

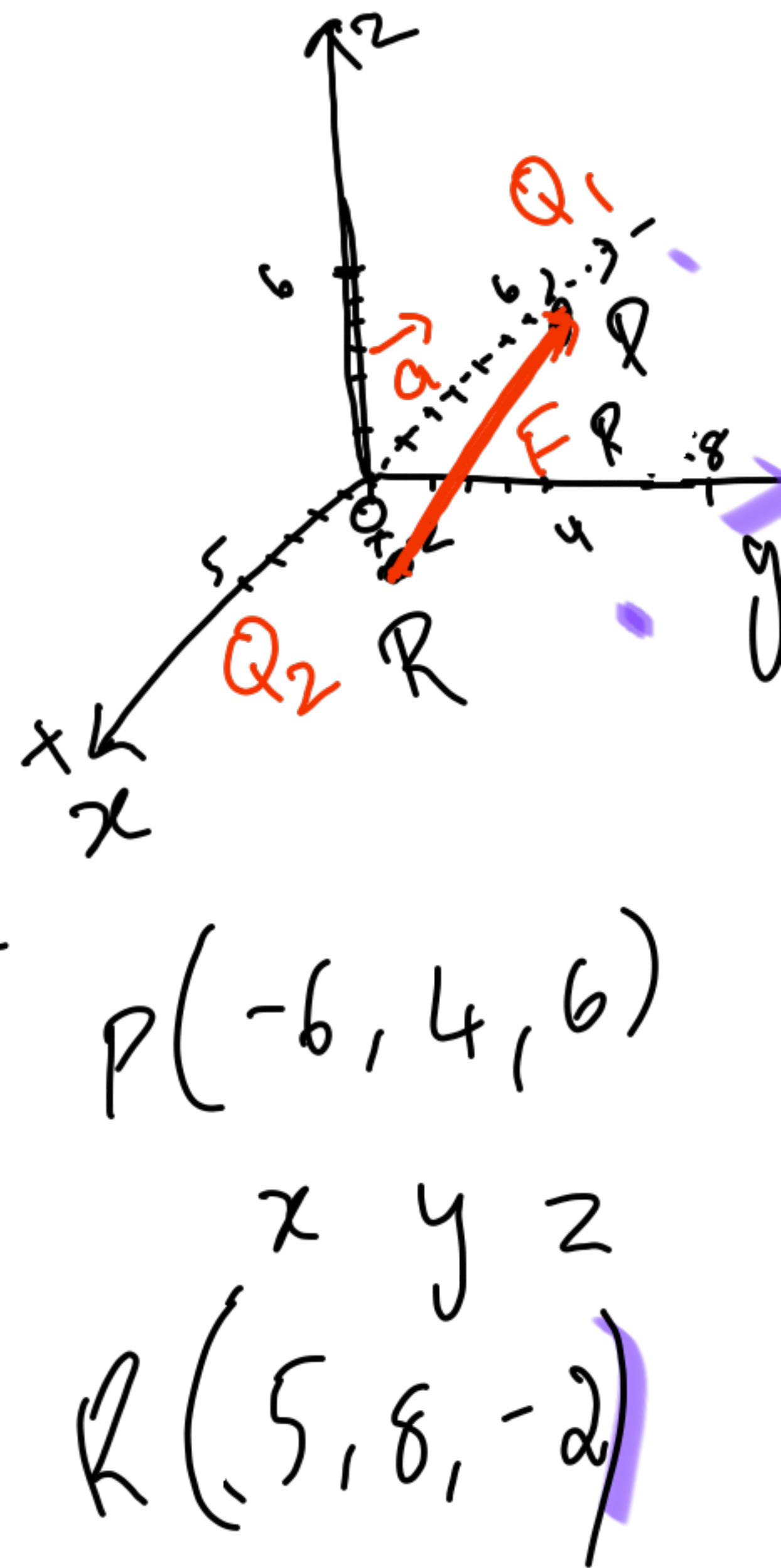
1) A charge  $Q_1 = -20\mu\text{C}$  is located at  $P(-6, 4, 6)$  and a charge  $Q_2 = 50\mu\text{C}$  is

located at  $R(5, 8, -2)$  in a free space

Find the force exerted on  $Q_2$  by  $Q_1$  in Vector form. The distance given are in metres.



$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \vec{a}_{PR}$$



$$\epsilon_0 = 8.854 \times 10^{-12}$$

$$\vec{a} = \frac{\vec{R}_{PR}}{|\vec{R}_{PR}|}$$

$$\vec{P} = -6\vec{a}_x + 4\vec{a}_y + 6\vec{a}_z$$

$$\vec{R} = 5\vec{a}_x + 8\vec{a}_y - 2\vec{a}_z$$

$$\vec{R}_{PR} \Rightarrow \vec{R} - \vec{P} = ((5 - (-6))\vec{a}_x + (8 - 4)\vec{a}_y + (-2 - 6)\vec{a}_z$$

$$\vec{R}_{PR} = 11\vec{a}_x + 4\vec{a}_y - 8\vec{a}_z$$

$$|\vec{R}_{PR}| = \sqrt{11^2 + 4^2 + (-8)^2} = 14.18$$

$$\vec{a} = \frac{11\vec{a}_x + 4\vec{a}_y - 8\vec{a}_z}{14.18}$$

$$\vec{a} = 0.78\vec{a}_x + 0.28\vec{a}_y - 0.56\vec{a}_z$$

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \vec{a}$$

$$F = \frac{-20 \times 10^{-6} \times 50 \times 10^{-6}}{4\pi \times 8.854 \times 10^{-12} \times (14.18)^2} \vec{a}$$

$$F = -0.0447 (0.78\vec{a}_x + 0.28\vec{a}_y - 0.56\vec{a}_z)$$

$$F = -0.035\vec{a}_x - 0.013\vec{a}_y + 0.025\vec{a}_z$$

To Find the magnitude of Force

$$|F| = \sqrt{(-0.035)^2 + (-0.013)^2 + (0.025)^2}$$

$$F = 44.6 \times 10^{-3} \text{ N} = 44.6 \text{ mN}$$

2) Determine the Electric field intensity vector at  $P(-0.2, 0, -2.3)\text{m}$  due to a point charge at  $+5\text{nC}$  at  $Q(0.2, 0.1, -2.5)\text{m}$  in air.

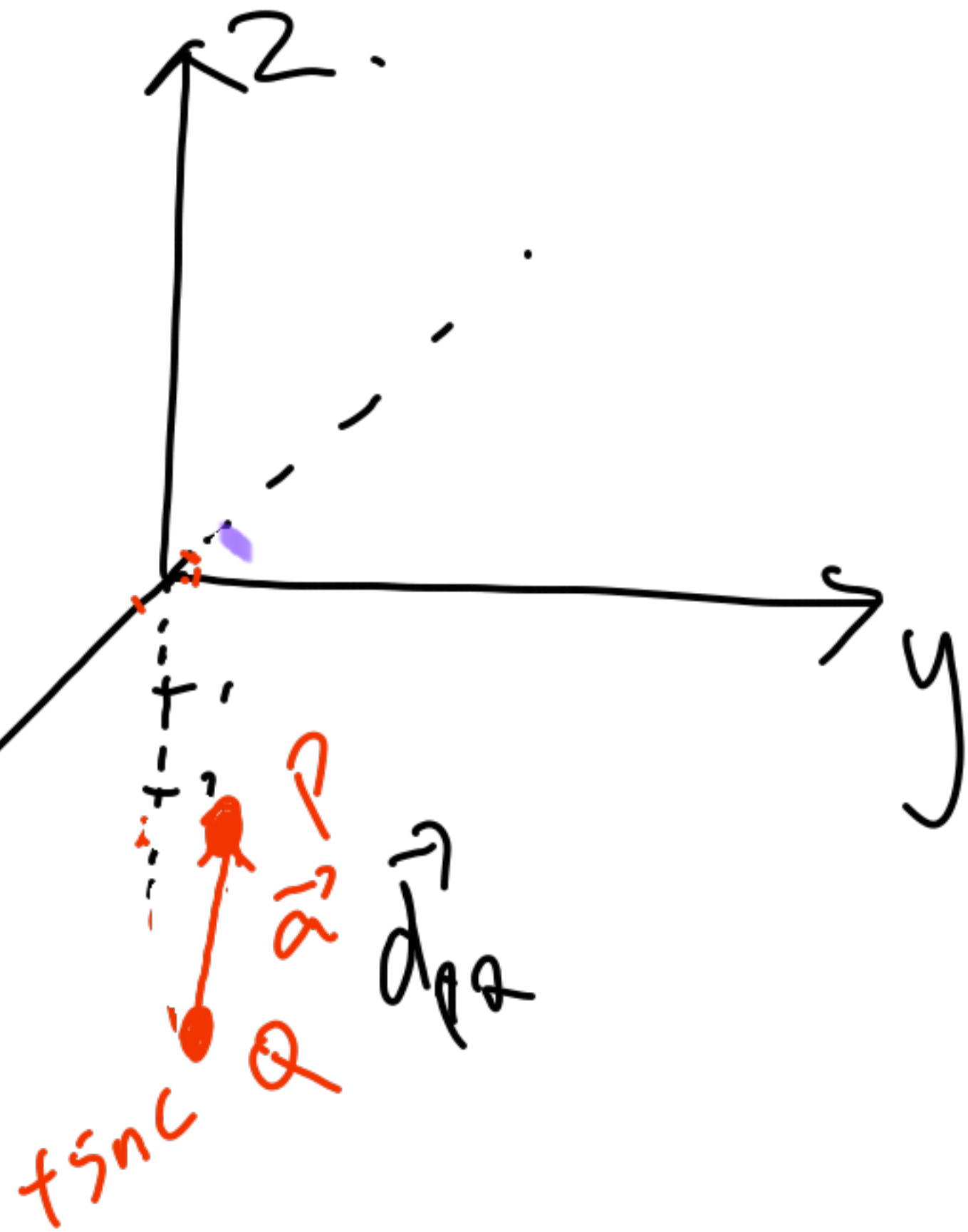
$$\vec{E} = \frac{Q}{4\pi\epsilon_0 d^2} \vec{a}$$

$$P \begin{pmatrix} -0.2 & 0 & -2.3 \end{pmatrix}$$

$x \quad y \quad z$

$$Q \begin{pmatrix} 0.2 & 0.1 & -2.5 \end{pmatrix}$$

$x \quad y \quad z$



unit vector

$$\vec{a} = \frac{\vec{d}_{PQ}}{|\vec{d}_{PQ}|}$$

$$\vec{d}_{PQ} = \vec{P} - \vec{Q}$$

$$\vec{P} = -0.2\vec{a}_x - 2.3\vec{a}_z$$

$$\vec{Q} = 0.2\vec{a}_x + 0.1\vec{a}_y - 2.5\vec{a}_z$$

$$\vec{d}_{pq} = \vec{p} - \vec{q} = (-0.2 - 0.2) \vec{a}_x + (0 - 0.1) \vec{a}_y + (-2.3 - (-2.5)) \vec{a}_z$$

$$\vec{d}_{pq} = -0.4 \vec{a}_x - 0.1 \vec{a}_y + 0.2 \vec{a}_z$$

$$|\vec{d}_{pq}| = \sqrt{(-0.4)^2 + (-0.1)^2 + (0.2)^2}$$

$$|\vec{d}_{pq}| = 0.46$$

$$\vec{a} = \frac{-0.4 \vec{a}_x - 0.1 \vec{a}_y + 0.2 \vec{a}_z}{0.46}$$

$$\vec{a} = -0.88 \vec{a}_x - 0.22 \vec{a}_y + 0.43 \vec{a}_z$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 d^2} \vec{a}$$

$$= \frac{5 \times 10^{-9}}{4\pi (8.854 \times 10^{-12}) (0.46)^2} \vec{a}$$

$$\vec{E} = 212.38 \vec{a}$$

$$\vec{E} = 212.38(-0.88 \vec{a}_x - 0.22 \vec{a}_y + 0.43 \vec{a}_z)$$

$$\vec{E} = -184.67 \vec{a}_x - 46.17 \vec{a}_y +$$

$$92.34 \vec{a}_z.$$

$$N/C.$$

$$V/m.$$

.46)<sup>2</sup>

3) A Volume charge density is expressed as  $\rho_v = 10z^2 x \sin \pi y$ .

Find the total charge inside

the volume  $(-1 \leq x \leq 2)$

$(0 \leq y \leq 1), (3 \leq z \leq 3.6)$

Soln:

To find total charge

$$\rho_v = 10z^2 x \sin \pi y$$

$$Q = \int_V \rho_v \, dv$$

$$Q = \int \int \int \rho_v \, dx \, dy \, dz$$

$$Q = \int_{x=-1}^2 \int_{y=0}^1 \int_{z=3}^{3.6} 10z^2 x \sin \pi y \, dx \, dy \, dz$$

$$Q = 10 \left[ \int_{x=-1}^2 x \, dx \int_{y=0}^1 \sin \pi y \, dy \int_{z=3}^{3.6} z^2 \, dz \right]$$

$$Q = 10 \left[ \left[ \frac{x^2}{2} \right]_{-1}^2 \left[ \frac{-\cos \pi y}{\pi} \right]_0^1 \left[ \frac{z^3}{3} \right]_3^{3.6} \right]$$

$$Q = 10 \left[ \left[ \frac{2^2}{2} - \frac{(-1)^2}{2} \right] \left[ \frac{-\cos \pi(1)}{\pi} - \frac{(-\cos \pi(0))}{\pi} \right] \left[ \frac{(3.6)^3}{3} - \frac{(3)^3}{3} \right] \right]$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

$$\int \sin \pi y \, dy = \left[ \frac{-\cos \pi y}{\pi} \right]$$

$$\int \sin y \, dy = -\cos y$$

$$= 10 \left[ 2 - \frac{1}{2} \right] \left[ -\frac{\cos \pi}{\pi} + \frac{\cos 0}{\pi} \right] \left[ \frac{46.66}{3} - \frac{27}{3} \right]$$

$$= 10 [1.5] \left[ -\frac{(-1)}{\pi} + \frac{1}{\pi} \right] [6.55]$$

$$= 62.57 \text{ C}$$

# Exam

Mark Marks = 100

Duration = 2 hrs.

Questions- 4

Answer All

Each Question- 25 marks

$$4 \times 25 = 100$$

Questions, [25 marks]

a) - - [5 mark]

b) - - - [10 marks]

c) - - [2 marks]

d) - - - [8 marks]

