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EXAMPLE 1:

Problem stated as 2-D, hence $\sigma_2 = 0$.

(a) TRESCA failure criterion is;

$$\sigma_1 - \sigma_3 = \sigma_y$$

and, Factor of Safety, $n = \frac{\text{Yield Stress}}{\text{Allowable working stress}}$

$$\therefore \sigma_1 - \sigma_3 = \frac{\sigma_y}{n} \quad \left\{ \begin{array}{l} \sigma_1 - \sigma_3 \text{ is allowable} \\ \text{working stress.} \end{array} \right\}$$

So, $n = \frac{\sigma_y}{\sigma_1 - \sigma_3}$ where, $\sigma_1 = 140 \text{ MN/m}^2$ tensile
 $\sigma_3 = 45 \text{ MN/m}^2$ compressive.

$$= \frac{240 \times 10^6}{(140 \times 10^6) - (-45 \times 10^6)}$$

$$\therefore n = \underline{1.297}$$

(b) VON-MISES failure criterion (including safety factor);

$$2 \left(\frac{\sigma_y}{n} \right)^2 = [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

$$\text{So, } 2 \left(\frac{240}{n} \right)^2 = [(140 - 0)^2 + (0 + 45)^2 + (-185)^2]$$

$$\frac{115,200}{n^2} = 55,850$$

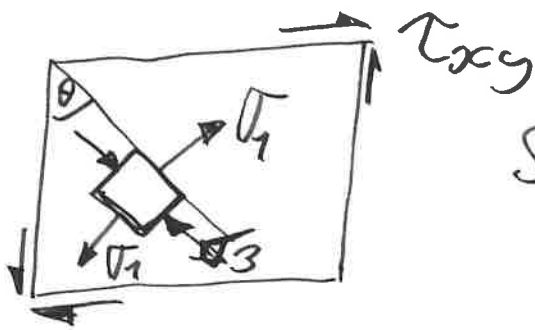
$$\therefore n = \sqrt{\frac{115,200}{55,850}} = \underline{1.436}$$

EXAMPLE 2:

FROM COMPLEX STRESS FOR AN ELEMENT
SUBJECTED TO SHEAR STRESS ONLY, i.e.
WHEN TORQUE IS TRANSMITTED;

$$\sigma_{\theta} = \tau_{xy} \cdot \sin 2\theta \quad \text{where maximum principal stress will occur when } \theta = 45^{\circ}$$

$$\text{Therefore, } \sigma_{1,3} = \pm \tau_{xy} \sin 90^{\circ}$$



$$\text{So, } \sigma_1 = \tau_{xy}$$

$$\sigma_3 = -\tau_{xy}$$

NOTE: Assuming 2D stress plane $\therefore \sigma_2 = 0$
and σ_x & σ_y are zero because shear.

FROM SIMPLE THEORY OF ELASTIC TORSION;

$$\frac{T}{J} = \frac{\tau}{R} = \left(\frac{G\theta}{L} \right) \therefore \tau = \frac{T \cdot R}{J}$$

MEAN DIAMETER OF STEEL TUBE IS 100mm;

Hence, Outside diameter, $D = 103\text{mm}$

Inside diameter, $d = 97\text{mm}$

Now, polar moment of inertia, J ;

$$J = \frac{\pi}{32} \cdot (D^4 - d^4) = 2.358 \times 10^{-6} \text{ m}^4$$

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$$\text{Now, } \tau = \frac{T.R}{J} = \frac{T \times (51.5 \times 10^{-3})}{2.358 \times 10^{-6}}$$

$$\therefore \tau = 21,840.54 T$$

This relationship will allow determination of transmitted torque once τ is found.

(a) Maximum shear stress theory - TRESCA.

Given; Factor of Safety, $n = 2.25$

Elastic limit, $\sigma_y = 225 \text{ MN/m}^2$

$$\text{Now, } \sigma_1 - \sigma_3 = \frac{\sigma_y}{n}$$

$$\therefore \tau_{xy} - (-\tau_{xy}) = \frac{225 \times 10^6}{2.25}$$

$$\text{So, } \tau_{xy} = 50 \times 10^6 \text{ N/m}^2$$

$$\text{Knowing } \tau = 21,840.54 T$$

$$\therefore T = \frac{50 \times 10^6}{21,840.54}$$
$$= 2,289.3 \text{ Nm}$$

Hence, transmitted torque as per Tresca theory is 2.29 kNm.

(b). Maximum strain energy per unit volume - HAIGH. ③
Failure criterion is;

$$\sigma_y^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\lambda(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)$$

$$\text{Now, } \left(\frac{\sigma_y}{n}\right)^2 = \tau_{xy}^2 + 0 + (-\tau_{xy})^2 - [2 \times 0.3(0 + 0 + (\tau_x - \tau))]^2$$

$$\text{So, } 1 \times 10^{16} = 2.6 \tau^2$$

$$1 \times 10^{16} = 2.6(21,840.54)^2$$

$$\therefore T = 2,839.6 \text{ Nm} \quad (T = 2.84 \text{ kNm})$$

(c). Maximum shear strain energy - Von Mises.
Failure criterion is;

$$2\sigma_y^2 = [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

$$2(1 \times 10^{16}) = [(\tau_{xy} - 0)^2 + (0 - (-\tau_{xy}))^2 + (-\tau_{xy} - \tau_{xy})^2]$$

$$\text{So, } 2 \times 10^{16} = 6 \tau_{xy}^2 = 6(21,840.54 T)^2$$

$$\therefore T = \sqrt{\frac{2 \times 10^{16}}{6 \times 21,840.54^2}}$$

$$= 2,643.5 \text{ Nm} \quad (T = 2.64 \text{ kNm})$$

EXAMPLE 3:

FROM THE ENGINEER THEORY OF BENDING
(OR THE EULER-BERNOULLI EQUATION);

$$\frac{\sigma}{y} = \frac{M}{I} = \left(\frac{E}{R}\right) \quad \therefore \sigma = \frac{M \cdot y}{I}$$

For a circular cross-section;

$$I = \frac{\pi d^4}{64}$$

$$\therefore \sigma = \frac{64 M \cdot y}{\pi d^4} \quad \text{where, } y \text{ will be radius of the shaft.}$$

$$= \frac{64 \times 1.9 \times 10^3 \times 25 \times 10^{-3}}{\pi \times (50 \times 10^{-3})^4}$$

$$\text{So, } \sigma = 154.83 \times 10^6 \text{ N/m}^2 \text{ (bending stress)}$$

FROM SIMPLE THEORY OF ELASTIC TORSION (See Ex 3)

$$\tau = \frac{T \cdot R}{J} \quad \text{where, } J = \frac{\pi d^4}{32}$$

$$\therefore \tau = \frac{32 \cdot T \cdot R}{\pi d^4} = 40,743.7 \cdot T$$

(a) Maximum shear stress theory - TRESCA.

$$\sigma_1 - \sigma_3 = \sigma_y.$$

FROM COMPLEX STRESS THEORY (Level 5 study)

$$\sigma_1 = \frac{1}{2} \left[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4 \tau_{xy}^2} \right]$$

$$\text{Let } (\sigma_x + \sigma_y) = a$$

$$\text{and, } \sqrt{(\sigma_x - \sigma_y)^2 + 4 \tau_{xy}^2} = b.$$

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$$\text{So, } \sigma_1 = \frac{1}{2}(a+b) = \frac{1}{2}a + \frac{1}{2}b.$$

$$\begin{aligned} \text{And, } \sigma_3 &= \frac{1}{2} \left[(\sigma_x + \sigma_y) - \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \right] \\ &= \frac{1}{2}(a-b) = \frac{1}{2}a - \frac{1}{2}b. \end{aligned}$$

From Tresca failure criterion;

$$\sigma_1 - \sigma_3 = \frac{1}{2}a + \frac{1}{2}b - \frac{1}{2}a + \frac{1}{2}b = b$$

$$\therefore \sigma_1 - \sigma_3 = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\text{So, } \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} = \sigma_{\text{yield}}$$

where, $\sigma_y = 0$ (2-D stress system).

$$\therefore \sigma_{\text{yield}}^2 = \sigma_x^2 + 4\tau_{xy}^2$$

$$\tau_{xy} = \sqrt{\frac{\sigma_{\text{yield}}^2 - \sigma_x^2}{4}}$$

where, σ_x is the bending stress determined earlier (154.83 MN/m^2)

$$\text{So, } \tau_{xy} = 63.3 \times 10^6 \text{ N/m}^2$$

$$\text{But, } \tau_{xy} = 40,743.7 \cdot T$$

$$\therefore T = \frac{63.3 \times 10^6}{40,743.7} = \underline{\underline{1553.62 \text{ Nm}}}$$

EXAMPLE 4:

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FROM THIN-WALLED CYLINDER THEORY:

There are 3 mutually perpendicular wall stresses;

$$\text{Hoop stress} = \frac{Pd}{2t} = \sigma_1 \text{ (MAXIMUM).}$$

$$\text{Longitudinal stress} = \frac{Pd}{4t} = \sigma_2 \text{ (MIDDLE).}$$

$$\text{Radial stress} = 0 = \sigma_3 \text{ (MINIMUM).}$$

(a) Consider TRESCA theory;

$$\sigma_1 - \sigma_3 = \frac{\sigma_y}{n} \Rightarrow \frac{Pd}{2t} = \frac{\sigma_y}{2}$$

$$\therefore t = \frac{Pd}{\sigma_y} = \frac{2.5 \times 10^6 \times 2}{400 \times 10^6} = 0.0125 \text{ m} \\ (12.5 \text{ mm})$$

(b) Consider VON MISES theory;

$$[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] = 2 \cdot \sigma_y^2$$

If $\sigma_3 = 0$, and Factor of Safety, $n = 2$;

$$[(\sigma_1 - \sigma_2)^2 + \sigma_2^2 + \sigma_1^2] = 2 \cdot \left(\frac{\sigma_y}{n}\right)^2$$

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$$\left(\frac{Pd}{2t} - \frac{Pd}{4t}\right)^2 + \left(\frac{Pd}{4t}\right)^2 + \left(\frac{Pd}{2t}\right)^2 = 2 \cdot \frac{\sigma_y^2}{4}$$

$$\left(\frac{Pd}{2t} - \frac{Pd}{4t}\right)\left(\frac{Pd}{2t} - \frac{Pd}{4t}\right) + \frac{P^2 d^2}{16t^2} + \frac{P^2 d^2}{4t^2} = \frac{\sigma_y^2}{2}$$

$$\frac{P^2 d^2}{4t^2} - \frac{P^2 d^2}{8t^2} - \frac{P^2 d^2}{8t^2} + \frac{P^2 d^2}{16t^2} + \frac{P^2 d^2}{16t^2} + \frac{P^2 d^2}{4t^2} = \frac{\sigma_y^2}{2}$$

$$\frac{P^2 d^2}{t^2} \left(\frac{1}{4} - \frac{1}{8} - \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{4} \right) = \frac{\sigma_y^2}{2}$$

$$\therefore \frac{3P^2 d^2}{8t^2} = \frac{\sigma_y^2}{2}$$

$$\frac{3P^2 d^2}{4t^2} = \sigma_y^2$$

$$\text{So, } t = \sqrt{\frac{3P^2 d^2}{4 \cdot \sigma_y^2}} = \sqrt{\frac{3 \cdot (2.5 \times 10^6)^2 \times 2^2}{4 \times (400 \times 10^6)^2}}$$

$$\therefore t = 0.01083 \text{ m} \\ (10.83 \text{ mm}).$$