$

To factorise this expression, find two numbers that have a product of +10 and a sum of +7.

There are a couple of ways of making +10 by multiplying two numbers. These are 1×10 and 2×5 . Only the combination of 2 and 5 will also give a sum of +7.

$$x^2 + 7x + 10 = (x + 2)(x + 5)$$

Now split these brackets up:

$$(x + 2) = 0 \text{ and } (x + 5) = 0$$

And solve these two equations

$$x = -2 \text{ or } x = -5$$

Remember you can expand the brackets out to check you get the original equation.

2. Factorise $x^2 - 2x - 15 = 0$

This time we need two numbers that have a product of -15 and a sum of -2. There are multiple ways of making -15 by multiplying two numbers. These are -1×15 , 1×-15 , -3×5 and 3×-5 . Only the combination of 3 and -5 will also give a sum of -2.

$$x^2 - 2x - 15 = (x + 3)(x - 5)$$

Now split these brackets up:

$$(x + 3) = 0 \text{ and } (x - 5) = 0$$

And solve these two equations

$$x = -3 \text{ or } x = 5$$

3. Factorise $6x^2 - 7x - 3 = 0$

This time we need two numbers that have a product of $6 \times -3 = -18$ and a sum of -7. .

There are multiple ways of making -18 by multiplying two numbers. These are -1×18 , 1×-18 , -2×9 , 2×-9 , -3×6 and 3×-6 . Only the combination of 2 and -9 will also give a sum of -7.

Rewrite the equation with the number combination: $6x^2 + 2x - 9x - 3 = 0$

Now we split the equation down the middle and factorise each half.

$$2x(3x + 1) - 3(3x + 1)$$

Notice that the two brackets are the same. The first bracket is therefore the common bracket, and the second bracket is the factorised terms outside of each bracket.

$$(3x + 1)(2x - 3)$$

Now split these brackets up:

$$(3x + 1) = 0 \text{ and } (2x - 3) = 0$$

$$x = -\frac{1}{3} \text{ or } x = \frac{3}{2}$$

Quadratic Formula

The general form of a quadratic equation is given by: $ax^2 + bx + c = 0$, where a, b and c are constants.

The solution of this equation is given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Examples:

1. Solve the equation: $3x^2 - 11x - 4 = 0$

Comparing with the quadratic formula, $a=3$, $b=-11$ and $c=-4$.

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4 \times 3 \times -4}}{2 \times 3} = \frac{+11 \pm \sqrt{121 + 48}}{6} = \frac{11 \pm \sqrt{169}}{6} = \frac{11 \pm 13}{6}$$

$$\text{Therefore: } x = \frac{11+13}{6} \text{ or } x = \frac{11-13}{6}$$

$$x = \frac{24}{6} \text{ or } x = -\frac{2}{6}$$

$$\text{Which gives: } x = 4 \text{ or } x = -\frac{1}{3}$$

2. Solve the equation: $x^2 - 6x - 10 = 0$

Comparing with the quadratic formula, $a=1$, $b=-6$ and $c=-10$.

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 1 \times -10}}{2 \times 1} = \frac{+6 \pm \sqrt{36 + 40}}{2} = \frac{6 \pm \sqrt{76}}{2} = \frac{6 \pm 8.7178}{2}$$

$$\text{Therefore: } x = \frac{6+8.7178}{2} \text{ or } x = \frac{6-8.7178}{2}$$

$$\text{Which gives: } x = 7.3589 \text{ or } x = -1.3589$$