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Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

1	Student Number	
2	Title of Examination Paper	TCA 2
3	Module Code	PCEE4000/PCB 4000
4	Date and Time of Examination	17/5/23 4.00
5	Examination Venue	
6	Seat Number	
7	Number of Answer Books handed in	

Complete, fold and seal
(see note 2 below)

SURNAME: KING

FORENAME(S): KIERAN

List the numbers of the questions you have attempted	Office Use
1	11
2	6
3	5
4	9
5	14
6	12
7	14
8	8
Total	79 ✓

Second Marker
50/100
18/05/23

1.
a. $y = 5e^{2x} \cos x$

Product rule

$$\begin{aligned} \frac{dy}{dx} &= 5e^{2x} \times -\sin(x) + \cos x \times 10e^{2x} \\ &= 5e^{2x} - \sin(x) + 10e^{2x} \cos(x) \\ &= 5e^{2x} (1 - \sin(x) + 2 \cos(x)) \end{aligned}$$

③ ✓

b. $y = \frac{2-3x}{x+1}$

Quotient rule

$$\begin{aligned} \frac{dy}{dx} &= \frac{(x+1) \times (-3) - (2-3x) \times 1}{(x+1)^2} \\ &= \frac{-3x-3-2+3x}{(x+1)^2} = \frac{-5}{(x+1)^2} \end{aligned}$$

① ✓

$$\frac{3x+3-2-3x}{(x+1)^2} = \frac{1}{(x+1)^2}$$

c. $y = 5 \sin(x^3)$

Function of function rule

$$= 5 \sin(x^3) \times (3x^2)$$

$$\frac{dy}{dx} = 3x^2 \times 5 \cos(x)$$

$$\begin{aligned} 15x^2 \cos(x) \\ = 15x^2 \cos(x^3) \end{aligned}$$

⑥ ✓

d. $y = \frac{2}{x^3} + \sqrt{x-x}$

$$\begin{aligned} 3x^2^{-3-1} + \frac{1}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{\frac{1}{2}} \\ = \frac{6}{x^4} + \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x}} \end{aligned}$$

① ✓

$$= \frac{6}{x^4} + \frac{1}{2\sqrt{x-x}}$$

→ DP

2. $V = 12(1 - 7e^{-3t})$

$$\begin{aligned} & 12 \times 1 - 12 \times 7e^{-3t} \\ &= 12 - 84e^{-3t} \\ &= -3 \times -84e^{-3t} \\ &= 252e^{-3t} \quad t=0 \end{aligned}$$

initial rate of growth of voltage
 $= 252 \text{ V/s}$

3. $y = 2x^3 - 15x^2 - 84x + 16$

a. $6x^2 - 30x - 84 \quad \checkmark \quad 6x^2 - 30x - 84$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(6)(-84)}}{2(6)} = \frac{30 \pm 84}{12}$$

$$x = 7 \quad \checkmark \quad x = -2 \quad \checkmark$$

$$\begin{aligned} y &\approx 2(7)^3 - 15(7)^2 - 84(7) + 16 = -621 \\ y &= 2(-2)^3 - 15(-2)^2 - 84(-2) + 16 = 108 \quad \checkmark \end{aligned}$$

b $\frac{d^2y}{dx^2} = 6x^2 - 30x - 84$
 $= 6x - 30$

$$x = 7 \quad 6(7) - 30 = 12 \quad \text{minimum} \quad \times$$

$$x = -2 \quad 6(-2) - 30 = -42 \quad \text{maximum} \quad \times$$

(5) $\checkmark$

4. a. $\int_1^4 (1 + 3x + 5x^2) dx = \int_1^4 (1x^0 + 3x^1 + 5x^2)$

$$= \frac{1x^{0+1}}{0+1} + \frac{3x^{1+1}}{1+1} + \frac{5x^{2+1}}{2+1} = \frac{1x^1}{1} + \frac{3x^2}{2} + \frac{5x^3}{3}$$

$$= \left[x + \frac{3x^2}{2} + \frac{5x^3}{3} \right]_1^4$$

At $4 = x = 4 + \frac{3(4)^2}{2} + \frac{5(4)^3}{3} = \frac{164}{3}$ or 54.66

At $1 = x = 1 + \frac{3(1)^2}{2} + \frac{5(1)^3}{3} = \frac{25}{6}$ or 4.16

$$\frac{164}{3} - \frac{25}{6} = \frac{101}{2} = 50.50$$

(41)

b. $\int_0^{\frac{\pi}{3}} (5 \cos \frac{x}{3} - 2 \sin x) dx = \left[5 \cos \frac{x}{3} - 2 \sin x \right]_0^{\frac{\pi}{3}}$

$$= \left[\frac{5}{3} \sin \frac{x}{3} - -\frac{2}{1} \cos(x) \right]_0^{\frac{\pi}{3}}$$

$$= \left[15 \sin \frac{x}{3} + 2 \cos(x) \right]_0^{\frac{\pi}{3}}$$

At $\frac{\pi}{2} = 15 \sin \frac{\pi}{3} + 2 \cos\left(\frac{\pi}{2}\right) = \frac{15+2\sqrt{3}}{2}$ or 9.23

At $0 = 15 \sin\left(\frac{0}{3}\right) + 2 \cos\left(\frac{0}{2}\right) = 2$

$$\frac{15+2\sqrt{3}}{2} - 2 = 7.23$$

(41)

$$S. \quad \frac{19x-5}{2x(x-1)} \quad dx = \frac{A+B}{2x(x-1)} = \frac{x-1+2x}{2x(x-1)}$$

$$19x-5 = A(x-1) + 2xB \quad \checkmark$$

$$x=1 \quad 19(1)-5 = A(1-1) + 2(1)B$$

$$= 14 = 2B$$

$$B = \frac{14}{2} = 7 \quad \checkmark$$

$$x=0 \quad 19(0)-5 = A(0-1) + 2(0)B$$

$$-5 = -1A$$

$$A = \frac{-5}{-1} = 5 \quad \checkmark$$

$$\frac{SA+7B}{2x(x-1)} = \int_3^5 \frac{19x-5}{2x(x-1)} dx$$

$$\frac{1}{a} \ln(ax+b) + c$$

$$5x \times \frac{1}{2}(2x) + 7x \times \frac{1}{1}(x-1)$$

$$= \left[5x \times \frac{1}{2} \ln(2x) + 7x \times \frac{1}{1} \ln(x-1) \right]_3^5 \quad \checkmark$$

$$At \ 5 = 5x \times \frac{1}{2} \ln(2x5) + 7x \times \frac{1}{1} \ln(5-1) = 15.4605 \quad \checkmark$$

$$A+3 = 5 \times \frac{1}{2} \ln(2 \times 3) + 7x \times \frac{1}{1} \ln(3-1) = 9.3314 \quad \checkmark$$

$$15.4605 - 9.3314 = 6.1291 \text{ units}^2 \quad \checkmark$$

14

- 5
6. $\frac{1920-5}{3} \times 20(x-1)$ Simpson's rule using 4 strips
4.d.p

3	3.5	4	4.5	5
$\frac{13}{3}$	$\frac{123}{35}$	$\frac{71}{24}$	$\frac{23}{9}$	$\frac{9}{4}$
y_0	y_1	y_2	y_3	y_4

$$\frac{1}{3} h (y_0 + y_4 + 4(y_1 + y_3) + 2 \times (y_2))$$

$$\frac{1}{3} \times 0.5 \left(\frac{13}{3} + \frac{9}{4} + 4 \left(\frac{123}{35} + \frac{23}{9} \right) + 2 \times \left(\frac{71}{24} \right) \right) = 6.1299 \checkmark$$

$$\text{Percentage error} = \frac{6.1299 - 6.1291}{6.1291} \times 100$$

$$= 0.0131\% \text{ error} \checkmark$$

12

7.

Time	0 < 2	2 < 4	4 < 6	6 < 8	8 < 10	Σ
Freq(f)	1	16	26	5	2	50
(x) mid p	1	3	5	7	9	
$f \times x$	1	48	130	35	18	232
$(x - \bar{x})^2$	$\frac{8281}{625}$	$\frac{1681}{625}$	$\frac{81}{625}$	$\frac{3481}{625}$	$\frac{11881}{625}$	
$f(x - \bar{x})^2$	$\frac{8281}{625}$	$\frac{26896}{625}$	$\frac{2106}{625}$	$\frac{3481}{125}$	$\frac{23762}{625}$	125.52

$$\bar{x} = \frac{\Sigma f x}{\Sigma f} = \frac{232}{50} = 4.64 \checkmark$$

$$\sigma = \sqrt{\frac{\Sigma f(x - \bar{x})^2}{\Sigma f - 1}} = \sqrt{\frac{125.52}{50 - 1}} = 1.60 \checkmark$$

14

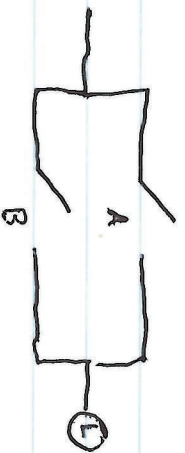
8. A works is 0.97 B works is 0.89

a series



connection = $0.97(A) \times 0.89(B) = 0.86$ ✓
 Probability that there is a connection across both switches.

b,



in parallel

Chance there is a connection = $0.03 \times 0.11 = \frac{33}{10000}$ ✓
 $1 - \frac{33}{10000} = 0.997$ ✓ chance there will be a connection in parallel

(8)

ADP